# RESEARCH PROBLEMS EMERGING FROM A TEACHING EPISODE: A DIALOGUE BETWEEN TDS AND ATD

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*When approaching an empirical teaching episode or data related to it, theoretical approaches always select and highlight some aspects in detriment of others, globally interpreting the episode using their own conceptual categories and methodological tools. Therefore, different theoretical approaches often construct different research problems, often making their comparison difficult or even impossible. The fact that the Theory of Didactic Situations and the Anthropological Theory of the Didactic share their main assumptions and their 'research programme' (in Lakatos' terms) makes it easier to contrast them in the way each one reinterprets and reformulates the problems raised by the other. Starting with 'neighbouring approaches' thus appears as a sensitive way to approach the complexity of networking theories.*

According to Rodríguez et al. (2008), we assume that any strategy to compare, contrast or network theories has to take into account the way theories question reality and formulate problems about it. This assumption leads us to consider as a *networking methodology* the comparison between the reformulations proposed by different theories of a research question raised by one of them. In this case, the question emerges from an empirical episode and a given set of data. We start this 'exercise' with the case of two theories close to each other, the Theory of Didactic Situations (TDS) and the Anthropological Theory of the Didactic (ATD). We first present the context where this study takes place, and then analyse the exchanges between the co-authors of this contribution around a particular research question, before entering a more general discussion about the potential of this methodology.

## 1. THE CONTEXT FOR THIS STUDY

This study is part of the work on the comparison of theoretical frames of a collective that emerged at CERME4, and whose first outcomes have been presented at CERME5 (Arzarello et al. 2008, Kidron et al. 2008, Prediguer 2008). Since CERME5, the group has orientated its work towards the development of networking methodologies. Different strategies are used for that purpose. One of these, which presents some similarity with the strategy used in the ReMath European project (Artigue 2007, Mariotti 2008), is the comparison between the formulations proposed by different theories when confronted to a given set of data and a research question raised by one of them. In our case, the research question emerged from the analysis of a video, which, from the very beginning, played a crucial role in the work of the group. It corresponds to a classroom session at grade 10 in Italy on the exploration of the properties of exponential functions in the Cabri-géomètre environment, and more

precisely to the observation of a group of two students. In a first phase of the work, the different teams involved in the group analysed the video from their respective theoretical perspectives, what made clear that all of them, except the Italian colleagues, could not find what they needed for completing the analysis they aimed at in the information initially provided: the video and some documents about the classroom session. Each team was thus asked to make clear the kind of information it needed, and the demands of the different teams were discussed at a post-CERME5 meeting. One of the results of this discussion was a questionnaire to be answered by the teacher in charge of the class observed. When the extra information agreed upon, including the teacher's answers to the questionnaire, was disseminated, each team tried to complete its analysis, and the results were presented during a joint meeting in Barcelona. In their respective presentations, several teams referred to a particular answer made by the teacher, pointing out that, from their perspective, such an answer raised important and non trivial issues and deserved further discussion. The question and the answer were the following:

> *"During a lesson of this type, under what circumstances do you decide to get involved with a pair of students, and what kinds of things do you do?"*

> "I try to work in a zone of proximal development. The analysis of video and the attention we paid to gestures bring me to become aware of the so called 'semiotic game' that consists in using the same gestures as students but accompanying them with a more specific and precise language in relation to the language used by students. A semiotic game, if it is used with awareness, may be a very good tool to introduce students to institutional knowledge."

This episode of our collaborative work and the potential we soon suspected it could have if analysed in depth, was the source of the networking methodology we then developed. This methodology obeyed the following organization: the team working in TDS formulates a research problem using its own terminology; each team converts the problem according to its theoretical perspective; the team working in TDS comments on the new formulations, looking at the generic and specific issues; each team works on its specific question and reflects on the process followed.

In what follows we describe the exchanges that this methodology generated between the TDS and ATD perspectives, and analyse their networking potential.

## 2. EXCHANGES ON "SEMIOTIC GAMES"

### 2.1. A first perspective inspired by TDS

As mentioned above, a series of comments regarding the teacher's answer and the articulation of some precise questions was first elaborated within the TDS perspective by two of the co-authors of this contribution (MA & AL). We summarize the main lines of their argumentation below, the teacher being denoted by T.

First, MA & AL observe that the answer expresses the confidence that T has in the so called semiotic games to face a major didactic problem: the connection between, on the one hand, the mathematics produced by students in an adidactic situation, through the interaction with the adidactic *milieu* of this situation,[1] and on the other hand, the institutional mathematical knowledge aimed at. They add that this connection generally requires at least changes in the ways the mathematics at stake are expressed in order to progressively tune them with more conventional forms of expression; and that T obviously considers that he has a specific mediating role to play for making this connection possible and uses semiotic games as a tool for that purpose. In other terms, semiotic games can be considered as components of the praxeology (or more certainly of the different praxeologies) that T has developed in order to solve this didactic task. It is interesting to point out that this last sentence uses terms coming from ATD and not TDS, organizing a first bridge between them.

MA & AL then point out that this answer raises two interesting didactic issues:

The first one is that the situations proposed to students for building new mathematical knowledge do not necessarily have the adidactic potential that is necessary to enable the students to produce the mathematics to be produced under the constrained conditions of the classroom. What is achievable and achieved through an adidactic interaction with the *milieu* is often far from allowing the teacher to easily establish a meaningful connection with the mathematical knowledge aimed at. The discrepancy leads to different phenomena that have been discussed in TDS research (for instance Jourdain effects or "*dédoublements de situation*"), all the more as the teacher feels obliged to maintain the fiction that the mathematics knowledge he or she is expecting has to be produced by the students.

The second one is that the situations proposed to students for building new mathematical knowledge are very often what the TDS calls situations of action. They can lead to a linguistic activity but language issues are not their main concern. The characteristics of the *milieu*, the feedback available, do not make the productivity of the interaction with the *milieu* strongly dependent on the language used by the students. This is a fundamental difference with situations of communication often associated with the dialectics of formulation in the TDS.

Referring to their analysis of the video, MA &AL claim that the associated situations have a rich adidactic potential but also that this potential is a priori not sufficient to ensure the production of all the mathematical knowledge aimed at according to T's answers to the whole questionnaire. They also add that, even if the students have to produce narratives, the three situations which can be identified in the observed ses-

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[1] The notion of *milieu* was introduced by Guy Brousseau as a main element of the Theory of Didactic Situations (Brousseau 1997). It refers to a system without any didactic intention that constitutes a key element of any adidactic situation. The reader unfamiliar with the TDS can find a very accessible introduction in Warfield (2006).

sion are closer to situations of action than to situations of formulation. They thus conclude that these situations constitute a priori good material to examine in context the potential and limits of semiotic games.

They also point out the specific status of T who is an expert teacher, but much more than that, due to his research engagement. According to them, this means that the confidence he expresses in the potential of semiotic games certainly has a solid experiential basis both in his personal practice and also in the practices of the research community he is involved in. Nevertheless, MA & AL's personal experience leads them to look at these semiotic games carefully, all the more when they are said to provide techniques for solving what are considered difficult didactic problems, and to try to understand under what conditions and why they can become efficient didactic techniques helping teachers face the difficulties described above.

The research question resulting from this analysis is the following:

> How to identify characteristics of the semiotic game technique that would help us to understand its potential for:
> - Compensating the possible limits of the interaction with the adidactic milieu to achieve the expected mathematical goals?
> - Fostering the linguistic evolution linked to the needs of institutionalization processes?
> - Identifying conditions required to activate this potential?
> How to identify possible difficulties in the management of such semiotic games and possible effects of their possible malfunctioning?

## 2.2. Conversion of the research question within the ATD perspective

The answer to this question analysed below comes from the other two co-authors of this contribution (MB & JG) who work in the ATD perspective.

### 2.2.1. Some preliminary considerations

The ATD describes human practices (including doing mathematics and its teaching and learning) in terms of *praxeologies* composed by two complementary folds: a *praxis* or practical block (the "know-how") made of *types of tasks* and *techniques* to carry out these tasks; a *logos* or theoretical block (the "knowledge" in its narrow sense) that appears as an assemblage of discourses to describe, explain and justify the *praxis*.[2] The question formulated by MA & AL starts from a rather vague notion of 'semiotic game' that, in the ATD, can be considered as a *didactic technique* that T describes as follows: "the teacher starts using 'the same gestures as students but accompanying them with a more specific and precise language in relation to the language used by students". T's comments on the episode also reveal some *theoretical components* explaining and justifying the use of this technique, formulated in terms

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[2] For the reader unfamiliar with the ATD, see Bosch & Gascón (2006).

of 'working in a zone of proximal development'. At the same time, the comments refer to a type of *teaching task* that is supposed to be performed with this technique: 'introducing students to institutional knowledge'. We are thus considering a *didactic praxeology* as it is evocated by the teacher.

Any 'didactic problem' (that is, a problem related to the teaching, learning, studying or diffusion of knowledge) can be generally identified both with a 'teaching problem' (that is, a question or difficulty that appears in the teacher's practice and that requires an appropriate *didactic praxeology*) and with a 'research problem' (that is, an open question for research in mathematics education). In both cases the problem is formulated in relation to a *teaching and learning process* and connected to a given mathematical content (which is a *mathematical praxeology* or a set of mathematical praxeologies). In this sense, the 'expected mathematical goal' that appears in the formulation of the question, as well as the 'proposed institutional knowledge' are mathematical praxeologies that can have different 'size': point, local, regional or even global.[3] According to Bosch & Gascón (2005), the ATD postulates that the minimal unit of analysis of didactic processes has to contain at least a *local* mathematical praxeology. Furthermore, this local level is considered as privileged or basic because, in order to be studied in an operative way, any didactic problem formulated beyond this level of analysis needs to be 'projected' into its local components. For MB & JG, in the ATD perspective, the initial research question can thus be situated in the very general problem of the study of *the conditions that make the building of local mathematical praxeologies in a given institution possible and the restrictions that hinder it*.

### 2.2.2. *The dialectic media/milieu*

At the beginning, the process of building local mathematical praxeologies can start from questions that arise within a point praxeology or in a small set of them. In any case, the driving force of the didactic process, what provokes the need to study or build a local praxeology integrating and completing the point praxeologies, is the emergence of questions that cannot be answered within the point praxeologies. How these questions arise in a given didactic process? What conditions are needed for a study community to 'take them seriously'? What '*media*' can help the study community to generate provisional answers and what '*milieu*' is available to test and modify these answers? These are still open questions and an in-depth analysis of what is called the 'dialectic media/milieu' seems essential to answer them.

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[3] A *point* praxeology is generated by a unique type of tasks and is often characterized by a unique technique to deal with them; a *local* praxeology is generated by the integration of several point praxeologies within the same technology; a *regional* praxeology is obtained by coordinating, integrating or linking several local praxeologies through a common mathematical theory and a *global* praxeology is a connection of some regional praxeologies (Rodríguez *et al*. 2008).

According to Chevallard (2004), the elaboration of an answer to a real question supposes 'resources' or 'milieus'. In close connection with the TSD terminology, a 'milieu' is a system without any didactic intention in the interaction we can have with it during the study process. In this sense a milieu behaves as a fragment of 'nature'. Besides the notion of 'milieu', the ATD introduces the notion of 'media' as any system the main goal or intention of which is to supply information about a given issue. In any knowledge construction process a dialectics between a media providing new knowledge or information and a milieu able to give evidence of the validity of this information takes place. An extreme situation is when one takes the message coming from the *media* as it appears, without any need for testing it. The opposite side is the construction of knowledge from scratch, through only the confrontation with a *milieu*. The existence of a vigorous (and rigorous) dialectics between *media* and *milieus* appears to be a crucial condition for a study process not to be reduced to a simple copy of previously elaborated answers spread over different social institutions.

### 2.2.3. Formulation of the question in the ATD frame

(a) *'Semiotic games' and the limitations of the adidactic milieu*

The general didactic problem we are considering is the study of the didactic tools, devices or praxeologies that are necessary for the teacher to lead and for the students to carry out the process of building local praxeologies. With respect to the problem of the 'limits of the interaction with the adidactic milieu', it is important to notice that, from the perspective of the ATD, the dialectic media/milieu supposes that any milieu has limitations in the didactic process consisting in building a local praxeology as the progressive answer to a problematic initial question. Even if a given milieu can help contrast a partial answer to the initial question, it will always provoke the need of new *media* introducing new information having to be tested with new milieus, and so on. In this context, T's 'semiotic game' considered as a didactic technique, may be interpreted as a resource used by the teacher – acting as a 'media' – to supply students with praxeological components of the praxeology that is to be built.

(b) *Institutional didactic praxeologies underlying the 'semiotic games'*

Beyond the didactic techniques a given teacher can 'create', research in the ATD frame is interested in the *didactic techniques a given institution makes available* to the teacher and the students to manage the construction of mathematical praxeologies and, more particularly, to manage the media/milieu dialectics.

This institutional dimension is essential because it strongly determines the ecological conditions required by these didactic techniques to normally evolve in the considered institutions. More particularly, the existing institutional conditions influence the kind of *technical gestures* that can usually be made in the institution, as for instance the 'semiotic games'. Like any other didactic technique, 'semiotic games' need an insti-

tutionalised didactic *technology* to describe, justify, interpret and control their role in the didactic process. Beyond the technological level, it is also interesting to study what *theoretical* foundation supports this teaching technique and technology.

## 2.3. Back to the TSD and the formulation of new research questions

After the re-formulation of the research question raised by TDS, a second exchange took place between the teams. We extract from it what concerns the TDS and ATD perspectives. In their comments, MA & AL first point out that, considered as didactic techniques both in TDS and ATD, semiotic games are given two different functionalities according to the theoretical perspective chosen. They also suggest that from this situation can emerge interesting insights regarding the relationships between ATD and TDS:

> "According to ATD, a condition for a study process not to be reduced to a simple copy of previously elaborated answers is the existence of a strong dialectic between appropriate media and milieus. Such a theoretical position presupposes that any milieu has limitations in the didactic process consisting in building a local praxeology, a process which is seen as the progressive answer to an initial question. Within this approach, T's semiotic games find their place as a didactic technique used for the management of the media/milieu dialectic. We think that it will be interesting from this point of view to compare the vision that will be proposed concerning these semiotic games on the one hand by the ATD analysis projecting them in the media/milieu dialectic and on the other hand by TDS projecting them at the interface between adidactic and didactic processes. Having its origin in a theoretical context both distinct from ATD and TDS, it may provide a good opportunity for understanding better the similarities and differences between these two theoretical approaches regarding these crucial aspects."

Another element stressed by MA & AL is that the conversion of the initial questions within an ATD perspective makes a new dimension move from the periphery to the centre: the institutional dimension. ATD indeed obliges the researchers to consider that the study of any kind of didactic technique has to be situated within an institutional perspective. It cannot exist and develop without any institutional legitimation, any institutionalised didactic technology used to describe, justify, interpret and control its role in the didactic process. Within this perspective, what is of interest for research is clearly not the study of semiotic games as practices of individual teachers but the study of their institutional status and ecology, of their relationships with other institutional techniques available to teachers for managing the dialectics between media and milieus. MA & AL add that, in this particular case, the experimental status of the course to which the observed session belongs means that at least two institutions are involved and should be considered: the research institution and the high school institution.

Finally, the exchanges also make MA et AL reflect more globally on the first phase of the work, and the limitation of the perspective underlying it. The first phase con-

sisted of using the TDS and ATD theoretical constructs to reflect about semiotic games, their didactic potential and limit, but the converse movement is also possible, leading to investigate what can be offered to TDS and ATD by having the ideas of semiotic game and the 'zone of proximal development' as functioning in T's 'practical theory' (Ruthven, 2006) entering the scene. This converse movement can also be insightful regarding relationships between TDS and ATD, and the possibilities of networking between them.

## 2.4. Main features of a didactic research problem

At this point of the networking between TDS and ATD, and in order to pursue the network with other theoretical frameworks, it seems necessary to locate the dialogue in a new position, more general and relatively neutral from an epistemological point of view. Three main features seem important to distinguish.

### 2.4.1. Institutional dimension of the didactic problems

In the ATD perspective, the expression 'semiotic game' appears as an element of the *teacher's didactic theoretical discourse:* it helps him interpret what happens in the classroom, take decisions, etc. In this sense, we are dealing with a component of the *spontaneous didactic praxeology* of a *concrete teacher*. A first difficulty appears concerning the personal or institutional dimension of this didactic praxeology.

Institutional praxeologies (and their ecology) are the ATD's primary object of study. To study them, we take as an empirical basis the personal manifestations of these praxeologies as well as their more collective or institutional manifestations: regular practices, discourses, texts, official documents, etc. The dialectic between *persons* and *institutions* can be made more explicit in the following terms. The institutions where praxeologies take place are composed of persons. Reciprocally, persons are always *subjects* of a complex of institutions and, as such, have a personal relation to praxeologies that can be explained to a great extent by the analysis of the institutional praxeologies they have encountered.

### 2.4.2. Mathematics as a core component of didactic problems

Taking into account the educational institutions' vision of teaching and learning processes is a basic methodological principle of the ATD. Otherwise, we run the risk of taking for granted the description of phenomena proposed by each institution – which can furthermore differ from one institution to another. More particularly, research on *didactic transposition processes* (Bosch & Gascón 2006) has shown the necessity for research to construct its own models of mathematical knowledge (or mathematical activities) in order to avoid taking for granted the models imposed by the dominant institutions. These models of mathematical knowledge should include the description of its construction, development and diffusion (and, thus, the mathematics teaching and learning processes).

### *2.4.3. The importance of the unit of analysis*

Any essay to contrast or compare theories has to face a dilemma. On the one hand, to contrast theories, we need a 'common' empirical universe and, thus, we have to remain close to the educational institutions. On the other hand, each theoretical perspective constructs its own vision of this empirical universe, moving away from the educational institutions (to 'escape' from their dominant vision). This detachment is necessary in order to approach problems related to the teaching and learning of mathematics in a more operative way. However, it has always to maintain an accurate distance to the reality one wishes to study – and modify!

The notion of 'minimal unit of analysis' (section 2.2.1) appears as a basic aspect of the modelling of educational phenomena proposed by each theory. Starting from the way each perspective formulates MA & AL's question, we could make explicit what units of analysis are considered in each case and how they can be connected. This could be a good way to improve our capacity of describing and comparing not only the concrete research or practical problem formulated by each frame but also the type of problems that can be proposed and the kind of empirical data needed.

## 3. CONCLUSION

This contribution illustrates a methodology of 'networking theories' based on the study of a question, considering how the different research frameworks engaged in the networking can formulate and approach it, through a sequence of exchanges and progressive refinements. We have taken the interaction between TDS and ATD as a study case, considering two close frameworks that share the same scientific project. This proximity makes the networking easier because the discussion on the fundamental background of the theories can be avoided. It is important to recall that ATD emerged within the TDS, thus integrating the original research programme, its basic assumptions, the nature of the considered problems and phenomena and, more particularly, the need to question and model mathematical knowledge (that is, to take it as a specific object of study). Making this methodology productive with more distant approaches raises the necessity to make the basic assumptions of each one explicit and to contrast them. Another positive consequence of this methodology stems from the fact that the theories involved are questioned from an external construction, which in our case has given rise to two main contributions. The first one is the institutional dimension assigned (by the ATD) to the 'semiotic games' and the way it can be taken into account by the TDS. This issue has long been explored and largely discussed by research in both the TDS and ATD perspectives (Sensevy et al 2005). The second contribution is the comparison between the projections 'didactic – adidactic' and the 'media and milieu dialectics'. They emphasize an obvious difference in the way both theories take into account the milieu's insufficiencies and the changes in our relation to knowledge led by the technological evolution. It is important to note finally that, till now, little advantage has been taken from the inverse networking movement: considering the contributions made by the external perspectives to the development of

our own one. For instance, by formulating problems which are not of first priority in our research programmes but the study of which can open unexpected lines of development. We finally postulate that making explicit the position adopted by research perspectives to the features considered in section 2.4 constitutes an essential step for the networking. This position is important because it delimitates what is considered a 'didactic research problem' and, consequently, contributes to characterise the object of study of our discipline.

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