AN INTERPLAY OF THEORIES IN THE CONTEXT OF COMPUTER-BASED MATHEMATICS TEACHING: HOW IT WORKS AND WHY

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***Abstract**: I analyze the interplay of theories within a study on computer-based mathematics teaching. I will address divergences in their conceptualization of the empirical realities, influences on the interpretation of data, characteristics of my linking strategies, and issues of compatibility.*

Keywords: impact of theories on data analysis, theory development, compatibility of theories, micro-sociology, linguistic activity theory

INTRODUCTION

Amongst many others (Lester, 2005; Mason & Waywood, 1996; to name two only), "interpretative" research in the German speaking community of mathematics education has highlighted the crucial role of theory in research (Bikner-Ahsbahs, 2003; Jungwirth & Krummheuer, 2008; Maier & Beck, 2001). Accordingly, on the one hand, this research invests much in the development of theoretical frameworks, on the other hand, it aims at a development of locally limited, grounded theories. The outcome of research is thought of as a reconstruction of phenomena that is always theoretical in the sense that it transcends data and thus is an ideal type of reality (Bikner-Ahsbahs, 2003; Jungwirth, 2003). The Austrian research project "Gender – Computers – Maths&Science Teaching" by H. Jungwirth & H. Stadler was based on the above position. The aim was to reconstruct participants´ "relationships" to mathematics, physics and computers in computer-based classrooms, and the role gender plays within their interactive development (Jungwirth, 2008b; for the mathematics-related part). Apart from theorizing those relationships, a theoretical approach to classroom processes being appropriate for a comparison of both subjects had to be developed. It had to provide a notion of teaching as an ongoing process (in order to scaffold the investigation of the establishment of relationships) and as a whole (in order to be able to specify the contextual conditions of both subjects). My previous research suggested a use of micro-sociological theories and of a supplementary theory that was located in the context of activity theory. In this paper I want to deal with these theories and their networking restricted to mathematics teaching (Jungwirth, 2008a; for the related findings). As my aim is not to present the study itself I just mention briefly that the data consisted of 21 common Austrian, mostly CAS-based mathematics lessons, that all were videotaped and transcribed, and analyzed according to the standards of that "interpretative" research which means that interpretation follows hermeneutics and text theory in order to go beyond participants´ (i.e. teachers´) subjective understandings, and beyond everyday life readings of the analyzed events. The overall procedure to elaborate the final set of hypotheses is borrowed from grounded theory (Glaser & Strauss, 1967).

MICRO-SOCIOLOGICAL THEORIES

A micro-sociological perspective on mathematics teaching and learning has already proven fruitful in a variety of studies. To be precise, the attribute does not denote a single perspective but refers to different theories that share a basic understanding of social reality. Its structures are assumed to be established by the members´ of society mutually related acting. Those theories that figure in the project are symbolic interactionism (Blumer, 1969), and ethnomethodology (Garfinkel, 1967).

According to symbolic interactionism, interaction is the key concept to grasp social reality. Within interaction objects (anything that can be pointed, or referred to) get their meanings, and meanings are crucial for people´s acting towards objects and, in that, for establishing reality. Interaction is thought of as an emergent process evolving between the participants in the course of their interpretation-based, mutually related enactment. Thus, social roles, content issues, or participants´ motives as well are not seen as decisive factors; rather, they are also objects that undergo a development of their meaning. Consequently, neither the course of an interaction nor its outcome is predetermined. The term "interaction" is not restricted to events having outstanding qualities in respect to number of participants, topics, kinds of exchanges a.s.o. This means that classroom processes do not need to meet special demands in order to be a proper research object. From the perspective of symbolic interactionism, attention will always focus on the meanings objects get in local interaction, and on the very development of that interaction. As all participants matter from the standpoint of that theory, students are considered to be equally important as the teacher.

Ethnomethodology, too assumes that social reality is made into reality in the course of action but addresses the issue that despite of its formation social reality is taken as a given reality. This is due to the reflexive character of everyday activities. By accomplishing their affaires the members of society provide explanations for their doing and thus make it the normal way of doing. Ethnomethodology tries to reconstruct those methods. Accordingly, it helps in taking into account the methods by which teachers and students make computer-based mathematics teaching a matter of course whatever it will be about. Because of the shared stance towards reality the micro-sociological theories are treated here as "one" approach.

However, both theories are not sufficient. First, they address even large joint actions under the aspect of formation by separate acts of the participants; that is, they do not foreground the idea of a whole that has its specifics and thus can be spoken of as an entity. Hence it is difficult to think of teaching as a business that has an overall orientation. Secondly, both theories may induce a bias towards verbal events. There is a tendency to focus on verbal processes because of the prominent role of participants´

indications to each other which are indeed often verbal. Yet in an analysis of computer-based mathematics and, even more, experimental physics teaching all kinds of doing have to be covered.

LINGUISTIC ACTIVITY THEORY

The added theory (Fiehler, 1980) is a linguistic branch of activity theory (Leont´ev, 1978) that is not specialized on teaching and learning issues. Its basic concepts are activity, and activity complex. Activities are not merely actions but lines of conduct aimed at outcomes, or consequences. An activity complex can be thought of as a network of, not necessarily immediately, linked activities of some people that is oriented towards a material, or a mental outcome; that is, the concept always indicates a purposive stance. Linguistic activity theory in particular elaborates on the idea that there are three types of activities: practical activities (being accomplished by manipulations of material objects, or by bodily movements), mental activities, and communicative activities (in the sense of verbal activities). It foregrounds the interplay of these types of activities; actually between practical and verbal ones as the involvement of mental activities is a matter of inference. Two kinds of activity complexes – verbally, and practically dominated ones – are postulated in which the orientation towards verbal, or practical outcomes shapes the interplay in specific ways. As for my concern, linguistic activity theory helps me think of computer-based mathematics classrooms as entities having their own character. In particular, attention is turned to their global objectives. This is a relevant issue since in computer-based mathematics teaching IT plays an important role and could become a matter of teaching of its own right. Thus, there might be a further objective. The micro-sociological point of view is open to this option. But linguistic activity theory is in particular conducive to an identification of such cases as it helps in recognizing modes of activities and their interplay.

STRATEGIES FOR NETWORKING

As for the strategies of networking (Prediger, Bikner-Ahsbahs & Arzarello, 2008), “contrasting” theories has taken place so far and revealed that they play rather complementary roles. In particular, this holds for the micro-sociological approach on the one side, and for linguistic activity theory, on the other side. Each of them provides perspectives that are not covered by the other one but are needed to form a better whole: on situational adjustment and formation, on the one hand, and on certain aspects of structure and overall sense, on the other hand.

This two-sided approach has been used for a certain conceptualization of computer-based (mathematics) teaching: Its overall appearance depends in particular on predominating activities and objectives that are put into effect. These features give evidence of certain activity complexes that are the outcome of a multitude of similar negotiations among participants. Different types of computer-based mathematics teaching can be assumed to be established, ranging from a highly verbal teaching emphasizing mathematical aspects to a teaching that is totally devoted to carrying out manipulations at a computer. That conceptualization can be seen as a nucleus of a theory of computer-based mathematics teaching.

Thus, because of combining theories for the sake of the development of a local theory, synthesizing is a networking strategy in my research. The micro-sociological theories contribute by a "close-up": the step-by-step formation of an activity complex becomes visible. Linguistic activity theory provides a "long shot": a multitude of interactions can be spoken of and treated as an entity.

However, in order to elaborate that nucleus of a grounded theory it has to be applied to the data. Empirical phenomena are interpreted in its light. This means that the basic theories are also co-ordinated. Networking also serves the purpose to reconstruct concrete computer-based mathematics teaching. But as the research aims at a local, grounded theory, co-ordinating turns out to be synthesizing.

NETWORKING OF THEORIES: AN ILLUSTRATIVE EXAMPLE

The transcript is taken from an 11th grade classroom. During the lesson the class was given an introduction into maximum-minimum problems in which Derive should be used. The initiating task was: "A farmer has 20 metres of a fence to stake off a rectangular piece of land. Will the area depend on the shape of the rectangle?" A table should help to systematize the findings. In a first step, the students developed a conjecture based upon examples being subject of the first part (lines 01-26). In the following section of teaching (which is disregarded here) Derive was used to note the examples and to build the table. At the beginning of the second part (lines 134 ff) that table, containing columns for length (x), width (y), and area within the range of the examples, is visible to the students by a data-projector showing the solution of Erna who had to provide the official solution in Derive in interaction with the teacher.

01 Teacher:	Our question is. All these rectangles with circumference 20. Do
02 Sarah:	[inarticulate utterance]
03 Teacher:	they have the same area. For example which ones can we take.
04 Boy1:	No.
05 Boy2:	No.
06 Teacher:	Which range can you give an example length width
07 Boy:	Six and four?
08 Teacher:	Six times four is
09 Boy:	24
10 Teacher:	Another example
11 Eric:	Five times five this is the square
12 Teacher:	Five times five would be a square having which area
13 Eric:	25
14 Teacher:	Or a smaller one. Is there a smaller area as well
15 Carl:	For instance three times seven
16 Teacher:	Three times seven is 21. Or another one.

17 Carl:	One times two sorry one times ten
18 Teacher:	One times ten is ten or if we make it still smaller half a meter
19 Girl:	[inarticulate]
20 Teacher:	No. One times ten does not work one times nine would be OK. If the length will be ten what will happen.
21 Boy:	I see
22 Teacher:	Length ten what will we get if we take ten for the length
23 Arthur:	It is a line, a line [smiles], an elongated fence
24 Boy:	Not at all [continues inarticulately]
25 Teacher:	A double fence without an area thus the area can range from zero to. What was the largest so far
26 Eric:	25
<...>	
134 Teacher:	OK. This is OK. [to Erna] We can see if x is zero the width
135 Boy:	Ten
136 Teacher:	The area
137 Student:	Ten?
138 Teacher:	Yes. But now I like to have names for the columns x y z sorry x y the area. This we can do in the following way. We did it never before. Through a text object. Insert a text object [to Erna] this is not the proper place [it is above the table] but it does not matter no delete it. [she does] We want it below the table please click into the table and a text object above. Yes. And now you have to try. Use the cursor to place x y and area x in order that it is exactly above yes x y and the area. [she has finished] I do not know another way. I have figured out just this one. OK. We can see now the area change from zero 9 16 21 24 25 24. Hence the areas differ.

The episode 01-26 is about a response to a question. An analysis following symbolic interactionism can work out what participants´ taken-to-be-shared consensus concerning that response actually is. Participants deal with the question in the way that they first present a concluding answer (04, 05, maybe 02, too) and then demonstrate its correctness by giving several examples. Thus the response becomes a moot point again, and participants establish an everyday argument of the kind “statements about parts of a whole hold for the whole as well” (Ottmers, 1996) that confirms the initial response. As for the development of the interaction, specifying length, width, and area serves as a format for giving examples but the binding character of the format does not come about at once. For instance, the second student foregrounds his own point and brings into play the shape as well (11). The teacher is always just one party in an interaction. Also his dealing with the wrong combination of length one and width ten (20) is a reaction to the events.

Ethnomethodology enables me to reconstruct the ways in which the whole process of responding becomes a matter of course. For instance, students keep to presenting length and width as factors (11, 15, 17); or, in the case of disturbance (11), the teacher´s ineffective acknowledgement of the square, consisting of a confirmation and an immediate question about the area (12), proves appropriate for stabilizing the format. In the end, it is quite normal that responding is about making sure that the areas differ and about finding out their range. The reference to the square (although not irrelevant at all) turns out to be already beyond the established scope.

Both theories do not provide a more global understanding of the event. In particular, the question may arise what this episode is good for in the light of the research it belongs to. Linguistic activity theory helps to recognize a general purpose of the first part of the episode. It can be taken as a part of an activity complex: of an introduction to maximum-minimum problems. Accordingly, in the presented part a mathematical matter is made plausible that constitutes a problem that, in a generalized version, will have to be solved by means of calculus involving Derive. Besides, linguistic activity theory makes the solely verbal accomplishment of the response task a more remarkable fact; it springs to mind that, for instance, the table is not drawn on the blackboard. Conversely, however, this theory does not provide insight into the specific way of arguing that turns out to be the solution of this task in the end.

In a nutshell, in a co-ordinated theoretical perspective a mathematical event is established that has the role of a preparatory step in a computer-supported task solving. The subject matter-related potential of the interaction is realized as far as it answers this purpose of preparation though, in the light of that role, the pseudo-reasoning about the difference of the areas appears somewhat artificial. Participants produce that event through a fine, inconspicuous verbal adjustment of their acting.

At the beginning of the second part of the episode (134-137) participants demonstrate how the table has to be read. The values in the first line are used to explain what the output means. In a smooth-running process the teacher and two students establish a shared understanding of the table. After the reading has been clarified the table could be used (and this actually happens afterwards) to check the maximum area conjecture by further examples that are not confined to integer-sized rectangles (to be precise: an adapted version has to be used that provides numerical values in between). However, beforehand headings for the columns in the given table are produced. A second meaning of the table emerges. The table that was designed as a means for the solution of a mathematical task turns into a mere scheme being subject to completeness. The switch is initiated by the teacher, and shared by the students (for example, Erna´s immediate adjustment to the new task; 138). All the time manipulations are carried out, and the utterances refer to them. That makes a difference to the first part of the episode. There is much talking again but the accomplishment of the practical activities shapes the verbal process. The completion of the table in Derive becomes the subject of the episode. The situation offers an occasion for such a change; apart from that options of a program will always have to be introduced in some task context. However, as the table was already interpreted well and should help to systematize the findings, the switch is rather a surprise. But: If teaching in that introduction to maximum-minimum problems aimed at accurate products at a computer this turn towards the completion of the table would not be an extraordinary event. It just had to have priority then. This interpretation hypothesis grounding on linguistic activity theory would neither reject the possibility that those products at a computer could be conducive to mathematical ambitions nor exclude that there could be entirely mathematics-related negotiations. Thus, in its light the first episode need not be an exceptional event; it can even get an important role: it gives the computer-oriented business a mathematical air.

In a modified version, this hypothesis is the overall résumé of my research: Computer-based mathematics teaching of the observed type is a technologically shaped practice. The connection of the theories has also given insight into the particular features of that practice (Jungwirth, 2008a).

To combine theories of different grain sizes seems to be rather a successful strategy for co-ordinated data analysis and theory development (Prediger, Bikner-Ahsbahs & Arzarello, 2008; for some examples). In the following sections I want to address aspects of the theories featuring in my research that may further explain the fruitfulness of networking of theories in my case, and even beyond.

EMPIRICAL LOAD OF THEORIES

The first aspect is the “empirical load” of a theory (Kelle & Kluge, 1999). Accordingly, theories can be classed by the risk of empirical failure: whether or not they comprise concepts and statements from which categories and hypotheses can be deduced that can be examined, and thus refuted through data. In the first case a theory has empirical substance, in the second one a theory has no empirical substance. These are the poles of a spectrum of states.

Symbolic interactionism is at the second pole. It is a stance towards the world that can be hold, or rejected. It is not possible, for instance, to formulate refutable hypotheses for the position that objects get their meanings in the course of interaction, or to deduce categories for those meanings from the theory. Ethnomethodology too is a theory that lacks empirical substance, There is no empirical decision-making whether or not people´s methods to settle their everyday affairs make these commonplace affairs, and to fix in advance those methods.

Empirically empty theories have the role of “sensitizing concepts” (Blumer, 1954), that is, of mere perspectives from which data can be looked at. The outcome in the given case has to be worked out in the data analysis. Data can never make such a theory plausible; rather, conversely, interpretations of the data can be plausible in the light of the theory. Qualitative research often draws upon sensitizing concepts because they favour its approach to reality that tries to take into account participants´ own interpretations of that reality (Schwandt, 2000).

Linguistic activity theory has some empirical substance. Observable hypotheses can be built and examined through data. The category of activity and its properties “verbal” and “practical” can be used for this. For instance, it is possible to decide whether or not practical activities dominate and replace talking at all in certain manipulation contexts.

A use of empirically rich theories is characteristic, or even necessary, for quantitative research as the hypotheses to be formulated need a ground they can be deduced from. Within qualitative research referring to such theories may go beyond expectations concerning the rules for that kind of research. Accordingly, literature on methodology (Kelle & Kluge, 1999) points to the risk that properties of categories and hypotheses formulated in advance could dominate and interfere with the intended reconstruction of reality. However, it is not necessary to use empirically rich theories as it is done in quantitative research (Hempel, 1965); a researcher is not obliged to restrict her/himself to examinations of fixed properties and hypotheses.

My study gives evidence that empirically empty and empirically rich theories are compatible, and, moreover, that combining them is a practicable mixture. It seems that this does not hold in my case only. Such a constellation can make connecting theories on a level involving empirical analysis particularly effective. Certainly, applying solely theories without an empirical substance has proven fruitful in qualitative research (in mathematics didactics as well); however, it may be harder to elaborate typologies. Besides, empirically rich theories enhance the development of grounded theories as they help to carry out the check of interpretation hypotheses being strictly demanded in Strauss´ version of grounded theory (Strauss 1987).

CONCORDANCE OF BASIC ASSUMPTIONS (PARADIGMS)

The second notable aspect is the compatibility of basic assumptions theories make for the subject under investigation. To put this concern more clearly I present it in well-established terms: it is about theories´ belonging to paradigms. The concept of paradigm has quite a lot of meanings; I will adopt here the broad view of Ulich (1976) in which a paradigm is thought of as a socially established bundle of decisions concerning the basic understanding of the section of reality a theory wants to cover.

According to him, the duality of stability and changeability of social phenomena is a crucial aspect for theories that deal with social processes and settings. Consequently, he has made it a starting-point for a typology of paradigms. “Stability-oriented” paradigms regard regularities as manifestations of stable, underlying structures. Theories in that tradition try to grasp invariablities. “Transformation-oriented” paradigms ascribe regularities to conditions that are changeable because they are seen as having been established by the members of society. Thus, theories try to reconstruct the constitution of regularities and to find out conditions for change.

The theories I refer to differ in their origins and their concerns. Yet despite of all differences they share the idea that regularities are established regularities; that is, that they are outcomes of practice that can change if inner conditions change. This is obvious for the micro-sociological theories but it holds for linguistic activity theory as well. According to activity theory in general, society is a man-made society; order and stability of societal phenomena reflect the cultural-historical development of human labour and living conditions (although there is an inner logic in that development). Thus, all theories belong to the transformation-oriented paradigms. Symbolic interactionism and ethnomethodology are usually considered to be representative of the "interpretative" paradigm (Wilson, 1970) but that is, in the given typology, simply the micro-sociological version of the transformation-oriented ones.

This common ground justifies an approach to activity complexes under the aspect of local development and, as a consequence, the above conceptualization of computer-based mathematics teaching. If linguistic activity theory thought of human practice as an invariable, "given" entity, networking would not be honest at least. Actually, the idea that an interaction is determined by the roles of the participants, and the idea that an interaction is a negotiation process from which (also) roles emerge could not be combined to an integrated view on interaction serving as a base for analysis.

The general issue arising from the discussion above is which elements of their respective grounds theories have to share in order that networking on the level of some synthesis of theories, or of an integrated analysis, can take place.

To summarize: The last sections should shed some light on the compatibility of theories. It seems that it depends on, or at least benefits from the aspects addressed.

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