USING MATHEMATICS AS A TOOL IN RWANDAN WORKPLACE SETTINGS: THE CASE OF TAXI DRIVERS

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The present study is part of an ongoing study of which the aims are twofold; to provide knowledge about why and how mathematics is involved in specific workplace settings, and to provide student teachers with culturally relevant examples to contextualise school mathematics for secondary school students. Observations and semi-structured interviews were conducted in the workplaces of two taxi drivers, one house constructor and one restaurant manager. The focus here is on taxi-drivers. The analyses draw on ideas from socio-cultural theory and the anthropological theory of didactics. A common main concern was economic profit and risk of loss; level of justification, mathematical problems to solve and techniques used differed. Among the taxi drivers, silent and taken-for-granted cultural knowledge were used.

INTRODUCTION

After the 1994-genocide, the Rwandan society was destroyed and disorganised in all sectors. In order to cater for capacity building, the Government of Rwanda has undertaken several measures in all economic sectors through its Vision 2020 for developing Rwanda into a middle-income country (Republic of Rwanda: Ministry of Finance and Economic Planning, 2000). For instance, in the educational sector, the Ministry of Education (MINEDUC) has embarked on prioritising the teaching and learning of science and technology (including mathematics) to provide human resources useful for socio-economic development through the education system. MINEDUC recommends that learning should be context-bound. This means that in order to serve the local society, teachers and researchers are encouraged to bring material to the students that are taken from national contexts. For instance, exploring mathematics via tasks from workplaces may support students to learn in ways that are personally meaningful (Taylor, 1998). Contextualising mathematics allows students both to understand the role of mathematics in solving different workplace problems and see ways in which mathematics is used out of academic institutions. They can also realize that such activities can be translated into mathematical language that is taught in different institutions.

However, before we embed mathematics in workplace settings, we should have a clear picture of the use of mathematics in such contexts. This is of crucial importance especially in Rwanda where this kind of research is relatively new and where mathematics is mostly seen as an abstract and hidden science which does not provide visible applications in workplaces (Niss, 1994; Williams & Wake, 2007).

In this study the use of mathematics as a mediating tool (Vygotsky, 1978) supports workers to solve problems related to the earning of their income, using culturally relevant concepts and experiences (Cole, 1996; Abreu, 1999) when seeking survival means is investigated. Therefore, the current study will provide knowledge about why and how mathematics is involved in three workplace settings: daily taxi driving, house construction, and restaurant management. Although the workplace settings are quite different and subject to change over time, the choice was made with the intention to understand mathematics in use in workplace settings where the actors perform differently but aim to achieve the same goal – to earn a good living. Within this study, the present paper will focus on the taxi driving context.

STUDIES ON SITUATED MATHEMATICS

Over the last thirty years, researchers have investigated how mathematics in everyday practices differs from what was taught at school and in academic institutions. In this endeavour Lave (1988) found that mathematics practice in everyday settings is structured in relation to ongoing activities. Based for example on the use of shoppers' "best-buy" strategies, she points out that mathematical practices in work places do not require any imposed regulation. Rather, adults use any available resources and strategies which could potentially help to solve a problem. Also, in a collection of studies related to informal and formal mathematics, Nunes, Schliemann and Carraher (1993) found that there was a discrepancy between street mathematics and school mathematics. This is demonstrated through a mathematical test which was given to the same children who performed better out of school than in a school setting. This discrepancy is due to the fact that at school children tried to use formal algorithms whereas in real situation they did arithmetic based on quantities. It should be noted though that the requested arithmetic procedures were quite simple. In results from a study related to college mathematics and workplace practice, Williams, Wake and Boreham (2001) found that the conventions of school and workplace graphs might be different. Indeed, in a chemical industry, school graph knowledge was not enough to allow a college student to interpret a graph of chemical experiments. However, the college student was able to interpret it with the help of an experienced employee. In a recent study Naresh and Presmeg (2008) followed a bus conductor in India in his daily practice, where they observed that though he performed significant mental mathematical calculations the bus driver's attention was fully concentrated on the demands of his job, making his mathematical work more or less invisible to him.

From the results of the above studies, we conclude that when it comes to solve a particular problem, the way mathematics is used at work is different, however logically organized (Abreu, 2008), compared to how it is used in academic institutions. At a workplace the problem solvers keep the meaning of the problem in mind while solving it in the real situation. In contrast, in the academic institution, the meaning of the problem is often dropped because of the imposed curriculum regulation where the problem solver is expected to employ certain mathematical symbols and conventions.

Researchers have also studied mathematical concepts and processes that are used in different workplace settings. In a study on mathematical ideas of a group of carpenters, Millroy (1992) found that not only are many conventional mathematical concepts embedded in the everyday practices of the carpenters, but their problem solving is enhanced by their stepwise logical reasoning similarly used in mathematical proofs. Abreu (1999) also found that Brazilian sugar cane farmers used indigenous mathematics to control their income. However, over time, technological innovations in measuring quality requested change to more school-like problem-solving strategies which made farmers prone to abandon traditional units of analysis and value their children's success at school mathematics. A study by Massingila (1994) revealed that mathematical concepts and processes are crucial in carpet laying practices such as estimation and installation activities. Furthermore, she found that measuring and problem solving are two major processes in the carpet laying practice. In their exploratory study related to how mathematics is used and described in workplaces in the context of employees in an investment bank, paediatric nurses, and commercial pilots, Noss, Hoyles and Pozzi (2000) found that practitioners use mathematics in unpredictable ways. Hence, their "strategies depend on whether or not the activity is routine and on the material resources at hand" (p. 17).

A common point to all these studies is that mathematical strategies that are used at workplaces differ to those taught at academic institutions. A mathematical strategy for solving a problem refers to a 'roadmap' that consists of identifying the problem to be solved and the appropriate technique(s) that allow solving that kind of task. However, in the above mentioned studies mathematical strategies are described as applied by workers without details about how they are or may be underpinned by mathematical justifications. Mathematics is seen as a tool to mediate human activity through the lens of workers' goal achievement. None of them looked at mathematics through the lens of its knowledge organisation, including types of problems worked on, as well as methods used to solve them and their justification (cf. Bosch & Gascon, 2006). To fill this gap the current study emphasises mathematical practices and its justifications embedded in mathematical activities found in specific Rwandan workplaces and their relation to academic mathematics.

MATHEMATICS AS TOOL TO MEDIATE WORKPLACE ACTIVITIES

Human activity is always goal-oriented and characterised by two major parallel actions: thinking and acting. The action is shaped by thinking and inversely through available socio-cultural tools for goal-oriented activity. Human mind and activity are always unified and inseparable. This means that the "human mind comes to exist, develops, and can only be understood within the context of meaningful, goal-oriented, and socially determined interaction between human beings and their material environment" (Bannon, 1997, p. 1). In activity theory, social factors and interaction between agents and their environment allow us to understand why tool mediation plays a central role. Tools shape the ways human beings interact with reality and reflect the experiences of other people who have tried similar problems at an earlier time (Bannon, 1997). Tools are chosen and transformed during the development of the activity and carry with them a particular culture. In short, the use of tools is a means for the accumulation and transmission of social knowledge. At the same time, they influence the nature of external behaviour and the mental functioning of individuals.

Engeström's (1993) model of basic human activity systems comprises six main elements: subject, object, tools, rules, community, and division of labour. He also suggests that such systems always contain "subsystems of production, distribution, exchange, and consumption" (ibid., p. 67). The present study is located in the subsystem of production which is mainly characterised by interactions between subject, tools and object. Within the production activity, subjects chose and transform useful tools that match a prior defined object to achieve a desired outcome.

However, our study will not elaborate on the production process as such. It will rather focus on the sub-production related to the selection and transformation of useful mathematics that facilitates the concerned subjects to achieve their goal on their respective workplaces. In other words, the study will investigate how the selected mathematics is organised so that the workers may interpret it in terms of the outcome of their activities. At that stage, it was imperative to add a complementary theory which explains deeply about the organisation of mathematical knowledge.

We will thus use a theoretical model from the anthropological theory of didactics (ATD), viewing teaching and learning as an activity situated in an institutional setting (Chevallard, 1999; Bosch & Gascon, 2006). By engaging in this activity, the participants elaborate a target piece of knowledge for which the activity was designed. This perspective sets a focus to the knowledge itself as an organisation system (a *praxeology*), including a *practical block* of types of tasks and techniques to work on these tasks, and a *theoretical block* explaining, structuring and giving validity to work in the practical block (Barbé, Bosch, Espinoza, & Gascon, 2005). This praxeological organisation of knowledge can be used to describe very systematic and structured fields of knowledge (such as mathematics or any experimental or human science) and its related activities, with explicit theories, a fine delimitation of the kind of problems that can be approached and the techniques to do so. Considering the mathematics teaching and learning process, we can find two different (intimately related) kinds of praxeologies: mathematical ones, corresponding to the subject knowledge taught, and didactical ones, corresponding to the pedagogical knowledge used by teachers to perform their practice. For the purpose of the present paper we will look into the mathematical praxeologies (or *mathematical organisations*) observed at the different workplaces.

Aims and research questions

The study reported in this paper is from the first part of an ongoing research project aimed at finding ways to contextualise school mathematics within cultural mathematical practices in Rwanda. In this project, the researcher documents the rationale and characteristics of mathematical practices in local workplace settings, to serve as a source to design contextualised mathematical activities for student teachers in a teacher education programme. From the experiences of working on such problems, the student teachers will design tasks contextualised in the local culture for secondary school students, whose work on these tasks will then be analysed. In this three-stage process, the didactical transposition (see Bosch & Gascon, 2006) of the workplace mathematical practice, via the mathematical tasks designed for and solved by student teachers, to the school students' contextualised mathematical work will be analysed.

The general question about why and how mathematics is involved in specific Rwandan workplace settings was split into specific research questions. First it was important to clarify what motivates the workers to involve mathematics in their daily activities (the why-question). In this regard, the interest was on what problems workers solve at their workplaces. Next there was a need to look at how those mathematical problems were solved. The answer to these questions raised the issue of justification of mathematical techniques used (the level of logos in the mathematical organisation observed). Using the ATD framework the following research questions were thus set up: What types of mathematical problems do workers solve at their workplaces? What techniques do they use to solve their mathematical problems? How are the techniques used justified?

THE EMPIRICAL STUDY

Method

In this interview study the data-collection was performed by the first author who is familiar to the field. Four workers from the three workplace settings volunteered to participate in the study, a female restaurant owner, a male constructor and two male taxi drivers. Three visits were conducted to each workplace. The purpose of the first visit was to inform the participants why and how he wanted them to be involved in the research. On this occasion, they agreed that he was permitted to observe and interview them about the use of mathematics in their daily activities. On the second occasion, after three weeks, the purpose was to observe and conduct the first semi-structured interview in order to understand how mathematics helps the workers to achieve their goals in their respective work sites. Three months later, a third visit was conducted to strengthen the understanding of the mathematical organisations. On that occasion, supplementary semi-structured interviews and observations were conducted. The interviews were performed in Kinyarwanda, a common language to all involved parties. Field notes were taken and interviews were tape recorded and transcribed at all visits. In the analysis we have used ideas from activity theory in which we draw on the object of activity to elucidate mathematics as one among the involved mediating tools in the activity. The analysis does not encompass the whole activity system; rather it focuses on the subsystem of production. The reason is that the purpose of the study is specifically to shed light on mathematics as a tool to help the participants to achieve their outcome. This part of the analysis illuminates the mathematical problems that are embedded in the workers' activity. Regarding how mathematics is used by workers on workplaces, the analysis draws on ideas of ATD, especially on its notion of mathematical organisation (MO). To perform this analysis we will build on a reference MO (Bosch & Gascon, 2006, p. 57), based on our own knowledge of academic and applied mathematics, in order to be able to analyse the observed MO in the workplace settings and on the interview data.

Findings

Due to space limitations detailed data on the observed mathematical organisations will be reported only from the taxi driving workplace. We will provide knowledge about the mathematical basis they use to determine the estimated transport fee charged to the customer. The taxi driving profession in Rwanda is mostly exercised by citizens with limited school background. The majority of taxi drivers consider the driving license as their core means of generating income. Some of them drive their own cars whereas others are employed. Taxi driving is mostly done in towns where you find financially potential people able to use taxi as a means of transport. Rwanda has not yet any explicit policy or norms and regulations that taxi drivers should follow to charge their customers. Because of lack of taximeters in the cars, the cost is negotiated between the taxi driver and the costumer.

From the transcripts of the interviews conducted with two taxi drivers, an employed (A) and a car owner (B), their main concern seems to be a non fixed level of profit and to avoid the risk of loss. Due to the difficulty of determining the number of customers every day, the estimation of costs depends mainly of considering control of factors such as road condition (good/bad), trip distance (in kilometres), quantity of petrol that the car consumes for a given trip (measured by money spent), waiting time (if necessary), and the time of the day (different day and night tariffs). Following an agreement between driver A and the employer, A was not responsible for expenses such as taxes, insurance, spare parts and so on. Also, A and his employer had agreed that A must deposit 5000 Frw every day to B and A's monthly salary was 30000 Frw. When the drivers were asked about their mathematical reasoning process while estimating costs, they always referred to authentic examples like pre-fixed estimations and rounded numbers without detailed calculations. In the interview, A gives an example of how he calculated the costs for a trip Kigali – Butare on a high quality tarmac road.

Interviewer: Ok.. let's take an example. Has it happened to you that you have taken a client from here [Kigali] to Butare?

Driver A: Yes, many times.

Interviewer: Could you explain to me how you have estimated the price?

Driver A: A one way of that trip is about 120 kilometres. The estimated cost for that trip was 30000 Frw. It means that I considered the cost of the petrol about 12000 Frw and I remained with 18000 Frw …

But sometimes it happens that while I am on my way of returning back, I meet customers and depending on how we negotiate the cost I charge him

3000 or 5000, it depends … But when estimating the price with the customer before the departure, I ignore this case because there is no guarantee to have this chance

This extract shows that the estimation of cost was made with respect to the cost of petrol and the driver's profit only. Road conditions were probably not mentioned as both interviewer and interviewee were assumed to be familiar with it. Transports between Kigali and Butare are frequent as contacts between the National University in Butare and official administrators or foreign aid agencies and others in Kigali take place on a daily basis. The next example is taken from a less frequented distance.

Interviewer: OK. Ok let's take the case of a Kigali – Bugesera trip. Although the road is now becoming macadamized it was always used as a non macadamized road. How much do you estimate for instance when you bring somebody there?

Driver A: …distance is almost 50 kilometres…then the return trip is 100 kilometres. But because of the poor road conditions, the cost is estimated at 15000 Frw. In that case I assume that the car is going to consume petrol for 5000 and I remain with 10000.

In the above extract, the estimation of the trip cost was made according to road condition, cost of petrol and the driver's profit. A seems to assume that more petrol is needed if the road is of bad standard but looking at Example 1 the same unit (10 km for 500Frw) is used. However, in Example 2 the driver does not seem to expect to be able to pick up a new passenger for the return trip.

In the second interview with B, the owner of the taxi, he explains how he estimates costs in relation to distance, price of petrol and time.

Interviewer: Let me ask you one explanation… for example when you charge a customer a cost of 1500Frw … what is your basis for that price?

Driver B: Do you remember I told you that with the petrol of 1000 Frw, I usually go 20 kilometres? Now when the customer tells me the destination I start to think of the number of kilometres to reach there. Then you say this time one litre of petrol costs for example 550 Frw… Approximately my car consumes 50 Frw to go one kilometre. This means that to go a distance which is not more than 10 kilometres for a return trip my car uses 500 Frw. So if I transport the customer to that destination without any waiting time I should have 1000 Frw for a work time less than 20 minutes... Do you get my point?

Like driver A, B calculates with rounded thirds, one third for petrol, one third for time spent and one third as a profit. As he is the car owner he could also have calculated with taxes and other costs involved with keeping a car.

Analysis of the observed MO

To characterise the MO observed in this taxi driving workplace setting, the type of problems involved could be described as varying versions of calculating the value of a function symbolically written as $W = F(x, y, z, t) + P$, where W is the estimated cost that the driver suggests to the customer. This cost consists of a non-fixed profit P and a cost F for the driver, estimated from all or a few of the four variables road condition (x), covered distance (y), petrol consumption (z) and time (t). Referring to the examples shown above, in the case of waiting for the customer the problem simplifies to $W = F(t) + P$, while the case with a short distance on a bad road will increase both the time and petrol needed: $W = F(z(t(x))) + P$. When the road is good but the distance longer it is the distance which is the deciding variable, $W = F(z(t(y))) + P$, which in the case of also a bad road changes to $W = F(z(t(x, y))) + P$. The techniques used by the drivers to solve these different types of problems are based on rounded estimations of basic costs, without providing a rationale of the amounts mentioned, and when needed elementary arithmetic operations are performed on these rounded numbers. For example, for the Kigali-Butare trip the model $W = F(z(t(y))) + P$ was used, with $y = 2 \times 120\,\text{km}$ and $W = 30000\,\text{Frw}$ with $z = 12000\,\text{Frw}$ and $P = 18000\,\text{Frw}$. In the case of the Kigali-Bugesera trip the road was not macadamized and thus in a bad condition and the model $W = F(z(t(x))) + P$ was applied, where $W = 15000\,\text{Frw}$ and $P = 10000\,\text{Frw}$ with $y = 2 \times 50\,\text{km}$. Technologies included number facts of addition and subtraction of natural numbers, and simple multiplication facts such as doubling. All numbers used were contextualised with units of distance and currency and no justification of the mathematical techniques used was referred to. Rather, it could be described as silent knowledge, adopted by experience and exchange with colleagues.

CONCLUSIONS

In Rwandan society as well as elsewhere in the world, the utility of mathematics is recognized through several activities. Those activities are seen on the one hand in academic institutions such as in schools and universities, where mathematics is used and learned for the purpose of developing knowledge about the subject per se; and on the other hand at different workplaces, where mathematics is used as a mediating tool to facilitate production within the workplace. The present study is partly an answer to policy departments' demands for a more contextualized mathematics education with a move away from using pseudo-problems to more culturally adapted problems. However, one aim is also to meet a theoretical challenge that attempts to combine sociocultural theories with Chevallard's anthropological theory of didactics. The latter makes possible an analysis of the observed knowledge organisation of workplace mathematics (in this case of taxi driving in Rwanda) that deepens the understanding of the purpose and function for the worker of using mathematics.

In the current study our focus was on taxi driving. A pre-determined common object for the drivers was to avoid any risk of loss while generating their income. The taxi drivers chose an appropriate mathematical organisation (MO) among other tools to mediate their activities, as described above. The observed techniques used by the subjects build on basic arithmetic related to addition and subtraction. Taken-for-granted cultural knowledge is seen in the example when the drivers request a higher profit for the distance Kigali – Butare as most local people travel this distance by frequently running minibuses. Taxis are for those who can pay. For community members the return fee to Kigali is subject to negotiation.

The way in which elementary arithmetic is applied should be understood in the context of continuous control of changing situational and cultural factors which make up a fundamental basis for the drivers' success. The observed MO is characterised by techniques which are functional to the problems at hand, the cultural constraints and the educational background of the drivers. As long as they are pragmatic for the goals of the activity, no further justification of the techniques is needed, resulting in a MO with undeveloped logos. This is reflected in the evident fact the drivers' goal is not to develop knowledge in the discipline of mathematics. What is functional at workplaces may in some cases be less functional in an educational context, where levels of justification often play an important role. However, these sets of constraints will form a background to the series of didactic transpositions that will occur before workplace mathematics can be used to contextualise school mathematics. This is a challenge for continuing research in this field. Moreover, the documentation of constraints and possibilities with which taxi drivers operate contribute to the ecology of mathematical and didactical praxeologies.

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