THE POTENTIAL TO ACT FOR LOW ACHIEVING STUDENTS AS AN EXAMPLE OF COMBINING USE OF DIFFERENT THEORIES

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In dealing with low achieving students one needs a fine grained measure for their gain in knowledge. I will show that the concept "potential to act" helps to understand the students' difficulties and to support their construction of knowledge. The concept connects parts of theories of different scope: a model for abstraction in context, self-determination theory and a psychological theory of action. The relevant parts of the theories will be discussed, and, more specifically, to which extend they are compatible. I shall utilize an example to illustrate the concept of the "potential to act" and to show the interplay of the different theories at work. Further, I will explain how their combining use gives rise to additional insight about the construction of knowledge.

INTRODUCTION

As part of an on-going project at the mathematics education department of the University of Bremen, I am working on a theory of support for low achieving students in Hauptschule[1], aged between 13 and 18. In the project, we want to identify what kind of potential to act in certain situations these students have in order to be able to adapt the supporting lessons better to them, and to understand how they construct and reconstruct mathematical knowledge. For this it is necessary to get finer information about the students' gain of knowledge than is possible by error analysis of direct tasks or questionnaires.

We are not discussing the phenomenon of low-achieving students in terms of "dyscalculia" or similar notions (cf. (Moser Opitz, 2007) for a recent review). Those studies concentrate mainly on primary school students and on typical problems with arithmetic and numeracy tasks. In contrast, I am interested in the problems of motivation for low-achieving students, which seem to have gained little interest so far. A notable exception is the article of Pendlington (2006), where the author describes the effect of supporting lessons on self-esteem.

I will not use the concept of self-esteem in this paper, but I will make use of self-determination theory for the motivational aspect. Furthermore, I complement this approach with the theory of abstraction in context and a theory of action. By applying these parts of different theories we can accomplish a more complete understanding of the learning process for low-achieving students.

In this paper I present a case of combining three different theories that in their cores may not be fully compatible and this case raises the question what compatibility means in this context.

THEORETICAL BACKGROUND

I will restrict the description of the three theories to their main parts.

Abstraction in context – the RBC model

Hershkowitz, Schwarz & Dreyfus (2001, p. 202) regard abstraction as "an activity of vertically reorganizing previously constructed mathematics into a new mathematical structure". This means that abstraction is an activity in the sense of Leont'ew's activity theory that comprises actions. Hershkowitz et al. identify three characteristic epistemic actions, namely recognising (R), building-with (B), and constructing (C). Recognising is described as an action in which a student becomes aware of a familiar mathematical structure in the situation, and building-with as "combining structural elements to achieve a given goal" without gaining new complex knowledge about the situation. When this happens constructing takes place.

These epistemic actions are observable in social interaction and provide evidence that a process of abstraction is taking place. The actions are nested, e.g. constructing requires that the subject has already recognised and built with existing structures to construct a new mental structure.

Self-determination theory (SDT)

The self-determination theory (Deci & Ryan, 2000; Ryan & Deci, 2000b) explains how different kinds of motivation emerge. For this the existence of three innate psychological needs is postulated: the need for autonomy, the need for competence and the need for social relatedness. These needs "specify the necessary conditions for psychological health or well-being" (Deci & Ryan, 2000, p. 229) and are indispensable for intrinsic motivation or integration of extrinsic motivation. Following Bikner-Ahsbahs (2005) I specify the innate needs for students in mathematics as follows: *autonomy* as the experience of being able to initiate learning processes and decide about them, *relatedness* as the experience of integration in the social environment and of social support. Bikner-Ahsbahs's definition of *competence* as experience of broadening or deepening one's mathematical abilities seems to be too narrow for our purpose, because low achieving students might get a feeling of competence simply by successful application or reproduction of their mathematical knowledge.

Theory of action

Oerter (1982) discusses the notion of action and the relation of objects and action. He follows the tradition of Leont'ew's activity theory and considers action to be of "primary reality" for each subject, i.e. action is the sole link between an individual and its environment.

> "There is no remembering, imagining or thinking as such, other than with respect to the objects of the environment." (Oerter, 1982, p. 103, transl. by the author)

This implies that any kind of relation to objects or between different objects can only be accomplished by action. There are three layers of object relations[2].

1. *no separate object*, i.e. the object is a fixed part of the situation and cannot be thought of after the current action. It will not even be recognised as an object.
2. *object separated from subject*, i.e. a relation beyond the current action. A subject can recognise the object and name it after the current action but it may still be dependent on the given situational context.
3. *abstract, formal object*, i.e. the common structure of the contextualized objects.

Our experiences with low-achieving students lead to the hypothesis that these students often fail at the transitions from one level to the other. For example, let us take a quarter of a certain cake. At the first level, the student does not realize a separate object at all, i.e. this quarter has no meaning by itself and after it has been eaten there is nothing left to think about. At the next level, the meaning of a quarter of this cake can be transferred to similar situations. So, we might think of a quarter of a piece of chocolate, but all of those quarters are still tied to their context. Finally, at level three a student might have a concept of a quarter of something, meaning one of four equal parts of an entity. Thus, this concept has become abstract and does not depend on the concrete action.

THE POTENTIAL TO ACT

We start with the definition: *The potential to act* consists of all possibilities a subject has to act in a given situation with respect to given objects. This rather abstract definition requires some explanation and we shall discuss it in a more concrete setting:

Imagine that you are working with a student on some mathematical concept, e.g. division of natural numbers. Using a traditional test you have already found out that he fails to solve most division tasks. Furthermore, you have experienced that he cannot make use of most basic ideas associated with division of natural numbers. However, if you ask him to explain how something might be divided in a certain family situation, he can explain some of these basic ideas. In this case his potential to act includes these concepts in the family situation, but not in the written test. So, using the family situation, you might be able to help him enlarge the potentials to act for division tasks.

It is obvious that it is impossible to describe the potential to act of a given student completely. Nevertheless, by looking at the real actions (in contrast to the potential ones) a researcher is able to identify indicators for them and can develop hypotheses about how the student's potential to act might look like in this specific situation and similar ones.

A potential to act can be described by two dimensions: the cognitive dimension and the motivational dimension. The RBC-model and the SDT provide tools to gain indicators in these dimensions. Let us briefly describe what these dimensions mean and how to get indicators for their description.

The motivational dimension is thought of as the degree of intrinsic motivation. Whenever an innate psychological need is satisfied, we interpret this according to SDT as an indicator for an increase in the motivational dimension. If the needs for competence, relatedness or autonomy are not satisfied, we infer that intrinsic motivation will decrease. At this stage of research we use the words *increase* and *decrease* in a qualitative sense without any quantification.

The epistemic actions of the RBC-model may serve as indicators for the cognitive dimension of the potential to act. This dimension inherits the hierarchy of the nested epistemic actions.

Besides the cognitive and motivational dimension, one has to cope with situational aspects of the potential to act including the objects involved. The layers of object relation are used as a tool to structure and categorize the objects in different situations.

Let me briefly comment why those three theories were chosen for the aspects of the potential to act. In order to have a framework for the notion “potential to act”, I chose the theory of action according to Oerter, which has the advantage to offer a description of relations to the objects. The theory of abstraction in context is used, because it allows gaining information about the process of construction of knowledge and fits well with Oerter’s framework of action. Self-determination theory was chosen, because it captures the motivational aspects of the potential and has already been successfully used in describing the motivational problems of low-achieving students in general (Skinner & Wellborn, 1997).

SOME DATA

The data shown below stems from an explorative study conducted at the University of Bremen to explore the potential to act for a group of low achieving students. The students where of age 14 to 18 and took part in weekly supporting lessons, which were done either for groups of three students or individually. The lessons were videotaped and the video was analyzed afterwards to reconstruct the potential to act and to set up the tasks for the next lesson based on this analysis.

The following transcript shows part of supporting lessons that were intended to help the student (S) to understand the concept of equivalence of fractions. This specific task was chosen to help S to develop connections between different representations of extending fractions. S is 14 years old and has been taught by a special school teacher in mathematics for over a year before she came into our project. In her math class fractions had already been introduced the year before and were again the topic of various lessons in class during the weeks before this episode was conducted. After S

has been given a worksheet showing figure 1 the teacher (T) asks her to explain the diagram.

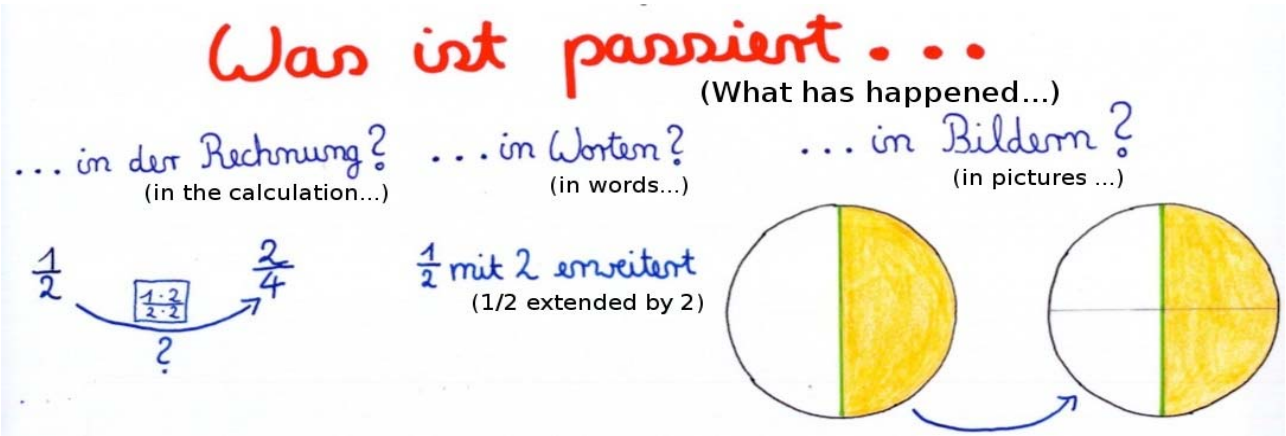

Figure 1: "What has happened …?" (translation by the author)

#	Speaker	
2	S	Well – erm – they have one half – times – they have calculate one times two – up here, haven't they? (S points at calculation in the denominator)
3	T	Hmm.
4	S	And – erm – what then four – erm – to get four as a result, they have calculated two times two.
5	T	Hmm, exactly.
6	S	Well, they have extended by two.
7	T	- And what is this picture?
8	S	Erm, that is one half and … quarter. Two quarters.
9	T	Hmm. And what exactly has this picture to do with – erm – the calculation?
10	S	This is one half and this – and these are two halves. (S points at ½ in calculation and left circle, 2/4 in calculation and right circle in fig. 1)
11	T	Hmm – exactly, fine, and – er – now in here there is this, this calculation described, isn't it? You have said this correctly already. Erm, can you find this, what has happened here, this calculation. Can you find it in here again?
12	S	(S pauses for 17 seconds) one times two is this (S points at left circle in fig. 1) and two times two this (S points at right circle in fig. 1) two and two (S smiles) –
13	T	(T shrugs, then smiles) Erm, two times two – where does it say that?
14	S	Down there.
15	T	Erm. And do you know, what it means, if it is written down there?
16	S	(S pauses for five seconds) If it says 2 times 2 below, then we must do "times two" above.

Transcript 1: "What has happened…?" (translation by the author)

Before we analyze the potential to act, let us first summarize the situation. The lines 11 – 13 point at the crucial situation. The student is asked to explain how the process of extension by two is visualized in the picture. She is expected to say that this is done by refining the given fraction. While the teacher explains to S after line 16, what the answer should have been, S is looking out of the window and seems frustrated. S does not engage herself anymore in the rest of the supporting lesson and is very serious.

We reconstruct S's potential to act in three steps. First, let us consider the epistemic actions. There are a number of recognising actions in lines 2, 4 and 6. S recognises the calculation in the numerator and the denominator of the fraction in the blue box in fig. 1. She also recognises the left circle as a half and the right circle as two quarters (8) and is able to relate them to the corresponding fractions in the calculation. In line 11 she is asked where to find the calculation inside the blue box in the picture. After a short pause, she identifies "one times two" as the left circle, and "two times two" as the right circle. This should be considered as a building-with action, because she puts together the things she has already recognised and she has to think about this question. In line 16 she also builds-with, because she states a general rule for the objects. Unfortunately, we do not know why she thinks this rule is valid.

What about the motivational component in this situation? There is no experience of autonomy in this transcript, because the task is very explicit and she has not been given much choice how to deal with it on her own. But we can see some experiences of competence here. She is able to identify the fractions in lines 8 and 10, and the teacher supports her by saying "exactly" and "fine". This experience of competence is deepened by S's answer to the question in line 11. S thinks for 17 seconds and manages to give an answer that makes her smile; she seems content with her own abilities. But the reaction of the teacher (shrug) and the teacher's later explanations reverse this experience of competence into the opposite. S realizes that her answer was wrong and may feel even more incompetent because she did not manage to understand that this answer was wrong. Likewise the need for relatedness might be fulfilled by the support S gets from the teacher and the smiling, a bit later this support might seem hollow and misleading. In summary, none of the three innate needs is satisfied here.

Using Oerter's layers of object relation we may interpret this episode further. For S the calculation is not one object, but likely she thinks of a pair of objects, i.e. two separate multiplications. Therefore she looks for a corresponding pair of objects that are given by the two circles in fig. 1. She uses the name "extend by 2" only once in line 6 and it may just be, because it is written on the sheet. Given she names the process of extension on her own, then her relation to this process as an object is in the second layer. But she does not even seem to be able to identify this process as an object of its own right (Oerter's first layer). Thus, her relation to the object "extension by 2" is somewhere between the first and second layer. Line 16 indicates that she

might actually be closer to the second layer, but we do not know, why S thinks one "must do 'times two' above". We do not know whether she is really able to understand this extension as an object of its own right, i.e. as a process that transforms one fraction into an equivalent one.

In summary, S is involved in the situation up to line 16, recognises and builds-with the corresponding mathematical objects. Her innate psychological needs are satisfied up to here. Since S is not able to identify the calculation in the picture correctly, T starts explaining how to understand the picture after this episode, which leads to the experience of incompetence for S. Using the layers of object relation we argue that S cannot correctly identify the extension process for the circles because she is only partially able to think of the extension by two as an object. Thus, she cannot recognise it or build-with. Moreover, this information in mind future supporting lessons can be planned to foster S in the transferring to the next layer of object relation.

The analysis above demonstrates that the use of only one theoretical perspective is not enough to understand the data in sufficient generality for the given purpose. Using the RBC-model we saw that S built-with the structures she recognised, i.e. she was engaged in the process so far. SDT can explain why her engagement stops and in terms of the layers of object relation we can understand her epistemic problem and why she could not construct or reconstruct the concept of "extension by 2" in the given situation. Leaving out one perspective results in serious loss of information, e.g., if the SDT was left out, we would know the epistemic problem but could not explain the sudden change in S behaviour.

SOME PRELIMINARY FINDINGS

It should be kept in mind that the following results are only some preliminary findings from the explorative study. They should be thought of as hypotheses for a larger study to be tested.

Low achieving students seem to make use of a large repertoire of avoidance strategies in order to cope with given tasks. Especially, if their basic psychological needs were not satisfied the students responded by withdrawal, denial or similar actions, as seen above.

Furthermore, the students' potential to act seems to be very dependent on the situational context. Frequently, their relations to the objects were found to be at the first or second layer, hence, the students had no abstract understanding of the objects. If the object relation was at the first layer, the students were not able to recognise the objects and thus could not do building-with actions. At the second layer students frequently developed different versions of an object depending on the context, e.g., a student had developed two different and unrelated object relations of a hexahedron having only the name in common.

TOWARDS THE USE OF THE DIFFERENT THEORIES

Prediger, Bikner-Ahsbahs & Arzarello (2008) suggest a landscape of strategies for connecting theories, which can be ordered by the degree of integration of theories. I shall now explain where the position of my approach in this landscape is.

I use the three theories as a way to understand the different dimensions and aspects of one concept. In terms of Prediger et al. I combined the different parts here "in order to get a multi-faceted insight into the empirical phenomenon in view" (Prediger et al., 2008, p. 173). It may even be that I coordinated, i.e. developed "a conceptual framework built by well-fitting elements from different theories" (ibid., p. 172). For this "a careful analysis of the mutual relationship between the different elements" is necessary and it "can only be done by theories with compatible cores" (ibid., p. 172). To decide the question whether I combined or coordinated let us consider the relationship of the theories:

From the broadest perspective, we have two psychological theories (SDT and the theory of action) and a theory originated in mathematics education research (RBC). SDT and RBC focus on the individual, Oerter's theory on social interaction, but there is no obvious contradiction at this level between these approaches.

The epistemic actions of the theory of abstraction in context have their roots in activity theory (Pontecorvo & Girardet, 1993). Oerter's concept of action is also motivated by activity theory and as far as foundations and basic assumptions are concerned, both theories are compatible.

How do these theories relate to SDT? SDT is a theory in cognitive psychology and at its core are the three innate psychological needs, which act as inner regulation processes that regulate and determine behaviour:

> "SDT describes and predicts the occurrence of distinct processes by which behavior is determined or regulated, some of which are characterized as autonomous and some as controlled or amotivational. We assume not only that these forms of regulation differ experientially, but they also differ in their antecedents, their consequences, and their neuropsychological underpinnings." (Ryan & Deci, 2000a, p.330)

It seems impossible to express the above quotation from Oerter's point of view. His fundamental critique is that action should not be thought of as an intentional but as the primary concept in psychology (Oerter, 1982, p.102). Every other concept has to be developed based on and connected to action. It is not clear to me, whether this implies contradicting basic assumptions, since the notion of "behaviour" by Deci and Ryan is not compatible with Oerter's actions.

What are the relations between different terms in the theories? The potential to act is a concept defined in the notions of Oerter's framework. The epistemic actions are expressed in terms of activity theory and can be understood in Oerter's framework without any change. The three innate psychological needs are defined through experiences of the subject that are the results of certain actions. Autonomy, for example,

was defined as the experience to be able to initiate learning processes and decide about them. This experience is the result of a successful initiation or decision action by the individual itself or by the social group, e.g. the class. In this way the potential to act and all terms used to investigate it can be coherently expressed in terms of the theory of action.

Since the main difference between coordination and combination of theoretical frameworks is whether the theories are compatible, which includes non-contradicting assumptions, I cannot say which one I did, although I have built up a coherent philosophical base above.

SUMMARY AND OUTLOOK

In this paper I presented the definition of the potential to act and applied it to an example using empirical data. It was utilized and helped to gain insight in the process of the construction of knowledge and the motivational aspects of it.

The interplay of the three theoretical parts in the potential to act was described and I tried to position myself into the landscape of connecting theories following Prediger et al. (Prediger et al., 2008).

Bearing in mind the difficulties I had to find the position of my approach, I ask what the meaning of the notions “compatibility of theories” and “non-contradicting cores of theories” is. Does it mean a theory is compatible with another one just because their terms are incommensurable? When do basic assumptions contradict? Cobb (Cobb 2007) remarks that there is no algorithm how to deal with different theoretical perspectives. I suppose that there is also no algorithm to guarantee enough compatibility such that one has not build up “inconsistent theoretical parts without a coherent philosophical base” (Prediger et al., 2008, p. 173), but there might be general strategies which can serve as guide lines for he process of analyzing compatibility.

The “potential to act” is part of my research on low achieving students. The long-term goal is to have a theory of support for low achievers which builds upon the enlargement of the potential to act. A first explorative study has been done on this and my next step is to use the experience gained there in a larger study on support for low achieving students.

NOTES

1. Hauptschule is a secondary school for children, which are supposed to be in the lowest achievement category

2. It should be noted, that these layers are simplified versions of Oerter’s layers adapted for the purpose at hand.

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