THE TRANSITION BETWEEN MATHEMATICS STUDIES AT SECONDARY AND TERTIARY LEVELS; INDIVIDUAL AND SOCIAL PERSPECTIVES

Erika Stadler

Växjö University

The aim of this paper is to illustrate how an empirical research interest in the transition between mathematics studies at secondary and tertiary levels generates a need for different theoretical approaches. From interviews with teacher students before and during their university studies in mathematics, three crucial aspects of the transition have been discerned; Mathematical learning objects, Mathematical resources *and* Students as active learners. *Whereas the two former have both individual and social dimensions, the latter can be regarded as relational, constituting a link between the learning environment and the student in his or her intention to learn mathematics.*

Keywords: teacher students, transition, individual, social, grounded theory

INTRODUCTION

My ongoing research project examines the transition between mathematics studies at secondary and tertiary levels, from now on termed “the transition”. This research interest stems from novice university students experiences with increased difficulties and changes in the conditions of mathematics studies at university, compared to upper secondary school. When novice university students begin their studies at university, they learn mathematics in a new learning environment. From a student’s perspective, this situation presents new challenges in terms of, or changes in, their knowledge, skills and self-image. Dynamic processes are going on, whereby students and their learning environment are mutually influencing each other. There are no obvious theories or methods at hand for dealing with this complex and extensive research area. Consequently, this study exemplifies the question raised by Arzarello, Bosch, Lenfant and Prediger of “how empirical studies contribute to the development and evolution of theories” (2007, p. 1620). Thus, an important part of the study has been to develop an analytical framework for the transition as seen from a student’s perspective. In this paper, I will give an account of the theoretical considerations this empirical problem brings to the fore.

TRANSITION-RELATED RESEARCH

Learning mathematics at university level is a well examined area. Many studies have focused on students’ learning and understanding of specific topics within university mathematics, for example limits of functions, derivatives, linear algebra and group theory (Dorier, 2000; Juter, 2006; Nardi, 2000). Other studies have considered how students struggle with advanced mathematical thinking, and with changes in the subject itself, including transformations from concrete and intuitive to more abstract, formal and general forms of mathematics (Tall, 1991) or demands for new ways of approaching the mathematical content (Lithner 2003, Schoenfeld, 1992). A common characteristic of these studies is an approach that focuses primarily on the students as individual learners. From a more situated and cultural perspective, the issue of transition between different contexts of mathematical practices has been carefully examined by de Abreu, Bishop and Presmeg (2002). They define transitions as individuals' experiences of movements between contexts of mathematical practices. The transition as seen from a student's perspective can be captured by studying students' actions and interactions in a learning situation, looking for traces of conflict between different learning cultures, or variations of meaning that students ascribe to phenomena in the learning situation.

Artigue, Batanero and Kent (2007) suggest that research in learning mathematics at the post-secondary level must go beyond notions of for instance advanced mathematical thinking and also involve more comprehensive perspectives on mathematical thinking and learning. In their article, they refer to Praslon, who states that the transition cannot be defined as a shift from school mathematics to formal mathematics, or from an intuitive approach to mathematics to a more rigorous one. Instead, the transition is rather a question of an accumulation of small changes in mathematical culture. It is a shift from studying specific mathematical objects towards an extraction of mathematical objects from more general conditions. It is a change from applying specific algorithms to a category of tasks towards general methods and techniques. According to Praslon this is a consequence of the increment of the mathematical content to be learnt, and the impossibility of learning a specific algorithm for every kind of task in a relatively short period of time.

By gathering many research studies from different areas with different perspectives, it is possible to grasp a more complete picture of the transition. This has been done in a recently published study by Gueudet (2008), who states that the transition involves individual, social and institutional phenomena that call for different theoretical approaches. From my brief overview of transition related research it can be concluded that research concerning the transition has been conducted both from individual (von Glasersfeld, 1995), situated (Wenger, 1998), and cultural perspectives (Säljö, 2000). To examine the transition from a student's perspective, where the transition is defined as learning in a new environment in light of previous experiences is to simultaneously consider individual and social perspective on learning. Thus, the challenge is to combine an individual a social perspective on a local level within one empirical study.

From a more general point of view, this issue refers to the discussion of whether individual and social perspectives on learning can be unified. Cobb and Yackel made an important contribution to this debate with their *Emergent perspective* (1996). Their notions of sociomathematical norms and mathematical beliefs and values coordinate an individual and a social perspective on the collaboration between the teacher and the students in classroom environments. The strong emphasis on interaction in the classroom can be regarded as a strength of this perspective. However, the transition from a student's perspective is not limited to the classroom. Instead, an essential part of the study must concern individual previous experiences of learning mathematics, requiring one to base findings on interviews. Thus, there is a mismatch between the methodological implications of Cobb and Yackel's Emergent perspective and the requirements of the research design of my study. My study requires a theoretical perspective that considers both an individual and a social perspective on the transition but from a methodological point of view, it requires more variety of data sources. Consequently, I was without a suitable theoretical framework and a pre-defined set of methods to follow to gather data and empirical considerations based on my definition of the transition had to serve as a starting point for the choice of research methods instead.

RE-ARRANGEMENT OF THE METHODOLOGICAL SEQUENCE

From a more general point of view this question also refers to a future challenge, raised during the Cerme 6 conference in Lyon, France, namely the discussion of how to find methodologies for networking theories, where the link between theory, empirical data and research results should be more highlighted. Methodological considerations link theoretical perspectives with appropriate research methods. Often, the formulation and intention of a research question is formulated within a theoretical discourse that results in a specific theoretical perspective. Thus, the research process, frequently used in mathematics education can schematically be described as follows:

Question → **T**heory → **M**ethod → **R**esult

Or alternatively:

Theory → **Q**uestion → **M**ethod → **R**esult

Here, theory may refers to a more comprehensive theoretical perspective, for example a social or situated perspective, but may also refer to a more local theoretical framework as the Emergent perspective. The point is that often decisions about method seem to follow almost automatically once the initial choices of research question and/or theoretical perspective have been described. My research approach has been somewhat different. The starting point for my study has been a real world situation, from which the aim and the definition of the transition were developed. Because the definition of the transition - the students' learning of mathematics in a new setting in the light of their previous experiences requires the study to combine an individual and a social theoretical perspective, there has not been a given choice of methodological approach. Instead, my intention to study the transition from a student's perspective has been used as a methodical starting point, whereby the results contribute to new theoretical approaches and relations.

This approach can be summarised as follows:

Aim → **M**ethod → **R**esult → **T**heory

With this rearrangement of the methodological sequence I want to emphasise how a real world problem implies a research process that ends up with a theoretical description of this phenomena. These descriptions have a local and specific character. However, based on their construction, they contain theoretical elements of both individual and social character. Thus, by studying them, conclusions can be drawn about how different theoretical perspectives come into play on a more general level. In accordance with my definition of the transition three main parts can be discerned, namely the students' previous experiences with mathematics studies, their learning of mathematics at university level, and the university as a new learning environment. To cover these parts empirically, I have collected different kinds of qualitative data from five teacher students during their first mathematics courses at university, i.e. individual interviews, observations from lectures and tutorials and written solutions to exercises and examinations. In this paper, I present some extracts from interviews with two of the students, Cindy and Roy. The pre-interviews were carried out after the students had enrolled at the university but before they had begun take courses in mathematics. The aim was to gain a picture of essential aspects of the students' understanding of mathematics studies in general and in particular of their experiences from upper secondary school. During their first courses in mathematics, the students were frequently interviewed to follow shifts in their thinking about mathematics and the learning of mathematics as they progressed through the courses. The interviews were audio-recorded and transcribed in full. Transcriptions have been analysed using methods inspired by Grounded theory (Charmaz, 2006). The data have been coded and sorted into categories, and axial coding has been used to analyse how the categories relate to each other. The result is a local theoretical description of essential aspects of the transition that could be discerned in the empirical data. However, these descriptions will contain aspects of individual and social theoretical perspectives from a more general point of view. How they interact within these concepts can also spread light of how different theoretical perspectives can be connected, coordinated, combined or networked.

RESULTS FROM INTERVIEWS

During the pre-interview, Cindy tells that she always liked mathematics and describes it as "her subject". She particularly enjoyed solving equations, which according to her demands accuracy and concentration. In lower secondary school, she was one of the best in her class, but in upper secondary school, she experienced that mathematics became more difficult. In her last courses, she had to "struggle to survive", and "integrals, strokes and such were not easy". A mathematics lesson usually started with a 10-15 minute lecture about the type of exercises the pupils were to work with. Next, the students would work individually with exercises from the textbook. During mathematics lessons, Cindy would collaborate with two classmates in a spontaneous group. By working together on the same exercise at the same time, they could explain to each other how to solve many exercises. To work on her own was meaningless to Cindy, because she would get stuck and could not continue on her own. When Cindy did not manage to understand the mathematical content, she simply tried to learn how to solve different types of exercises. She emphasises that there is a huge difference between knowing what to do and understanding mathematics, but her experience is that she often had to be content with the former. A new experience concerning exercises is that even if one finds the right answer, one cannot know if the solution is correct. For example, Cindy says that if she finds the limit of a function, she does not know if she has based her conclusion on the correct arguments or if she was simply lucky.

Cindy also thinks that another difference between mathematics studies at upper secondary school and university is that "it is harder" at university. She experiences that the mathematical content is more difficult and that everything is always completely new. During a mathematics lecture at the university an extensive amount of mathematics is covered, which results in many new things at the same time. This increases the risk of forgetting the first things that were said during the lecture. Cindy feels that the university teacher is good. When answering individual questions, he gives detailed explanations from the beginning. On the other hand, Cindy remarks that it is hard to get a straight answer or a simple explanation. Cindy feels that the most useful part of the lectures is when the teacher shows examples on the whiteboard, and when all steps in the solutions of the examples are demonstrated.

In the pre-interview, Roy tells that during upper secondary school he studied all available mathematics courses and got the highest grades. According to him, the first mathematics courses at upper secondary school were too easy. The majority of the mathematics consisted of using algorithms in a mechanical way and solving many similar exercises. This felt meaningless and bored him. It was not until later courses that Roy also met some challenges, which he defines as a need to "think for yourself". He tells that probability was one of his favourite subject areas, because it offered the opportunity to reason logically and to try different solution strategies.

Roy remembered that mathematics lessons usually began with a short demonstration by the teacher. During the remaining part of the lesson, the pupils worked individually or in spontaneous groups, solving exercises from the textbook. Roy's strategy was to look at the last exercises in the chapter. If he managed to solve them, he concluded that he could also solve the previous ones and that he had understood the content of the lesson. Most of the time, Roy worked on his own. However, if he did get stuck, he preferred discussing with his classmates instead of asking the teacher. He also frequently helped other students in his class and enjoyed explaining things to others. At university, Roy prefers working with peers rather than on his own, because it makes him more disciplined. From a social point of view, it is nice to meet with others and it makes studies more enjoyable. Often, he has solved more exercises than his peers have, but Roy likes to help the others solve exercises and feels it is a good opportunity to review the mathematical content.

When Roy compares mathematics studies at upper secondary school and university, he says that the main differences at university are longer lectures, a higher tempo, less time to work on exercises during lessons, the importance of "being in phase", and really understanding. Another difference is that mathematics is no longer only a question of understanding or not understanding; it is also necessary to read about mathematics and learn some things by heart. This results in a need to study mathematics, not only to work on exercises. It is also essential to truly understand what one is doing and not just work on exercises. Roy says that he is very satisfied with the teacher, who works thoroughly on "building up the concepts with understanding" and states that he can "buy his explanations". He also states that understanding is more important than ever, because if he is going to become a teacher, he needs a deep understanding to be able to explain even to gifted students. He feels very highly motivated.

ANALYSIS OF INTERVIEWS

From the interviews with Cindy and Roy, portraits of two individual students appear with very different experiences and abilities for mathematics studies. In the following, I will give an account of three central aspects that can be discerned from the interviews and that seem crucial to mathematics education in a learning environment, namely mathematical learning objects, mathematical resources and student as an active learner.

There are a number of objects and relationships that play an important role in students' mathematics education, for example the teacher, peers, the textbook and time. Cindy's and Roy's stories illustrate how these come into play in different ways and how they support their learning of mathematics. Thus, empirical data implies that students use both tangible and intangible issues to accomplish what they consider as learning of mathematics. Results also show that to obtain mathematical learning demands making use of different entities in the environment. The *Mathematical learning object* refers to the main target of mathematics studies in a wider sense from the student's point of view. This concept captures the very essence of what students think that mathematics is and what should be learnt. Though Cindy and Roy study the same mathematics courses, they give very divergent descriptions of the subject. While Cindy feels that mathematics gets harder and harder, Roy characterizes the increasing difficulty as a stimulating challenge. Cindy's statement about integrals and strokes can almost be considered drivel, which in turn indicates a superficial view and memory of the mathematical learning object. Students use M*athematical resources* to obtain mathematical learning objects. In the interviews, Cindy and Roy explain how they collaborated with peers during mathematics lessons. However, while peers were an essential resource for Cindy to be able to solve exercises, peers rather had a motivational and self-confirmational function for Roy. Thus, a mathematical resource is relational rather than absolute and is constituted by students' usage of it. Which mathematical learning objects students focus and what they experience as understandable and meaningful can also be related to which mathematical resources the students are able to use. Different ways of interpreting mathematical understanding, their assignments and what it means to learn mathematics will also influence their mathematical study methods, which mathematical resources they choose to use, and how they view themselves as learners. One example that is worthwhile to examine further is their view of what a mathematical problem is, and what it means to solve it. Thus, it is plausible that how students perceive the mathematical learning object affects them as active learners, which in turn actualizes diverse mathematical resources and puts them into play in different ways.

There is a mutual relationship between mathematical resources and mathematical learning objects. Students use mathematical resources to obtain mathematical learning objects, but on the other hand, a mathematical learning object requires students' use of different mathematical resources. How they come into play depends on the characteristics of *Students as active learners*, which can also be discerned from the interviews. Students as active learners highlight the activities and actions they undertake to learn mathematics, and the intentions behind them. In the interviews, Cindy and Roy tell how they participated in the mathematics education and their thoughts and feelings about it. From these narratives, central aspects are, for example, the students' self-conception, motivation and identity. Cindy and Roy show clear differences between most of their learning activities, but they also carry out the same activity with different intentions.

As an example of how these three aspects interact, and how they interact in different ways for Cindy and Roy, I will return to an empirical example from the interviews. Even though Cindy wants to study mathematics, she often experiences the mathematical content as difficult. From her perspective, the content can be described as *inaccessible.* As a learner of mathematics, Cindy can be characterized as *dependent* with a view of the mathematical content as sometimes unmanageable and hidden. From her perspective, peers and teacher constitute a basic condition for her mathematical learning by helping her to find solutions to exercises and explaining things. By using them as a mathematical resource, she gains access to her mathematical learning object. For Roy, the mathematical content is *accessible*. To gain access is rather a question of his motivation for, and time spent on, studying. Roy can be described as an *independent* learner with a great portion of self-confidence in relation to the mathematical content. In his interaction with peers, they serve as a source of self-confirmation. Thus, peers as a mathematical resource have a more social and motivational character for Roy. The words *dependent* and *independent* as a description of students as learners and the *accessibility* or *inaccessibility* of mathematical content may be interpreted as inherited properties. However, this is not the way they should be understood. Instead, these characteristics are activated in the dynamic and interrelational interplay between the individual and the social environment. The concept of dependent-independent can rather be interpreted as an individual concept used with a social meaning. In the same way, the notion of access is used from an individual perspective. Thus, this example emphasizes and confirms that the transitions merge an individual and social perspective on learning. In relation to previous studies of mathematics studies at university level and the secondary-tertiary transition, it is obvious that the transition cannot be understood by limiting to learning a specific topic, ways of reasoning or advanced mathematical thinking. Instead, the interviews show that it is rather a question of an accumulation of small changes in the mathematical culture (Praslon in Artigue, Bataneri & Kent, 2007). However, these changes occur as a consequence of both changes in the learning environment and students' intentions and abilities to relate to them in a favourable way.

To further elucidate mathematical learning object, mathematical resources and the students as active learners, I will relate my analysis to the theoretical framework of Wenger (1998) regarding communities of practice. According to him, a practice is about meaning as an experience of situated activities. There are two interactively constituted processes involved in the negotiation of meaning within a practice, namely participation and reification. While the former is used in a common sense, the latter needs some clarification. According to Wenger, reification refers to "the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (Wenger, 1988, p. 58). Thus, reification is tightly connected with the creation of meaning in relation to concrete or invisible objects and entities in the surroundings. From the above description, a parallel between Wenger's concepts of participation and reification on the one hand and my concepts of students as active learners and the mathematical content on the other can be discerned, whereby the mathematical resources constitute an interface between participation and reification or as the bridge between students as active learners and the mathematical learning object. Thus, it is clear that the concept of mathematical resources is more embracing than simply referring to something that gives rise to cognitive conflicts for the individual student from a constructivist point of view (von Glasersfeld, 1995). Neither does a mathematical resource equal a sociocultural artefact (Säljö, 2000). Instead, mathematical resources must be considered relational and dynamic. They come into play in the interaction between a student's intentional actions to learn mathematics in an actual situation, surrounded by a specific learning environment. From Cobb and Yackel's "Emergent perspective" (1996), students as active learners and the mathematical content can be related to both a social and a psychological perspective at all levels, while the mathematical resources appear between the individual and social columns in their model.

CONCLUDING REMARKS

My intentions with this paper is to show how a research interest can give rise to new theoretical concepts that do not fit in more established theoretical frameworks about thinking and learning. The case in question concerns secondary-tertiary transition.

The emergence of mathematical learning objects, mathematical resources and the students as active learners are a result of my initial statement that the transition is best understood from both an individual and a social perspective. For example, a mathematical learning object can be constituted by a specific mathematical concept or entity, but the shape of the learning object and which mathematical resources the student uses are both a matter of individual pre-knowledge, identity and overall aim with his or her studies, as well as the learning situation and availability of potential mathematical resources in the setting. There is a constantly ongoing interplay between these individual and social dimensions of the transition. The dynamical aspects of these categories capture essential aspects of the transition from the students' perspective. The transition may change the students' roles as active learners by contributing to shifts in their intentions with learning mathematics and in their actions in different learning situations. In turn these shifts may change the students' use of mathematical resources and their focus on different mathematical learning objects. This captures the core of the transition from the students' perspective, but also elucidates the interplay between individual and social theoretical aspects, raised from a complex "real world situation" that lacks an obvious choice of theoretical approach. The next step is to analyse observations of students working with mathematics in tutorials and in clinical settings, both when they work alone, under the guidance of the teacher and in collaboration with peers. These analyses are to contribute to a more sophisticated definition of the concepts, which can be used to characterize different learners and their paths through the transition.

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