# CERME 6 – WORKING GROUP 7

## TECHNOLOGIES AND RESOURCES IN MATHEMATICAL EDUCATION

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# INTRODUCTION

# TECHNOLOGIES AND RESOURCES IN MATHEMATICAL EDUCATION

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## INTRODUCTION

Technologies in mathematical education has been a theme present at CERME from the first edition. The available technologies have evolved a lot during these years. At CERME 5 conference, the conclusions of the technology Working Group (Kynigos et al. 2007), as well as Artigue's and Ruthven's interventions (Artigue 2007, Ruthven 2007), signal perspective evolutions towards more comprehensive studies, in several respects. Drawing on these previous works, CERME 6 WG7 intended to go further in the directions they have indicated.

An important issue, accounting for the introduction of the word "resources" in the name of a group which was previously called "tools and technologies in mathematical didactics", is the need for considering technologies within a range of resources available for the students, the teachers, teacher's trainers etc. These agents can draw on software, computers, interactive whiteboards, online resources, but also on more traditional geometry tools, textbooks, etc. Various kinds of digital material are now extensively used, and they can be viewed as belonging to a wider set of curriculum material (Remillard 2005) and teaching resources (Adler 2000). The papers in WG7 concern different kinds of resources, still with a specific focus on digital material. Another specific focus of WG7 is on theoretical approaches. Design issues need to focus on integration and impact, especially in the use of innovative technology. This entails the development of approaches framing research on fidelity, efficacy, and effective integration (Hegedus & Lesh, 2008). These approaches have been discussed in the group, and several issues linked with the articulation between research and development have been raised, as it is presented below.

The work in the group was organized into three parallel sessions, corresponding to three themes summarized below; specific slots were devoted to the presentation of the work done within three projects co-funded by the European Community, whose participants are represented in WG7: the Telma European Research Team, the Remath project, and the Intergeo project. The whole group was nevertheless gathered for the first session, with two important activities: the identification of questions considered important for the group's work by the participants (figure 1); and the "plenary" address of Jean-Baptiste Lagrange on the results of the ICMI17 study (Hoyles & Lagrange, to appear). The following trends in research and salient elements presented by Jean-Baptiste Lagrange were extensively present in the group's discussions:

- The integration and synthesis of previously fragmented theories and the development of broad approaches;
- The consideration of the design of tools and curricula as a major issue for mathematical education;
- The development of teacher-oriented research studies with a specific consideration on methodological issues such as the consideration of "ordinary" teachers by researchers

1. The word "dynamic" permeates mathematics teaching and learning activities which involve technology. Where and when is it appropriate?
2. How is it possible to restructure the maths curriculum to take advantage of new technologies to generate mathematical thinking?
3. How can we assess the applicability and the effectiveness of current theories?
4. Do we need specific theoretical tools or approaches to study the different ways in which teaching can be carried out using technologies?
5. What kind of professional development could support pre/in-service teachers to integrate new technologies in their classroom practice?
6. How can “old” and “new” resources interact each other? For example, how is it possible to incorporate e-technologies in textbooks?
7. If we take seriously into account a semiotic perspective considering the evolution of ICT, what new is offered in terms of creative power of semiotic means?
8. Can the European projects presented at WG7 contribute to create a “general theory” of teaching and learning with ICT which can be useful in different European countries?

Figure 1. Examples of questions raised by WG7 participants

## THEMES AND PROJECTS IN WG7

*Design, articulation of design and use*

The work within the « design » theme extensively dealt with the link between design and learning. In particular, the question on the way in which mathematical knowledge can be modified according to different environments was raised. Changes induced by vizualization (Ladel & Kortenkamp, Lois & Milevicich, Kortenkamp & Rolka), virtual reality and simulations (Dana-Picard et al., Bessot & Laborde) opportunities were discussed. Such changes are linked with the specific software tools considered and related features and their analysis is central for the design process. Tools can modify knowledge and learning as well, as it clearly happens for outdoor activities (Nilsson et al.). A broad view on the technology involved, but also on the appropriate associated mathematical tasks is essential (Buteau & Muller, Diakoumopoulos). Amongst the possibilities offered by technology and likely to affect learning and mathematical knowledge, the collective dimensions deserve a specific attention. Software connectivity features, enabling students' collaboration, modify their participation in the mathematical work (Geraniou et al., Hegedus et al.). Questioning collective dimensions also includes a focus on the link between designers and users, and the possible interventions of the users within the design process. This is one of the aspects tackled by the Inter2geo project that dealt with the interoperability of digital geometry systems in Europe (Kortenkamp et al., Trgalova et al.). Beyond the interoperability and indexation issues, this project produced resources for teachers. These resources were tested by users, and users feed-back was included in the design process, with a quality objective. The question of quality, that is the way to assess and validate design and use of technology, was central in the group's discussions. There is still a need for methods to evaluate efficacy of a given technology as far as the learning, engagement and motivation of the students. Even if the projects presented within the group were all grounded in research and linked to given theoretical framework, many questions remain open on the way in which research results can operatively contribute to successful design and use of educational technology.

*Technologies, tools and students mathematical activity*

Papers presented under this theme concerned a wide range of tools and technologies: online resources, software tools, as well as more traditional tools, such as textbooks. Amongst these technologies, Computer Algebra Systems were considered in several papers (Artigue & Bardini, Buteau et al., Weigand & Bichler). But even with a specific interest on CAS, the research works presented in WG7 consider in fact new complex artefacts, articulating CAS and graphing tools in particular, and raised the problem of designing resources to scaffold the use of these artefacts.

Another direction in which the papers showed richness and variety is the theoretical frameworks they draw upon. On the one hand, general theories of mathematical education were considered: functional thinking (Hoffkamp), situated learning, activity theory (Fernandes et al., Jacinto et al.), theory of didactic situations (Aldon & Durand-Guerrier). These theories were used to enlighten specific aspects of learning with technologies: the idea of functional dependency, the mediation provided by an artefact, the didactic contract. On the other hand, several papers refer to specific theories such as that of instrumental approach (Iranzo & Fortuny, Martigone & Antonini, Rezat). This framework, specially designed to study teaching and learning phenomena involving technology, proposes a genesis perspective on learning with technology. It leads to analyses of learning phenomena in terms of schemes. In WG7, precise classification of schemes and of operational invariants were discussed. Such discussions can contribute to a further progress of the instrumental approach framework.

*Interactions between resources and teachers' professional practice*

The acknowledgment of difficulties linked with the integration of technology in classrooms, identified in previous CERME conferences (Kynigos et al. 2007, Drijvers et al. 2006) was still present in CERME 6 WG7 together with the acknowledgment of the key role played by teachers. The need for investigating teachers' beliefs about technology adoption (Chrysostomou & Mousoulides) was recognized as well as the need for conceptualising systemic innovations of educational systems (Ulm). (Emprin, Cantürk-Günhan & Ozen, Faggiano) discussed the importance of setting up pre-service and in-service teachers' training programs taking into account, for in-service teachers, their pre-established repertoires of resources. An evolution from specific studies of individual teachers' practice to investigations of general integration issues was observed, thus moving a step towards theoretical evolutions. As the matter of fact, for example, teacher's use of ICT was examined with a semiotic mediation perspective in (Maracci & Mariotti), while some authors addressed the development of the instrumental approach to study the role of the teacher, drawing on the notion of instrumental orchestration (Trouche 2004), and introducing the consideration of teachers' instrumental genesis (Billington, Bretscher, Drijvers et al.). The acknowledgment of the variety of resources involved in the teacher's activity as well as the need to take into account the whole classroom context, led some authors to develop holistic approaches, such as a documentary approach to didactics (Gueudet & Trouche) and key structuring features of technology integration in the classroom practice (Ruthven).
Delicate methodological issues are attached to the implementation of these theoretical developments, in particular to the question of the "ordinary teacher", which remains open.

*TELMA/Remath projects*
The topic of the articulation of different theoretical frames is central in the TELMA and Remath European projects. In an effort for overcoming the national specificities, these projects developed a cross-experimentation methodology: The key idea around which this methodology was built was the design and the implementation by each team involved in one of these projects of experiments, carried out in real classroom settings, making use of an ICT-based tool developed by another team (Bottino et al., 2009). They also designed meta-tools, in particular scenarios for researchers and for teachers, and proposed developing an integrated theoretical framework (Bottino & Cerulli; Chiappini & Pedemonte; Maracci et al., Markopoulos et al., Moustaki et al., Trgalova & Chaachoua). In fact the three themes of WG7 are present within these large projects, which opened promising methodological and theoretical directions for research.

## CONCLUSION

What do we retain from the work in WG7? The three themes proposed for the contributions, oriented towards design, students and teachers were from the beginning presented as articulated. The design loops integrate more and more the users, students or teachers. The interactions between students and teachers in class are a focus of attention for the researchers. The articulations between different kinds of resources were also extensively discussed, confirming the need for a broad point of view on resources. Research presented in WG7 is focused on technology, but technology does not mean here a precise delimited tool; it includes meta-tools and complex sets of resources. Reflecting on this evolving meaning of technology can be a direction for the work in future CERME conferences.

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# REALISATION OF MERS (MULTIPLE EXTERN REPRESENTATIONS) AND MELRS (MULTIPLE EQUIVALENT LINKED REPRESENTATIONS) IN ELEMENTARY MATHEMATICS SOFTWARE

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*Assumptions of multiple mental representations lead to the presumption of an enhanced mathematical learning, especially of the process of internalization, due to MERs (Ainsworth 1999) and MELRs (Harrop 2003). So far, most educational software for mathematics at the primary level aims to help children to automatize mathematical operations, whereby symbolical representations are dominating. However, what is missing is software and principles for its design that support the process of internalization and the learning of external representations and their meaning themselves – in primary school these are in particular symbols. This paper summarizes the current state of research and presents a prototype that aims to the above-mentioned purpose.*

## INTRODUCTION

In this article we describe the theory and new achievements of a prototypical educational software for primary school arithmetic. After developing the guiding principles that are based on multimedia learning models, we present DOPPELMOPPEL[1], a learning module for doubling, halving and decomposing in first grade.

## THE COGNITIVE THEORY OF MULTIMEDIA LEARNING (CTML)

In the 1970s and 80s it was assumed that comprehension is limited to the processing of categorical knowledge that is represented propositionally. Nowadays, most authors assume the presence of multiple mental representation systems (cp. Engelkamp & Zimmer 2006; Schnotz 2002; Mayer 2005) – mainly because of neuro-psychological research findings. With regard to multimedia learning the Cognitive Theory of Multimedia Learning (CTML) of Mayer is to emphasize (Fig. 1).

[1] see http://kortenkamps.net/material/doppelmoppel for the software

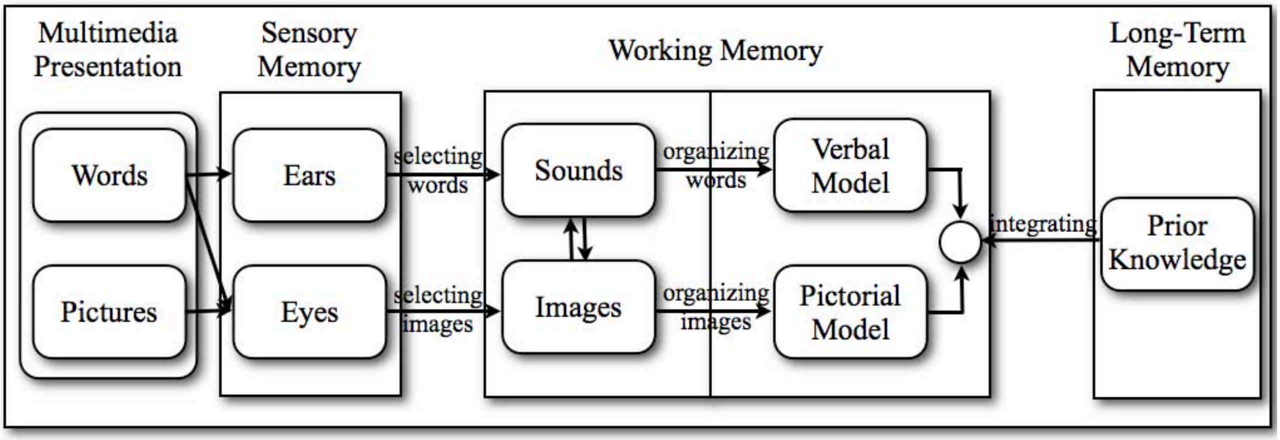


**Figure 1: The Cognitive Theory of Multimedia Learning (CTML) of Mayer**

Mayer (2005) acts on the assumption of two channels, one for visually represented material and one for auditory represented material. The differentiation between the visual/pictorial channel and the auditory/verbal channel is of importance only with respect to the working memory. Here humans are limited in the amount of information that can be processed through each channel at a time. Besides the working memory Mayer assumes two further types: the sensory memory and the long-term memory. Furthermore, according to Mayer humans are actively engaged in cognitive processing. For meaningful learning the learner has to engage in five cognitive processes:

(1) Selecting relevant words for processing in verbal working memory

(2) Selecting relevant images for processing in visual working memory

(3) Organizing selected words into a verbal model

(4) Organizing selected images into a pictorial model

(5) Integrating the verbal and pictorial representations, both with each other and with prior knowledge (Mayer 2005, 38)

Concerning the process of internalization the CTML is of particular importance. The comprehension of a mathematical operation is not developed unless a child has the ability to build mental connections between the different forms of representation. According to Aebli (1987) for that purpose every new and more symbolical extern representation must be connected as closely as possible to the preceding concrete one. This connection takes place on the second stage of the process of mathematical learning where the transfer from concrete acting over more abstract, iconic and particularly static representations to the numeral form takes place (Fig. 2). A chance in the use of computers in primary school is seen in supporting the process of internalization by the use of MELRs. This is the main motivation for the research on how the knowledge about MERs and MELRs in elementary mathematics and educational software is actually used and how it can be used in the future.

## TO THE REALISATION OF MERS AND MELRS IN ELEMENTARY MATHEMATICS SOFTWARE

Despite the fact that computers can be used to link representations very closely, it is hardly made use of in current educational software packages. Software that offers MERs and MELRs with the aim to support the process of internalization is very rare. This is also the reason why tasks are mainly represented in a symbolic form (Fig. 2).

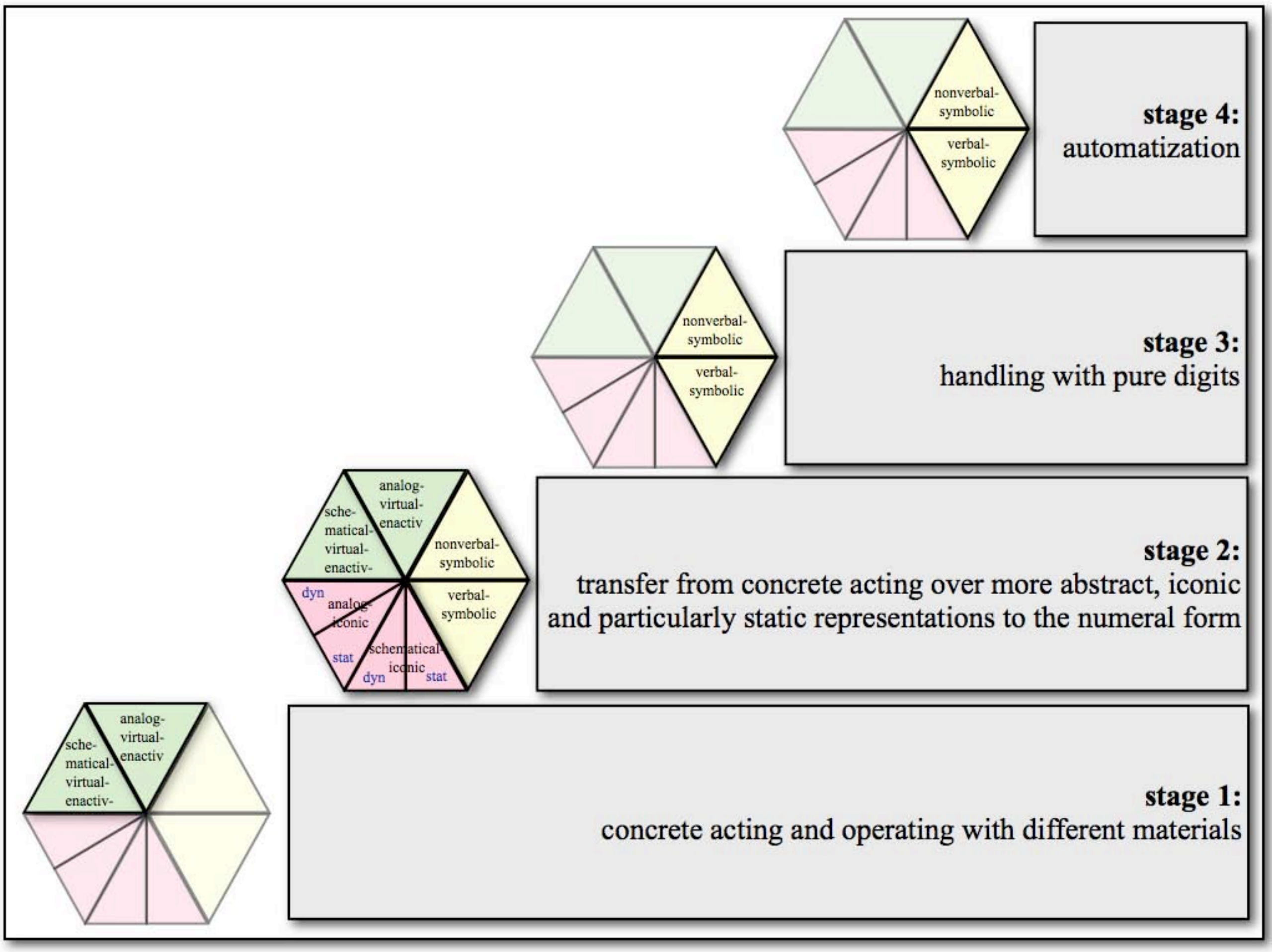


**Figure 2: Forms of external representations combined with the four stages of the process of mathematical learning**

Nevertheless, most software offers *help* in form of visualizations and thereby goes backward to the second stage. This is realised in different ways, which is why a study of current software was done with regard to the following aspects:

- Which forms of external representations are combined (MERs) and how are they designed?
- Does the software offer a linking of equivalent representations (MELRs) and how is the design of these links?

After this analyse, a total of sixty 1st- and 2nd-grade-children at the age of six to eight years were monitored in view of their handling of certain software (BLITZRECHNEN 1/2, MATHEMATIKUS 1/2, FÖRDERPYRAMIDE 1/2). Beside this own exploration – which will not be elaborated at this point - there is only a small number of studies that concentrates on MERs and MELRs on elementary mathematics software. In 1989, Thompson developed a program called BLOCKS MICROWORLD in which he combined Dienes blocks with nonverbal-symbolic information. Intention was the support of the instruction of decimal numeration (kindergarten), the addition, subtraction and division of integers ($1^{st}$ – $4^{th}$ grade) as well as the support of operations with decimal numbers (Thompson 1992, 2). Compared to activities with "real things", there were no physical restrictions in the activities with the virtual objects to denote. Furthermore the program highlighted the effects of chances in the nonverbal-symbolic representation to the virtual-enactive representation and reverse. In his study with twenty $4^{th}$-grade-children Thompson could show that the development of notations has been more meaningful to those students who worked with the computer setting compared to the paper-pencil-setting. The association between symbols and activities was established much better by those children than by the others.

Two further studies that examined multi-representational software for elementary mathematics are by Ainsworth, Bibby and Wood (1997 & 2002). The aim of COPPERS is to provide a better understanding of multiple results in coin problems. Ainsworth et al. could find out, that already six-years-old children do have the ability to use MERs effectively. The aim of the second program CENTS was the support of nine- to twelve-years-old children in learning basic knowledge of skills in successful estimation. There were different types of MERs to work with. In all three test groups a significant enhancement was seen. The knowledge of the representations themselves as well as the mental linking of the representations by the children were a necessary requirement. The fact that a lot of pupils weren't able to connect the iconic with the symbolic representation told Ainsworth et al. (1997, 102) that the translation between two forms of representations must be as transparent as possible.

The opinions about an automatic linking of multiple forms of representations vary very much. Harrop (2003) considers that links between multiple equivalent representations facilitate the transfer and thus lead to an enhanced understanding. However, such an automatic translation is seen very controversial. Notwithstanding this, it is precisely the automatism that presents one of the main roles of new technologies in the process of mathematical learning (cf. Kaput 1989). It states a substantial cognitive advantage that is based on the fact that the cognitive load will be reduced by what the student can concentrate on his activities with the different forms of representations and their effects. An alternative solution between those two extremes – the immediate automatic transfer on the one hand and its non-existence on the other hand – is to make the possibility to get an automatic transfer shown to a decision of the learner.

## PRINCIPLES FOR DESIGNING MERS

The initial point and justification of multimedia learning is the so-called multimedia principle (cf. Mayer 2005, 31). It says that a MER generates a deeper understanding than a single representation in form of a text. The reason for this is rooted in the different conceptual processes for text and pictures. In being so, the kind of the combined design is of essential importance for a successful learning. The compliance of diverse principles can lead to an enhanced cognitive capacity. Thus Ayres & Sweller (2005) could find a *split-attention-effect* if redundant information is represented in two different ways because the learner has to integrate it mentally. For this more working space capacity is required, and this amount could be reduced if the integration were already be done externally. Mayer (2005) diversifies and formulates besides his *spatial contiguity principle* the *temporal contiguity principle*. According to this principle, information has not only to be represented in close adjacency but also close in time. If information is also redundant, the elimination of the redundancy can lead to an enhanced learning (*redundancy-effect*). The *modality principle* unlike the split-attention principle does not integrate two external visual representations but changes one of it into an auditory one. Hence an overload of the visual working memory can be avoided.

In addition to the modality principle Mayer recommends the segmenting principle as well as the pretraining principle to enhance essential processes in multimedia learning. As a result of the *segmenting principle* multimedia information is presented stepwise depending on the user so that the tempo is decelerated. Thus the learner has more time for cognitive processing. The *pretraining principle* states that less cognitive effort will be needed if an eventual overload of the working memory is prevented in advance through the acquisition of previous knowledge. Finally, the abidance of the *signaling principle* allows a deeper learning due to the highlighting of currently essential information. Extraneous material will be ignored so that more cognitive capacity is available and can be used for the essential information.

In elementary instruction the children first of all have to learn the meaning of symbolic representations and how to link them with the corresponding activities. So the above-described principles cannot be adopted one-to-one. Based on an empirical examination of the handling of six- to eight-years-old pupils with MERs and MELRs in chosen software, we could identify new principles and the above-described ones could be adapted, so that their compliance supports the process of internalization. These principles are demonstrated and realized in the following example of the prototype DOPPELMOPPEL.

## THE PROTOTYPE DOPPELMOPPEL

### Didactical concept and tools

The function of the ME(L)Rs in DOPPELMOPPEL is the construction of a deeper understanding through abstraction and relations (fig. 3). The prototype was built using the Geometry software Cinderella (Richter-Gebert & Kortenkamp 2006) and can be included into web pages as a Java applet.

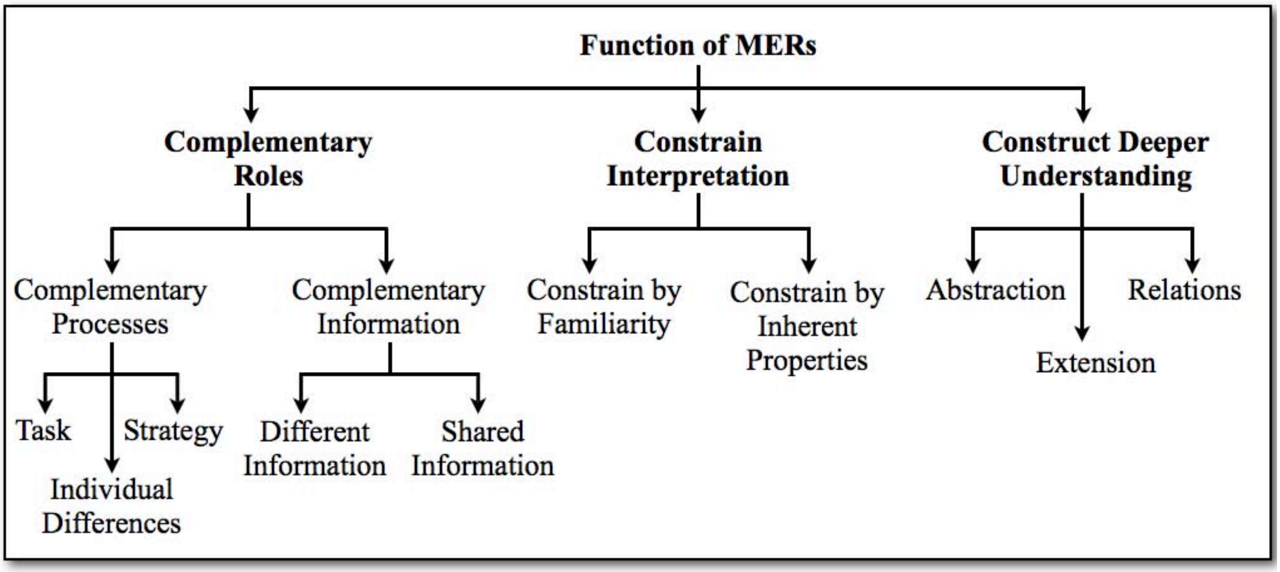


**Figure 3: Functions of MERs according to Ainsworth (1999)**

Using the example of doubling and halving the children shall – in terms of internalization – link their activities with the corresponding nonverbal-symbolic representation and they shall figure out those symbols as a log of their doing. The mathematical topic of doubling and halving was chosen because it is a basic strategy for solving addition and subtraction tasks. In addition, DOPPELMOPPEL offers to do segmentations in common use.

The main concern of the prototype is to offer a manifold choice of forms of representations and their linking in particular (MELRs). Two principles that lead the development are the *constant background principle* and the *constant position principle*. The first one claims a non-alteration of the design of the background but an always-constant one. Furthermore the position of the different forms of representations should always be fixed and visible from the very beginning so that they don't constrict each other.

DOPPELMOPPEL provides the children with the opportunity to work in many different forms of representations. On the one hand there is a zone in which the children can work **virtual-enactive**. Quantities are represented through circular pads in two colours (red and blue). To enable a fast representation (*easy construction principle*) and to avoid "calculating by counting" there are also stacks of five next to the single pads. According to our reading direction the five pads are laid out horizontally. The elimination of pads happens through an intuitive throw-away gesture from the "desk" or, if all should be cleaned, with the aid of the broom button. A total of maximal 100 pads fit on the table (10x10). The possible activities of doubling, halving and segmenting are done via the two tools on the right and the left hand side of the desk (fig. 4).

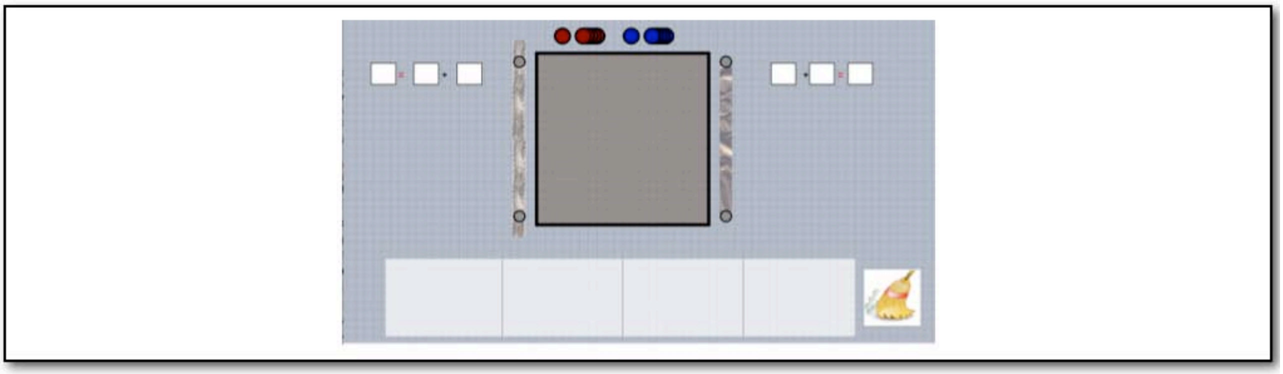

**Figure 4: Screenshot of the prototype DOPPELMOPPEL**

The doubling-tool (to the right) acts like a mirror and doubles the laid quantities. The saw (to the left) divides the pads and moves them apart. Both visualisations are only shown for a short time after clicking on the tools. Afterwards, the children only see the initial situation and have to imagine the final situation (mirrored resp. divided) themselves. The pupils can use the mouse to drag the circular points on the doubling-tool and the saw to move them into any position. A special feature of the saw is that it also can halve pads. At this point the program is responsive to the fact that already six-years-olds know the concept of halves because of the common use in everyday life.

The children can do **nonverbal-symbolic** inputs themselves in the two tables on the right and the left hand side. The left table enables inputs in the form _=_+_, the right one in the form _+_=_. The table on the right is only intended for doubling and halving tasks. That’s why the respectively other summand appears automatically after the input of one. In the table on the left any addition task can be entered.

If the pupils don’t fill in the equation completely they have the possibility to get their input shown in a **schematic-iconic** representation. Depending on the entered figures, the pads appears in that way that the children can’t read the solution directly by means of their colour. The doubling-tool respectively the saw are placed according to the equation so that the children – like in the virtual-enactive representation – are able to act with the tools (fig. 5).

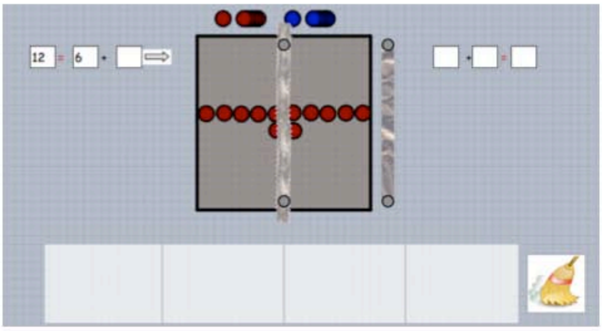


**Figure 5: Schematic-iconic representation of a task**

According to the *signaling principle* an arrow is highlighted when the pupils enter numbers in the free boxes. A click on this arrow initiates the **intermodal transfer**. A similar arrow appears below the desk after every activity done by the children (click on the doubling-tool respectively the saw). Here, the pupils have the possibility to let the software perform the intermodal transfer from the virtual-enactive and the schematic-iconic representation to the nonverbal-symbolic one. This is another special feature of DOPPELMOPPEL that is rarely found in current educational software. If external representations are linked, the linking is mostly restricted to the contrary direction. Depending on the activity the equation appears again in the form _=_+_ or _+_=_. Those equations aren't separated consciously, however a coloured differentiation of the equal and the addition sign (as in the tables above) point to pay attention.

Besides the forms of representations there are two more functions available. Both – the broom to clean the desk and the exclamation mark for checking answers – take some time in order to encourage considerate working and to avoid a trial-and-error-effect. If the equation is false the program differentiates on the type of error. In case of an off-by-one answer or other minor mistake the boxes are coloured orange otherwise red. If the equation is correct a new box appears below.

This prototype doesn't already respond to *modalities* but the concept already incorporates auditory elements.

## Testing of DOPPELMOPPEL

For the testing of DOPPELMOPPEL four versions of the prototype were created. Two of those feature multiple representations; the other two only offer single representations. One of the multiple representations provides an additional linking, that is an intermodal transfer in both directions (fig. 6).

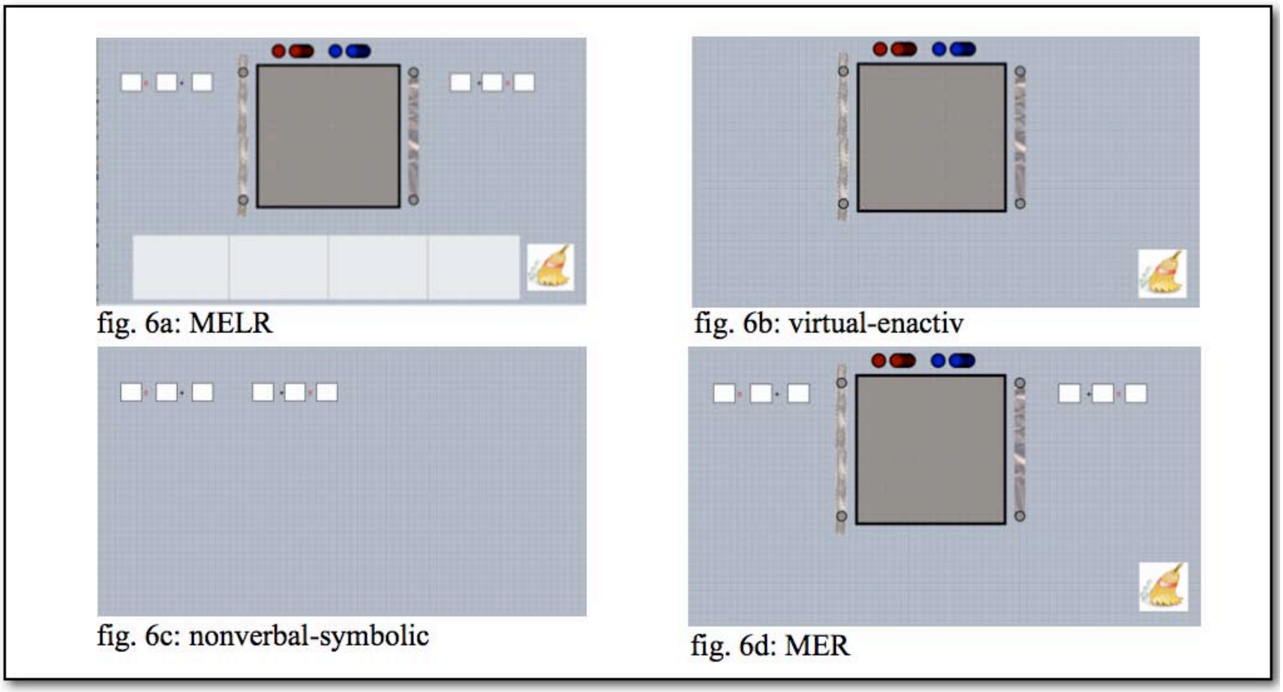


**Figure 6: 4 versions of the prototype**

The dedication of those four versions is to make sure that it is neither the medium computer nor the method of instruction that causes results of the testing.

28 pupils of a 1st class worked about 20 minutes per five terms with the program. During their work there was one student assistant who observed and took care of two children. In addition, the activities of the children were recorded with a screencorder-software. Furthermore a pre- and a posttest were done.

To the current point of time the data interpretation is still in progress but first results should be available to the end of January.

CONCLUSION

Educational software that is based on the primacy of educational theory, as claimed by Krauthausen and others, has to take both mathematics and multimedia theory into account. Carefully crafted software however, is very expensive in production. We hope to be able to show with our prototype that this investment is justified.

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# THE IMPACT OF TECHNOLOGICAL TOOLS IN THE TEACHING AND LEARNING OF INTEGRAL CALCULUS

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*There is still a tendency to see that mathematics is not visual. At University education, it´s evident in several ways. One of them, is an algebraic and reductionist approach to the teaching of calculus.*

*In order to improve educational practices, we designed an empirical research for the teaching and learning of integral calculus whith technological tools as facilitator resources of the process of teaching and learning: the use of predesigned software that enables to get the conceptualization in a visual and numeric way, and the using of a virtual platform for complementary activities and new forms of collaboration between students, and between teachers and students.*

## KEY WORDS

Predesigned software – virtual enviroments – registers of representation - social infrastructure - epistemological infrastructure

## INTRODUCTION

The ideas, concepts and methods of mathematics presents a visual content wealth, which can be geometrically and intuitively represented, and their use is very important, both in the tasks of filing and handling of such concepts and methods, and for the resolution of problems.

Experts have visual images, intuitive way of knowing the concepts and methods of great value and effectiveness in their creative work. Through them, experts are able to relate, most versatile and varied, often very complex, constellation of facts and results of their theory and, through such significant networks, they are able to choose from, so natural and effortless, most effective ways of solving the problems they face (Guzman, 1996). Viewing, in the context of teaching and learning of mathematics at the university, has to do with the ability to create wealthy images that individuals can handle mentally, can pass through different representations of the concept and, if necessary, can provide the mathematic ideas on a paper or computer screen (Duval, 2004). The creative work of mathematicians of all times has had “the visualization” as its main source of inspiration, and this has played an important role in the development of ideas and concepts of the infinitesimal calculus.

However, there is a tendency to believe that mathematics is not visual. At university education, it´s evident, particularly through an algebraic and reductionist approach of the teaching of calculus. One of the didactic phenomena which is considered essential in the teaching of Mathematical Analysis, is the “*algebrización*”, that is: the algebraic treatment of differential and integral calculation. Artigue (in Contreras, 2000) expresses this fact in terms of an algebraic and reductionist approach of the calculation which is based on the algebraic operations with limits, differential and integral calculus, but it treats the thinking and the specific techniques of analysis in a simplistic way, such as the idea of instantaneous rate of change, or the study of the results of these reasons of change.

We believe that the problems with Mathematical Analysis learning, in the first year of college, have to do with this context. These difficulties are associated with the formalism in dealing with the concepts and the lack of association with a geometric approach. Anthony Orton has worked for a long time about the difficulties in learning calculus. His research work at the University of Leeds confirmed that students had difficulty in learning the concepts of calculus: the idea of exchange rate, the notion of a derivative as a limit, the idea of area as the limit of a sum (Orton, 1979). Cornu (1981) arrived at similar conclusions regarding the idea of "unattainable limit" and Schwarzenberger and Tall (1978) regarding the idea of "very near". Ervynck (1981) not only documented the difficulties of the students in understanding the concept of limit but he also remarked the importance of viewing the processes by successive approximations. In this sense, wue can see that usual graphs met in textbooks of calculus have two problems: they are static, which can not convey the dynamic nature of many of the concepts, and also they have a limited number of examples, usually one or two, which leads to develop, in students, a narrow image of the concept in question. (Tall and Sheath, 1983). In this sense, taking into account our previous exploratory research (Milevicich, 2008), we can say that students can not understand the concept of definite integral of a function as the area under the curve, because they do not visualize how to build this area as a sum, usually known as Riemann Sum.

In terms of the educational processes, it should be noted that teachers usually introduce the concept of integral in a narrative way, avoiding the real purpose, which is to obtain more precise approximations. A simplistic approach to the concept is usually done, disconnected from integral calculus applications, which hinders the understanding of students, and consequently, the resolution of problems relating to calculation of areas, length of curves, volume of solids of revolution, and those dealing with applications to the engineering work, pressure, hydrostatic force and center of mass.

## JUSTIFICATION

Innovation in educational processes including th use of multimedia means demands not only on teachers´ professionalism but also new activity managing. Research work is currently being carried out at different universities aiming to find out what use teachers make of these tools and the specific competencies that they have to acquire for making effective use of them. From a didactic point of view, the usage of multimedia in teaching-learning process, presumably, should increase students motivation, and, in that sense, we ask ourselves: What should be the goals of education aimed at improving the university today? and How can we make it easier through the use of technological tools? The answers to these questions are not clear for us. Students, nowadays, have more and more information than they can process, so that one of the functions of the university education would be to provide them with cognitive and conceptual tools, to help them to select the most relevant information.

University Students should try to get skill and develop attitudes that enable them to select, process, analyse and draw conclusions. This change in the goals represents a departure from traditional learning. In this sense, the use of a predesigned software in the classroom, designed within the group research, can be a teaching facilitator resource of the process of teaching and learning:

- to convey the dynamic nature of a concept from the visualization,
- to coordinate different registers of representation of a concept,
- for the creation of personalized media best suited to the pedagogical requirements of the proposal.

## RESEARCH CHARACTERISTICS

### Population and sample

The population is made up of Engineering students from Technological University and the specimen is a Electrical Engineering commission of about 30 students. Regarding the characteristics of the population, some considerations can be made about their previous knowledge of integral calculus. Some students come from the Mechanic School of a known automotive Company and others, from a technical electricians school. Based on a detailed analysis of library materials used by teachers in these institutions, and the students' writings, we infer that integrals are taught as the reverse process of derivation, with the focus on the algebraic aspects. These students study the concept of integral associated with a primitive, practice various methods of integration, transcribe or solve hundreds of exercises in order to calculate integrals, and some of them even achieve a considerable level of skill in the use of tricks and recipes that help to be more effective in getting results. Another group of students come from near schools where geometric concepts are little, essentially the calculating of areas studied during primary and middle school. However, the largest group, is made up of students studying Mathematical Analysis for the second or third time. Some of them have completed the course in previous years but failed in the exams. It may be that those students have some ideas about integral calculus and its applications, or not. It is possible that those ideas interfere with the getting of new knowledge or hinder it (Bachelard, 1938), primarily on those students who associate the integral exclusively to algebraic processes. That is why it was very important to carry out a diagnostic test (pretest) that would allow exploration on the previous skills and students ideas about definite integral and thus, categorize according to the following levels of the independent variable:

Level 1: associate the concept of integral to the primitive of a function and calculates easy integrals.

Level 2: associate the concept of integral to the primitive of a function, calculates easy integrals and links the concept with the area under the curve.

Level 3: associate the concept of integral to the primitive of a function and links the concept with the area under the curve.

Level 4: has no specific pre knowledge associated with the topic.

**Focus**

*The general purposes of our research work were:*

*to determine* if students understand the concept of integral through the implementation of a proposal that would allow its teaching in a approaching process, using different systems of representation, according to the processes man has followed in his establishment of mathematical ideas,

*to analyze,* in a reflective learning context, the ways in which students solve problems related to integral calculus,

*and the specific purposes were:*

*to categorize* the students, involved in the experience, according to his integral calculation preconceptions, at the beginning of the intervention,

*to implement* a proposal that provided, on the one hand, the use of different systems of representation in the development of individual and group activities, and on the other, to promote conjeturación, experiment, formalization, demonstration, synthesis, categorization, retrospective analysis , extrapolation and argumentation, with the help of specific software, and feedback on students' early productions so they could reflect on their own mistakes,

*to review* progress achieved after the implementation of the didactic proposal, to *analyze* the impact of using a virtual platform for complementary activities.

**Methodology**

The design is pre-experimental type of pretest - treatment - postest with a single group. The independent variables in this study are: the design of teaching and pre knowledge of students on the definite integral. The dependent variable is: the academic performance.

Regarding these previous knowledge, a pretest at the beginning of the intervention allowed to place each student in one of the preset categories. After 8 weeks of intervention, a postest allowed to determine the levels of progress made in learning the concepts of integral calculus in relation to the results obtained in the past three years cohorts (2003, 2004 and 2005). In addition, an interview at the end of the experience was implemented, in order to gather qualitative information.

In order to improve educational practices, we designed a proposal for teaching and learning integral calculus according to the proposal of using a pre designed software as indicated in the goals. In this sense:

We designed a software package allowing the boarding of integral calculus from the concept of definite integral associated with the area under the curve, from a geometric point of view.

We selected the problems students should solve, in a way, that their approach would allow to establish a bridge between conceptualization of integration and problems related to engineering. In that sense, the use of the computer allowed to have a very wide range of problems, where the choice was not conditioned by the difficulty of algebraic calculus.

The students used pre designed software for:

a) The successive approximations to the area under a curve, considering left and right points on each of the subintervals. The software allows to select the function, the interval and the number of subdivisions. (See Figure 1).
b) The successive approximations to the area under a curve through the graph of the series which represents the sum of the approach rectangles (See Graphic 1) and the table of values (See Table 1).
c) The visualization of the area between two curves, it also allows to determine the points of intersection.
d) The representation of the solid of revolution on different axes when rotating a predetermined area. (See Figure 2)
e) The numerical and graphical representation (through table of values) of the area under the curve of an improper integral.
It was designed a set of activities with the purpose students conjecture, experience, analyze retrospectively, extrapolate, argue, ask their peers and their teachers, discuss their own mistakes and evaluate their performance. Assessment techniques were redesigned, so that the analysis of students productions would provide feedback about their mistakes.
We incorporated a Virtual Campus using Moodle supporting design, as an additional element, in order to keep continuity between two spaced weekly meetings. According to Misfeldt and Sanne (2007), communication on mathematical issues is difficult using computers and a weekly meeting is insufficient. In response to this problem, we used the virtual campus for communication, flexibility and cooperation, but the use of it was not a learning objective in itself. Instead, we used it to publish texts and exercises guides and also, students made active use of the forum for discussion groups.
We also had in mind that the challenges in creating an online learning environment might be different when working with mathematics than in other topics (see also: Misfeldt et. al, 2007 & Duval, 2006). Many of the signs that goes into building mathematical discourse is not available on a standard keyboard, and the way that mathematical communication often is supported by many registers and modalities that are used simultaneously, as writing and drawing various representations on the blackboard or paper is also not avalilable. Students, using the Virtual Campus, had the possibility to upload files showing the solving process and using every symbol they needed.

## Implementation of the proposal

Students were distributed in small groups no more than three, who worked in several sub-projects. Each of them included a significant number of problems.
Subproject No. 1: The concept of integral.
Subproject No. 2: Fundamental theorem of Calculus.
Subproject No. 3: Improper integrals.
Subproject No. 4: Area between curves.
Subproject No. 5: Applications of Integral Calculus.

Guidelines for systematic work for each of the meetings were made. In the first part, it was discussed the progress and difficulties of the previous practice, where the essential purpose was to ensure that students analyze their own mistakes, and the second part, teachers and students worked on new concepts at the computer laboratory. The first part of each meeting was guided by the teacher, but a assistant teaching and a observer teacher were present in the class. The second half had the same staff and an extra assistant teaching.

The assesment took place during the whole experience through:

- weekly productions of students reflected in their electronic folders and notebooks. These ones allow cells to keep comments, observations, etc.; very valuable material in assessing the level of understanding achieved by students.
- students interaction in classes and into working groups.
- Students participation in the discussion forums of the virtual campus.

In that sense, spreadsheets were used for monitoring activities, which proved to be an effective tool to assess different aspects relevant to student´s performance. Summary notes taken by the observer teacher along the 8 weeks allowed us to infer the change of attitude in an important group of these students. From the initial population, made up of 30 students, 24 of them showed increased commitment to the development of activities.

Some of these activities were:

*Subproject 1: Evaluate the following integrals by interpreting each in terms of areas*

*a)* $\int_1^3 e^x dx$ *b)* $\int_0^3 (x-1)dx$

Case a: because $f(x)=e^x$ is positive the integral represents the area. It ca be calculated as a limit of sums and a computed algebra system can be used to evaluate the expression.

Case b: The integral cannot be interpreted as an area because f takes in both positive and negative values. But students should realize that the difference of areas works.

*Subproject 3:Sketch the region and find its area ( if it is possible)*

a) $S=\{(x,y)/\ 0 \le x \le \pi,\ 0 \le y \le Tan(x)Sec(x)\}$
b) $S=\{(x,y)/\ x \ge 0,\ 0 \le y \le e^{-x^2}\ \}$

Case a: Probably students confuse the integral with an ordinary one. They should warn that there is an asymptote at x= $\pi/2$ and it must be calculated in terms of limits. At this point students must bear in mind that whenever they meet the symbol $\int_a^b f(x)dx$ they must decide, by looking at the function f on [a,b], whether it is an ordinary definite integral or an improper integral.

Case b: The integral is convergent but it cannot be evaluated directly because the antiderivative is not an elementary function. It is important students look for a way to solve the problem and although it is impossible to find the exact value, they can know whether it is convergent or divergent using the Comparition Test for Improper Integrals.

Both examples above show activities where students need to find out solutions and get conclutions without teacher telling them.

## RESULTS

The pretest was done by 30 students, the results allowed us to locate them as follows: 15 at Level 1, 1 at Level 2 and 14 at level 4. It should be noted that those who came from technical schools had achieved a considerable level of skill in the calculation of integrals but they didn´t know about the links with the concept of the area. The postest consisted of 6 problems related to the sub projects students had worked on, each of which was formed by several items. It was provided to the 24 students remaining at the end of the experience, and took place at the computer laboratory, where students usually worked. In general, the level of effectiveness was above 50%, except in the case where they were asked to determine the area between two curves and then the volume to rotate around different axes. The difficulty was to get the solid of revolution from a shift in the rotation axis. Although the students had no difficulty in getting the solid geometrically, they could not get an algebraic expression for it.

In a comparison with the three previous year cohorts, it was possible to emphasize the following differences:

a) There were no important difficulties in linking the concepts of derivative and integral.
b) An important group of students (83% of them) successfully used Fundamental Theorem of Calculus.
c) In general, there were no difficulties in algebraic developments, however it is possible to associate the lack of such obstacles to the use of the computer. All of students tested, could associate the concept of solid revolution with the concept of integral, and even more, they were able to correctly identify the area to rotate.
d) The 74% of the students tested could identify improper integrals, but only 43% of them, correctly, applied the properties.
e) Most of the students tested succeeded in establishing a bridge between the conceptualization of integration and problems related to engineering: 89% of them correctly solved problems relating to applications for work, hydrostatic pressure and force.

The written interviews at the close of the experience reflects the importance that students attribute to the use of virtual campus as an additional resource: most of students were very keen on having prompt responses from the teacher when asking questions in the forum and the help offered by other students.

One of the questions was:

*"How did teachers interventions at the forum helped, when you had difficulties in the development of practices? (A: they were decisive, B: they helped me to understand, C: they were not decisive. I managed without them, D: they did not contribute at all. Please explain your choice)."*

12 students selected A , 8 puplis selected B, 4 students selected C and D was not selected.

Some of the explanations given by students were:

Student a: *"…They helped me because teachers answered quickly and clearly"*

Student b*: "…Excellent, clear and concise answers that helped with the resolution of the problems."*

Student c:*"…There were many situations where I managed to solve a problem just reading the doubts of my fellow students. I have not done a lot of questions at the forum because someone asked my doubt before me…"*

It is worth mentioning that there were no substantial differences between the students belonging to different categories, according to the pretest. An analysis of results in relation to the initial categorization, suggests that pre conditioned ideas did not influenced the acquisition of new knowledge. There were no significant differences among the largest groups of students ranked in levels 1 and 4.

**CONCLUTIONS**

The failure of the students in understanding the concepts of calculation, more generally, and the definite integral, in particular, is one of the most worrying problems in the learning of Mathematical Analysis, in the first year of Engineering, as this hinders the understanding and resolution of problems of application. The way to search for the causality of this failure led us to raise the need for a change in the point of view. This is a change in the processes and representations through which students learn, in this case, the concept of integral.

Focusing our attention on the problem how students can understand more deeply the concepts using tools and technology, we can conclude that the recent evolution of digital materials leads to devote a specific interest to the change of activities induced by virtual learning environments which allow new forms of collaboration between students, and between teachers and students. Besides, the use of the computer is a valuable strategy with the aim of achieving significant learning. While learning the concept of definite integral, the computer facilitates making the important amount of calculations and displays the successive approximations, contributing to the concept of area under the curve. In that sense, the use of a predesigned package software allowed students to view the alignment between the smaller and smaller geometric rectangles and curvilinear area to be determined.

The carrying out of the activities required the use of the predesigned package software, specifically adapted to the needs of the experience. Students had to make numerous graphs, edit their guesses, propose new solutions, test, and analyze retrospectively the achieved results. Dynamic graph was valued for making student work with figures easier, faster and more accurate, and consequently for removing drawing demands which distract them from the key point of a problem. Various aspects of making properties apprehensible to students through dynamic manipulation were expressed in CERME V Plenaries: *"When a dynamic figure is dragged, students can see it changing and see what happens, so that properties become obvious and students see them immediately*" (Ruthven,2007: 56). In that sense, technology is seen as supporting teaching approaches based on guiding students to discover properties for themselves. We agree on suggesting that teachers might guide students towards an intended mathematical conclusion, but students could find out how it works without us telling them so that they could feel they are discovering for themselves and could get a better undestanding.

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# APPENDICES

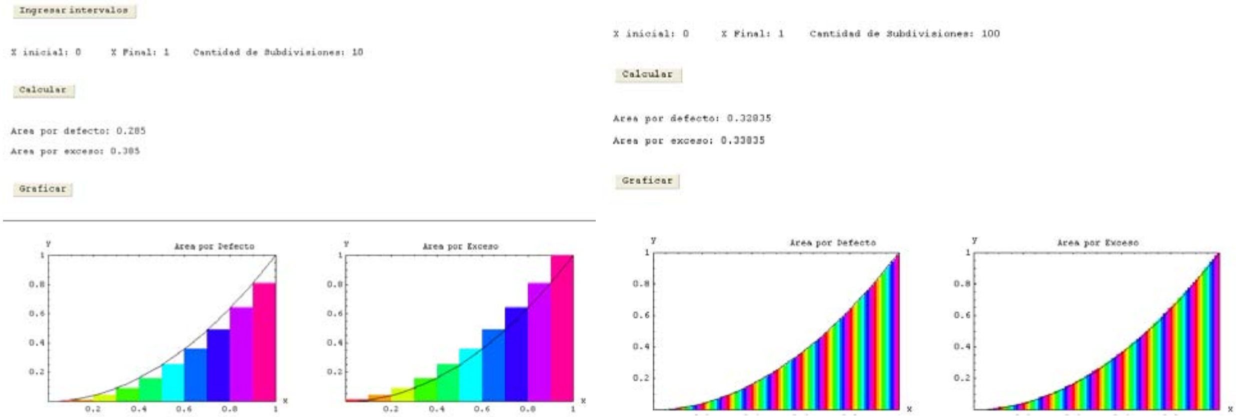
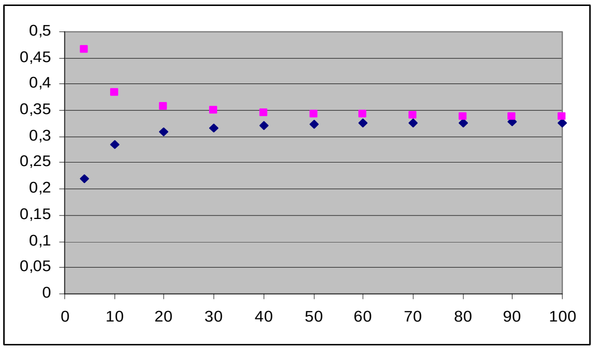



**Figure 1. Capture screen from the predesigned software about conceptualization of definite integral. Estimation of the area of $y=x^2$ using 10 subdivisions and 100 subdivisions, $0 \leq x \leq 1$**

| number of subdivisions | default sums | excess sums |
|---|---|---|
| 4 | 0,219 | 0,467 |
| 10 | 0,285 | 0,385 |
| 20 | 0,308 | 0,358 |
| 30 | 0,316 | 0,35 |
| 40 | 0,321 | 0,346 |
| 50 | 0,323 | 0,343 |
| 60 | 0,325 | 0,342 |
| 70 | 0,326 | 0,34 |
| 80 | 0,327 | 0,339 |
| 90 | 0,328 | 0,339 |
| 100 | 0,327 | 0,337 |

**Table 1. Sums for different subintervals increasingly small under the curve $y= x^2$ on the interval [0,1]**




**Graphic 1. the series which represents the sum of the approach rectangles, default sums are in blue and excess sums are in pink.**

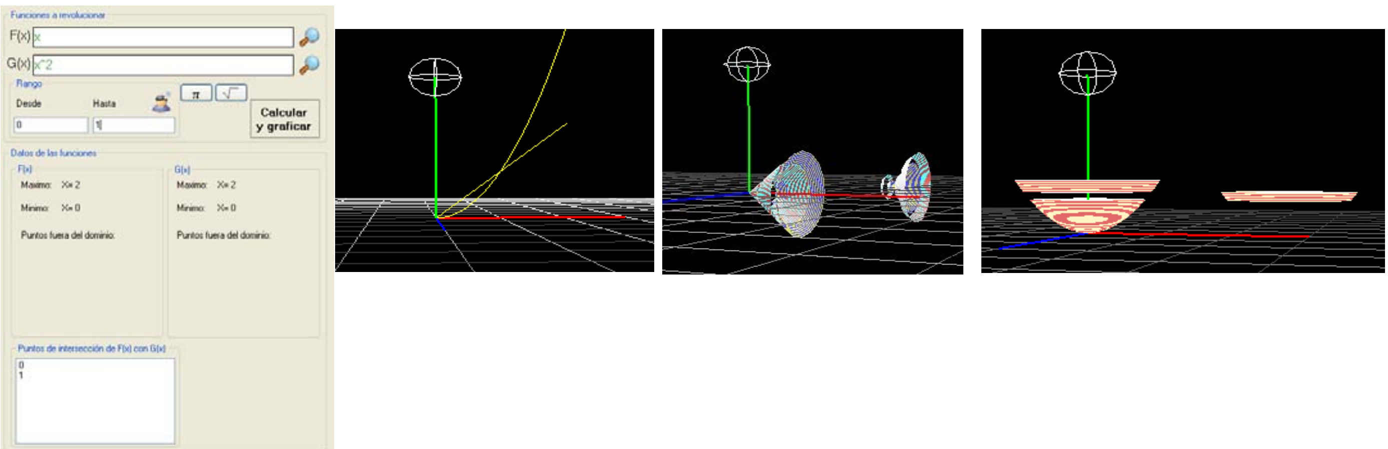



**Figure 3. Captured screen from the predesigned software about Solid of revolution. Area between the functions y=x and $y=x^2$ , and the solid of revolution that is generated to rotate on the x-axis and the y- axis.**

# USING TECHNOLOGY IN THE TEACHING AND LEARNING OF BOX PLOTS

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*Box plots (or box-and-whisker-plots) can be used as a powerful tool for visualising sets of data values. Nevertheless, the information conveyed in the representation of a box plot is restricted to certain aspects. In this paper, we discuss both the potential and limitations of box plots. We also present a design for an empirical study in which the use of a variety of tasks explicitly addresses this duality. The activities used in the study are based on an interactive box plot applet that surpasses the currently available tools and offers new ways of experiencing box plots.*

## MOTIVATION

Recently, the mathematics curricula of many parts of the world were revised in order to include more statistics and data analysis. In the literature, one can find an extensive discussion about this idea under the notion of "statistical literacy" (Wallman, 1993; Watson & Callingham, 2003). This reflects the growing importance of the ability to understand and interpret data that has been collected or is being presented by others. The NCTM (2000) standards, for example, state, "To reason statistically--which is essential to be an informed citizen, employee, and consumer--students need to learn about data analysis and related aspects of probability." The global availability of data through the Internet makes it easy to access and process huge data sets. For these, it is important that students have the skills and tools to summarise and compare the data, also by using the computer.

In this paper, we focus on *box plots* as a means to visualize statistical data. Box plots are used not only in textbooks, but are also available in graphing calculators. In order to use statistical information properly, the students have to develop a clear concept of what the information means, no matter whether it is given numerically or, in this case, visually.

The situation described also applies to Germany where some states have incorporated a larger amount of statistics and data analysis into the mathematics curriculum. Our personal experience with teacher students teaching in 8th grade (14-year-olds) has shown that both teachers and learners tend to ignore the mathematical concepts behind the statistical analysis and fall back to recipes that enable them to solve the standard exercises from the text books. In a similar way, Bakker, Biehler and Konold (2004) point out that some of the features inherent to box plots raise difficulties in young students' understanding and use of them. As a remedy, we developed a series of activities that should enable students to develop a clear understanding of the statistical terms. The ultimate goal of the activities is that students can not only draw box plots for given data, but also interpret box plots that describe real world situations.

## THEORETICAL BACKGROUND

Box plots are part of the field of Exploratory Data Analysis where data is explored with graphical techniques. Exploratory Data Analysis is concerned with uncovering patterns in all kinds of data. A box plot (or box-and-whisker-plot) is a relatively simple way of organizing and displaying numerical data using the following five values: the minimum value, lower quartile[1], median[2], upper quartile, and maximum value. Considering a set of data values like, for example, 52, 32, 29, 30, 35, 17, 42, 63, these five values are easy to calculate: minimum value = 17, lower quartile = 29.5, median = 33.5, upper quartile = 47, and maximum value = 63.

Using these five numbers, the related box plot can be constructed on a vertical (which we use in the following description) or horizontal scale (which is used in Fig. 1) by (a) drawing a box that reaches from the lower quartile to the upper quartile, (b) drawing a horizontal line through the box where the median is located, (c) drawing a vertical line from the lower quartile (the lower end of the box) to the minimum value, (d) drawing a vertical line from the upper quartile (the upper end of the box) to the maximum value, and finally (e) marking minimum and maximum with horizontal lines. Figure 1 shows the box plot corresponding to the data above, created with a box plot applet provided by CSERD.

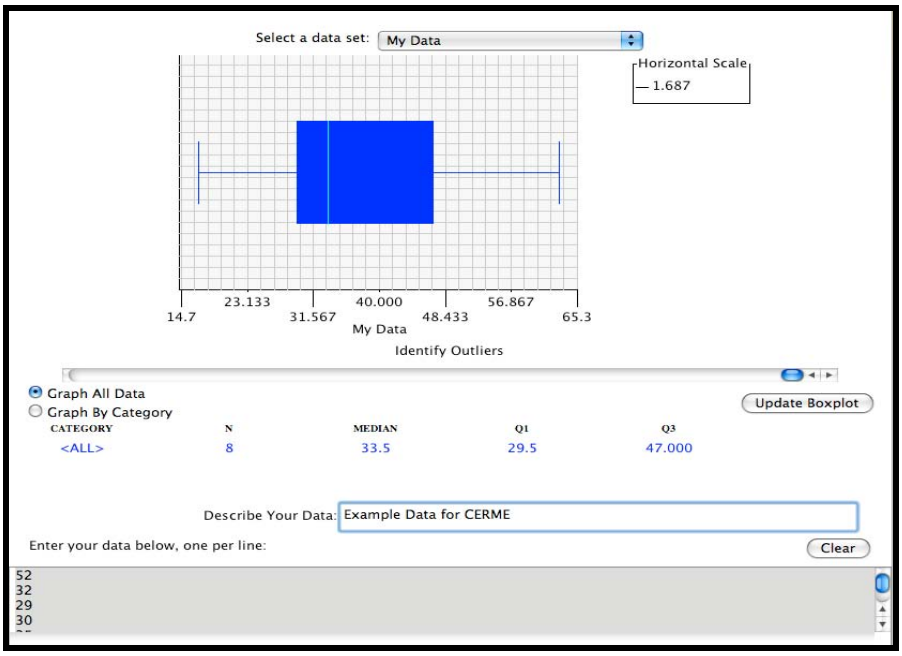



Figure 1: Box plot created online for the sample data in this article

At the same time, box plots contain *more* and *less* information. On the one hand, the representation of a box plot communicates certain information at a glance: The median and the quartiles can easily be recognized which is not the case for the

[1] As there is no universal definition of a quartile, we dedicated a whole subsection of this article to this issue. Also, the original box plot uses the lower and upper hinge instead of the quartiles.

[2] The median can be defined as the number separating the lower half of a data set from the higher half in the sense that at least 50% of the values are smaller than or equal to the median.

original set of data values. Moreover, the line indicating the median illustrates the centre of the data, the width of the box demonstrates the spread of the central half of the data, and the length of the two lines next to the box show the spread of the lower and upper quarters of the data. This enables skilled people to interpret the box plot and draw conclusions about the underlying distribution. Various authors have declared that box plots are particularly useful for easily comparing two or more sets of data values (e.g. Kader & Perry, 1996; Mullenex, 1990). In order to illustrate this idea, compare two data sets where the minimum and maximum values as well as the arithmetic mean are equal and reveal no hint of how to draw conclusions about the values as shown in Figure 2.

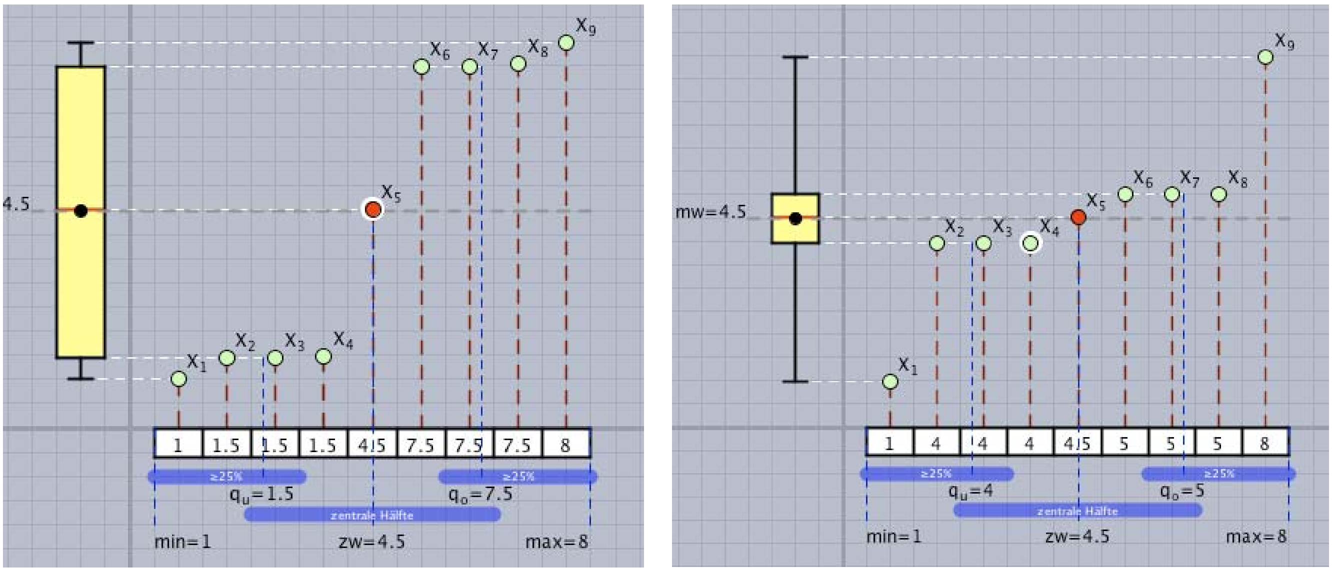



Figure 2: Two box plots with different interquartile ranges

It is obvious that in the second case, the box is much smaller than in the first one, indicating that the spread of the central half of the data is lesser. We use this technique extensively in the exercises that are part of the teaching unit.

On the other hand, the box plot representation is reduced to just five key values and the underlying individual values are not apparent any more – one considerable reason for students' difficulties with this kind of graphical representation (Bakker, Biehler & Konold, 2004). In addition, box plots – compared to many other graphical representations like, for example, histograms – do not display frequencies but rather densities (Bakker, Biehler & Konold, 2004). This means, the smaller a particular area is, the more values are contained in it.

## A Useful Quartile Definition

There is no universal definition of a quartile; actually, there are at least five different definitions in use (Weisstein 2008). The situation is even worse for software packages. According to Hyndman and Fan (1996) even within a single software package several definitions might be used concurrently. A visualization sometimes uses a different definition than a numerical calculation. One reason for this is that the original concept of box plots as introduced by Tukey (1977) used the *hinges* of a data set instead of the quartiles, which are different in one of four cases. Unsurprisingly, the concept of a quartile is obscure to most students and even teachers.

School textbooks in Germany usually do not give an exact definition of quartiles, but combine a colloquial description with a recipe to calculate the quartiles. All definitions are not based on the desired result (i.e., "the first quartile is a value such that at least 25% of the values are less or equal, and at least 75% of the values are greater or equal"), but on a specified way to calculate them (i.e. "the first quartile is the value that is placed at position (n+1)/4 if this is an integer, else…" or similar). Unfortunately, these recipes are incompatible with the QUARTILE function as provided by Excel, which is the most common tool for data analysis in German schools, besides the availability of special purpose educational tools for statistical analysis like, e.g., Fathom (Key Curriculum Press, 2008). The documentation of the QUARTILE function in Excel[3] is similar to the text book definitions of quartiles: it lacks a formal definition or explanation of the desired properties, and focuses on examples instead. It is not possible to explain the results of Excel on that basis.[4]

Most of the critique above only applies to small data sets. With larger amounts of data the actual definition used is not as significant as with less than, say, 20 values. Still, these data sets are the ones that are accessible to hands-on manipulation in the classroom.

For our study, we chose a definition that is both easy to understand and easy to use. A lower quartile[5] of a set of values is a number $q_u$ such that at least 25% of all values are less than or equal to $q_u$, and at least 75% of all values are larger than or equal to $q_u$. In many cases, this number is a value of the data set, but we do not restrict quartiles to be chosen from the values. The definition for the upper quartile $q_o$ is analogous. Using 50% instead of 25% and 75% we can also use it to define the median. All definitions are valid even if some values occur several times.

## Finding the Median and Quartiles

A very useful and action-oriented way to *find* the median and quartiles is the following one:[6] Order all values in increasing order, and write them down in a row of equal-sized boxes. The strip of ordered values may look like this (for 8 values):

[3] We used the German version of Excel 2004 on Mac OS X. There are explanations of the formulas used available, for example, in learn:line NRW at http://www.learn-line.nrw.de/angebote/eda/medio/tipps/excel-quartile.htm. Excel uses a weighted arithmetic mean for the quartiles.

[4] Büchter and Henn (2005) provide a definition of quartiles that is precise and matches the expectation that the lower and upper quartile are the smallest values that cut off at least 25% of the values.

[5] We are using the standard German notation here, instead of $Q_1$ and $Q_3$ for lower and upper quartile.

[6] A student teacher, Simone Seibold, came up with this method during her traineeship in school.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 7 | 14 | 26 | 31 | 33 | 42 |

Now, fold the strip in the middle by lining up the left and right border. The crease will be between 14 and 26, in this example, as is the median. We may use any number between 14 and 26 (not including them), for example the arithmetic mean, 20.

Finding the quartiles works by iterating the procedure described above. Folding the left and right half of the strip will create creases between 4 and 7, yielding a suitable lower quartile of 5.5, and between 31 and 33, which suggests choosing 32 as upper quartile.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 7 | 14 | 26 | 31 | 33 | 42 |

The appeal of this method is that it also applies to situations where the creases pass through the boxes instead of separating two of them (i.e., for odd numbers of values, or if the number is not zero (modulo 4)). In that case, the (only) suitable value for the quartile (resp. median) is the value in that box. The conditions of our definition above are fulfilled automatically.

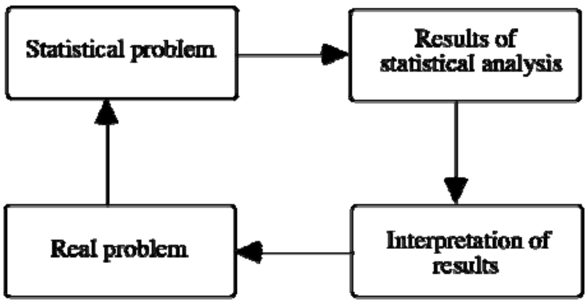
Of course, the method is not suitable for real computations with data sets of significant size, but only for the proper conceptualisation. It can easily be transferred to a formula for the quartile and medians, however.

**Advantages of Using Technology**

Computers are a major reason for the increasing importance of statistics, and vice versa. The whole field of *data mining* became feasible only through the computing power to analyse large sets of data easily. Actually, the first applications of mechanized computing were of statistical natures, for example in the 1890 United States census (Hollerith 1894). In general, multimedia learning bears advantages, in particular if several representations of a situation have to be connected mentally (see Schnotz & Lowe 2003; Cuoco & Curcio 2001). Relating to suitable design for multimedia learning, we refer to the book of Mayer (2003) that details some of the guiding principles. This being said, the existing online tools for creating box plots disregard these principles. Even the online tool that is officially endorsed by the NCTM (see Fig. 1) violates most of these rules. For example, the distant placement of the data entry and the box plot is in clear contradiction to the Spatial Contiguity Principle of Mayer. The quality of interaction is another measure for multimedia learning. The direct interaction with a simulation with *immediate* feedback supports the learner (Raskin 2000). Even if there is no such concept of a "level of interactivity," as it is not a one-dimensional scale, such interaction is considered a key ingredient of good software (Niegemann et al. 2003, Schulmeister 2007). Sedig and Sumner (2006) categorized the possible types of interaction in mathematics software. Again, the activities found on the web so far do not obey these rules.

### Data Cycle

Biehler (1997) suggests a "Cycle of solving real problems with statistics", similar to the typical modelling cycle (Fig. 3 left). However, we suggest that in our case another model is more suited. The typical way to work with data and data analysis in school can be described in a "data cycle" (Fig. 3 right), where data is created by, e.g. measurements in the real world, this data is processed to create a representation of it, the representation can be used for interpretation, and this should be connected to the original data. From top to bottom there is less information (in the information-theoretic sense), but more structure. On the left we work with the real world, that is concretely, on the right we work with a mathematized version of it, that is abstractly.



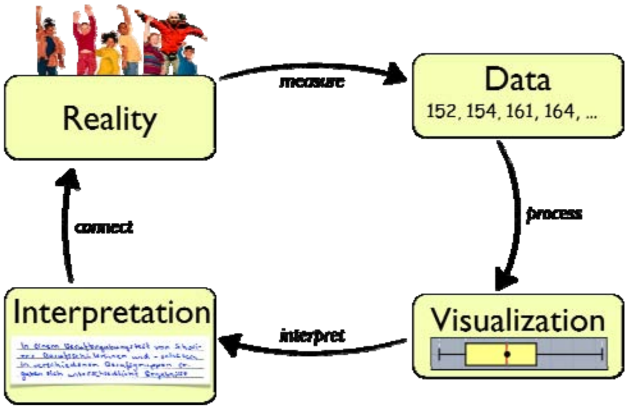




Figure 3: Problem solving cycle by Biehler (1997) on the left, and our proposed data cycle on the right

## DESIGN OF THE ACTIVITIES

The design of the study is used in order to answer our main research question: *To what extent are students able to interpret box plots related to real world situations if they work with them interactively on abstract data sets?* Based on the theoretical analysis given above we therefore designed a set of exercises that enables the students to experience both the power and the restrictions of box plots. In all exercises students use the same interactive applet.[7] The applet is embedded into a plain web page and can be used without prior installations using a standard Internet browser. Using this applet, students can view and manipulate data with up to 22 values (the limit is not due to technical reasons, but given by the screen size). They can add or remove data, change data by dragging the associated data point with the

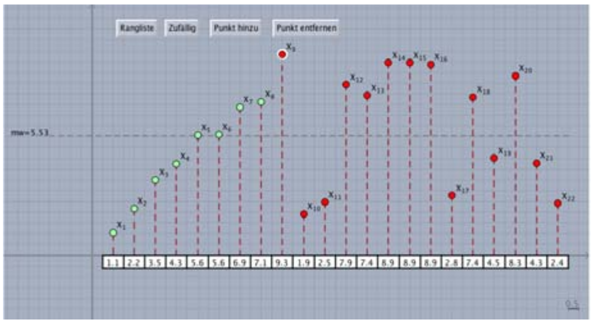
[7] See http://kortenkamps.net/material/stochastik/Quartile.html. The applet is based on Cinderella (Richter-Gebert & Kortenkamp 2006). In our box plot visualization we do not use outliers, as these are not used in the standard textbooks, either.

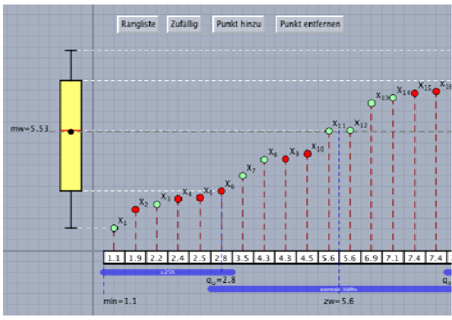
mouse vertically, and re-order values by dragging the points in direction of the *x*-axis. Points that have been added by the students are shown in red, others that were given are depicted in green.

According to Bakker, Biehler and Konold (2004), it is helpful for students if individual cases can be recognized within the box plot representation. This is granted in the applet that we use in our study. All data is visible at all times. While the students are manipulating the data, the current mean value is displayed both numerically and by a dashed horizontal line. The values that correspond to the data points are shown numerically in a white box below each point (Fig. 4 left).

If the values are ordered ascending the applet adds more statistical information to the visualization. To the left of the values the corresponding *box plot* showing the minimum, maximum, quartiles and median, is drawn. Those are connected through dashed lines with the corresponding "creases" and the values that are shown below the data. The blue bars mark the lower and upper quarters of the values as well as the central half (Fig. 4 right).

Figure 4: Applet with unordered values on the left, and ordered values on the right





## Exploratory Exercises

Assuming that the students cannot master the interpretation step if they already fail at processing the data, we designed a set of exercises that aim at connecting the visualized data and the concepts behind them with the original data. Using the applet, students can easily process data dynamically, while modifying it, with an immediate update of the visualization. The exercises focus on modifying data sets in order to change or preserve the measures of variation: (a) Change *only* the arithmetic mean by changing values, (b) Change *only* the minimum or maximum by changing values, (c) Change *only* the length of the whiskers, (d) Change *only* the size of the box (the interquartile range), (e) Add values without changing the box plot, (f) Remove values without changing the box plot, (g) Try to move the arithmetic mean outside of the box, and (h) Try to move the median outside of the box.

Our primary goal is that students understand that box plots are a compact visualization of five (or six, depending on the plot) statistical measures, which in turn describe the distribution of values in a data set. Based on these measures it is possible to draw conclusion about the original set. Students should be able to find as many conclusions as possible, while not over-interpreting the measures. The activities force the students to create data sets that differ only in certain aspects, while showing an interactive visualization of the data and the measures.

For example, while experimenting with (d) students will see that for a distribution with smaller box (i.e. a smaller interquartile range) the values in the central half are more densely distributed than for a distribution with a larger box. Also, common misconceptions like a correspondence between the size of the box and the number of values in the data set are addressed. Adding or removing values does not necessarily change any of the measures of variation.

## SUBJECTS AND METHODS

In line with the recommendations formulated by a group of stochastic educators in Germany (Arbeitskreis Stochastik, 2003), the participants in our study are aged at least 15 years. We conducted preliminary tests with the material in schools in two German states, Baden-Württemberg and North Rhine-Westphalia.

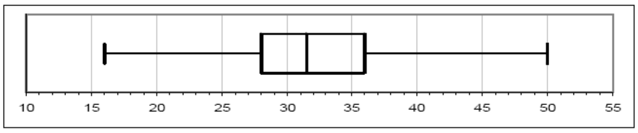
In Baden-Württemberg, we worked with 28 students in grade 9 at the “Realschule” level. They already received some training with box plots, but not with interpretation, in grade 8. In order to let them recall the basics they all received a hand-out about medians, quartiles, and box plots. First, they worked for 20 minutes in pairs with the applet and were asked to answer the exploratory exercises as given in the last paragraph in writing. Next, they were asked to analyze a series of box plots on another (paper) work sheet and interpret them in writing. Their answers were collected for further analysis.

In North Rhine-Westphalia, three students of grade 11 were involved in an interview-like situation where they had the possibility to explore the applet and work on the above presented exercises related to box plots. Beforehand, they had also received a hand-out providing an overview of medians, quartiles, and box plots. Subsequent to the exploration of the applet, they were given two interpretation tasks that they answered in written form.

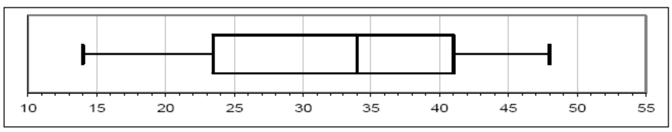
## EXAMPLE OF AN INTERPRETATION TASK

In class 10a, there are 30 students, in class 10b 29. In both classes, the same test was written. The two box plots are based on the scores achieved by the students:

Class 10a




Class 10b




a) Describe as detailed as possible which information you can extract from the two box plots and compare them with each other.

b) Which class wrote the better test? Justify your answer.

c) Give examples for scores of the 30 students from class 10a that fit the given box plot and explain your procedure.

## FIRST RESULTS

We only report on the results from one of the three students who took part in the interview-based exploration of the applet and then answered the interpretation task presented above. At first, the student describes the two box plots by simply listing the five key values respectively. This observation is in line with results reported on in the literature, and also our observations with the other student group in Baden-Württemberg. However, he does not remain at this merely descriptive level and formulates the following statement:

> In class 10a, a good portion of the students are located in the centre, whereas the points in class 10b are more distributed. However, here the higher points are more pronounced.

Being sympathetic to the student's answer, one could conclude that he has understood some basic principles of the box plot representation. However, in order to get more information about his competencies without construing too much, he was later asked by e-mail to clarify this answer. These are his additional explanations:

> The set of students is divided into four parts by the median and the two quartiles. In class 10a, the two middle areas are particularly small. This means that particularly many students are located there. In class 10b, the four areas are about the same size. This means that the students are distributed equally regarding to the score. The rightmost area in class 10b is considerably smaller than the one in class 10a. This means that the students in this area have achieved particularly high scores.

The additional explanations illustrate that the student has mastered some of the difficulties and challenges related to box plots that are described in the literature (Bakker, Biehler & Konold, 2004). He realizes that a box plot consists of four areas that approximately contain 25% of the data respectively. Moreover, he is able to formulate the relationship between the size of the particular areas and the density of the values contained in them.

## CONCLUSION

We agree with the NCTM (2000) standards that students should also be able to create and use graphical representations of data in form of box plots as well as discuss and understand the correspondence between data sets and their graphical representations. The applet presented in this paper and employed in our study does not need any further software packages and therefore provides a basic but powerful tool for students in order to explore the potential and limitations of box plots. The applet is definitely easy to implement in the classroom. However, at the moment we cannot say too much about the effects on the interpretation competencies of the students who worked with the applet in a classroom situation. For the interview-like individual exploration our results show that the work with the applet can support the ability of students to analyze and interpret box plots. Currently, we are concerned with using the promising experiences based on the interview-like situations in order to make the applet also accessible to the work in the classroom.

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# DYNAMICAL EXPLORATION OF TWO-VARIABLE FUNCTIONS USING VIRTUAL REALITY[1]

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*We present the rationale of an ongoing project, aimed at the development of a Virtual Reality assistant learning of limits, continuity, and other properties in multivariable Calculus. The Mathematics for which this development is intended is described briefly, together with the psychological and pedagogical elements of the project. What is Virtual Reality is explained and details are given about its application to the specific field. We emphasize the fact that this new technological device is suitable for self-teaching and individual practice, as well as for the better storing and retrieving of the acquired knowledge, and for identifying its traces whenever it is relevant for further advanced learning.*

## BACKGROUND

### The institution and its pedagogical situation

The Jerusalem College of Technology (JCT) is a High-Tech Engineering School. During the Spring Term of first year, a course in Advanced Calculus is given, mostly devoted to functions of two, three or more real variables. A problem for many students is a low ability to "see" in three-dimensional space, with negative consequences on their conceptualization of notions such as limits, continuity, differentiability. Another bias appears with double and triple integrals, as a good perception of the integration domain is necessary to decide how to use the classical techniques of integration. Sik-Lányi et al. (2003) claim that space perception is not a congenital faculty of human being. They built a Virtual Reality environment for improving space perception among 15-16 years old students. With the same concern we address a particular problem of space perception with older students, using the same digital technology.

Berry and Nyman (2003) show students' problems when switching between symbolic representation and graphical representation of a 1-variable function and of its first derivative. They say that "with the availability of technology (graphical calculators, data logging equipment, computer algebra systems), there is the opportunity to free the student from the drudgery of algebraic manipulation and calculation by supporting the learning of fundamental ideas". Tall (1991) notes that the computer "is able to accept input in a variety of ways, and translate it's flexibly into other modes of representation, including verbal, symbolic, iconic, numerical, procedural. It therefore gives mathematical education the opportunity to adjust the balance between various

[1] The first author is supported by Israel Foundation Grant 1340/05. The development is supported by a grant from the Israel Inter-University Center for e-Learning, number 5768/01.

modes of communication and thought that have previously been biased toward the symbolic and the sequential".

Until now, various technologies have been introduced as a tentative remedy to problems encountered with three-dimensional perception. Nevertheless, problems still remain. Numerous technologies have been introduced for the sake of visualization. Arcavi (2003) classifies the roles of visualization as a) support and illustration of essential symbolic results, b) provider of a possible way of resolving conflicts between (correct) symbolic solutions and (incorrect) intuitions, and c) a help to re-engage with and recover conceptual underpinnings which may easily be bypassed by formal solutions.

In the present paper, we focus on functions of two real variables, plotting and analyzing their graphs, considering especially the b) component in Arcavi's classification. A problem may appear inherent to all kinds of support: a graphical representation may be incorrect, either because of non appropriate choices of the user or because of the constraints of the technology (Dana-Picard et al. 2007). In order to overcome this problem we turn our attention towards another technology: Virtual Reality (VR). This technology is extensively used for training pilots or other professionals. Jang et al. (2007) discuss the usage of VR related to representation of anatomy, clearly a 3D situation too. But as far as the authors know, it has been implemented yet neither for Mathematics Education in general, nor for the Mathematics Education of Engineers. In this paper, we present the rationale for the authors to start the development of a VR assistant to learning Mathematics. We describe an environment where the learner is not passive and has some freedom to choose his/her actions. A VR environment offers cognitive assessment, spatial abilities, executive and dynamical functions which are not present in more traditional environments.

## Representations of a mathematical object

Among the characters articulated in mathematics teaching cognitive aspects:

- Multiple representations of the same objects: textual (i.e. narrative) presentations, literal formulas, graphical representations, tables of numerical values, etc. These presentations may either be redundant or leave empty holes. Note that every presentation has to be accompanied by a narrative presentation for embodying the rule and for the sake of completing the given description of a rule. Mathematics educators generally agree that multiple representations are important for the understanding of the mathematical meaning of a given notion (Sierpinska 1992).
- When using together multiple representations in order to give a concrete appearance of composite consequences of the rule under consideration, it can be necessary to perform a transfer between an abstract concept and concrete representations. For example, Gagatsis et al. (2004) present a hierarchy among the possible representations of a function, calling tables as a prototype for

enabling students to handle symbolic forms, and graphical representations as a prototype for understanding the tabular and verbal forms of functions (for a study of prototypes, see Schwarz and Hershkowitz 1999).

- The more numerous the rule's implications (in Physics, Biology, Engineering, Finance, etc.), the more important is the requirement of creative skills (e.g. interpolations, extrapolations, which the learner will have to apply). Here the teacher will generally try and guide the learner with examples, graphical representations, and animations.
- The more fundamental the rule, the more important for the learner to store it, to internalize it and its consequences for a long duration. This will enable him/her to build more advanced rules. More than that, the learner needs ways to extract the knowledge and to find its traces whenever it is relevant for further learning (Barnett et al, 2005).
- Regarding a mathematical rule with geometrical implications and representations, its complete mastering requires from the learner, according to the Gestalt conception, a permanent transfer from one kind of representation to another kind (see Hartmann and Poffenberger, 2007). On the one hand, it is necessary to understand how a change in the parameters of the rule influences the representation. On the other hand, abstraction skills enable to conjecture the rule from the graphical representation and to modify the parameters in the formula according to the changes in the graphical representation. This is the rationale for the usage of software for dynamical geometry.

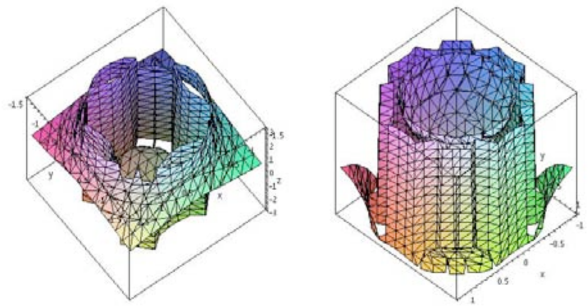
The graphical representation has been made using either Maple 9.5 or the free downloadable software DPgraph (www.dpgraph.com). Because of the dynamic character of a VR device, we do not include screenshots. Suitable presentations can be found at URL: http://ndp.jct.ac.il/companion_files/VR/home.html.

## LIMITATIONS AND CONSTRAINTS ON THE CONVENTIONAL REPRESENTATION TOOLS

Real functions of two real variables may have various representations: symbolic (with an explicit analytic expression $f(x,y)=...$), graphical (the graph of the function, i.e. a surface in 3D-space), numerical (a table of values), not necessary all of them at the same time. This last kind of representation is generally not easy to use in classroom; the plot command of a CAS uses an algorithm which provides numerical data, and the command translates this numerical data into a graphical representation. Generally the higher level command is used, and the user does not ask for a display of the numerical output. The VR device that we develop uses this numerical output to create a *terrain* (a landscape) over which the student will "fly" to discover the specific properties of the function, either isolated or non-isolated singularities, asymptotic behaviour, etc.

It happens that a symbolic expression is unaffordable. This creates a need, central for teaching, for suitable tools to illustrate the function and make it more concrete. An example is given by Maple's **deplot** command for plotting the solution of a Differential Equation without having computed an analytic solution; of course this command uses numerical methods. Within this frame, educators meet frequently obstacles for their students to achieve a profound and complete understanding of the behaviour of such functions. Examples of the limitations have been studied by Kidron and Dana-Picard (2006), Dana-Picard et al. (2007) and others.  The student's understanding of the behaviour of a given function depends on the representations which have been employed.

Dana-Picard et al (2008) show that the choice of coordinates has a great influence on the quality of the plot produced by a Computer Algebra System (CAS).  Compare the plots of $f(x,y)=1/(x^2+y^2-1)$, displayed in Figures 1 and 2. Cartesian coordinates have been used for Figure 1 and polar coordinates for Figure 2. The discontinuity at every point of the unit circle is either not apparent or exaggerated. Moreover Figure 1b shows a kind of waves which should not be there.

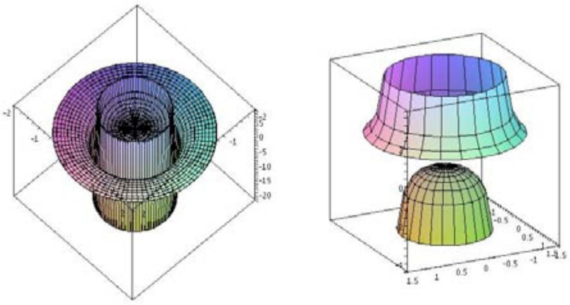


**(a)** **(b)**

**Figure 1: Plots of a 2-variable function, with Cartesian coordinates**

The choice of suitable coordinates is not the sole problem for getting a correct plot. Figure 2a shows that our discussion on "correct coordinates" is not the ultimate issue, and even with these coordinates, other choices influence the accuracy of the graph, whence the student's understanding of the situation. In Figure 2a the discontinuities are totally hidden, as a result of the interpolation grid chosen by the software. This issue is discussed by Zeitoun et al. (2008).

A "wrong" choice of coordinates may *hide* important properties of the function, but may *show* irrelevant problems, whence numerous problems with the figure and its adequacy to the study. A central issue is to decide what "correct coordinates" are and what a "wrong choice" is. It has also an influence on the possible symbolic proof of the properties of the function. A couple of students have been asked why they have hard time with such problems; they answered that the reason is a lack of basic understanding of the behaviour of the represented mathematical object (no matter whether the representation is symbolic, numerical, or graphical). A problem can arise when checking that data of two different kinds actually represent the same function. Experience must be accumulated by the learners.



**(a)** **(b)**

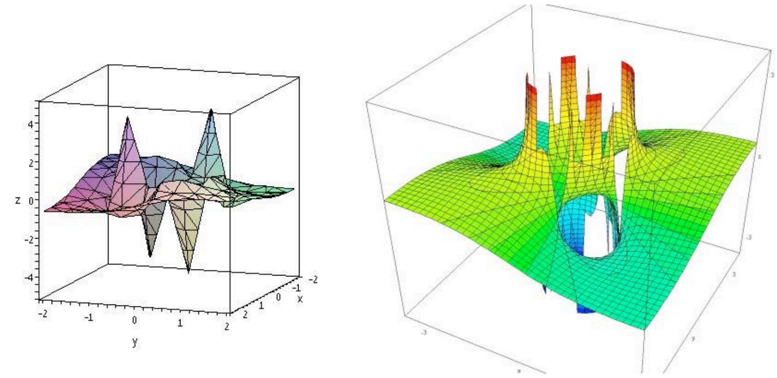
**Figure 2: Plots of a 2-variable function, with polar coordinates**

Moreover, the students may receive a proof of a certain property using an abstract-symbolic representation of the mathematical object under study. Despite the proof's precision, it happens that the student needs a more concrete presentation. In a practice group of 25 students, the teacher chose the function defined by $f(x,y)=1/(x^2+y^2-1)$ and showed plots like those displayed in Figure 1. Two thirds of the students saw immediately that the function has a lot of discontinuities (intuitively, without giving a proof), but could not explain immediately what is wrong with Figure 1.

The graph of a 2-real variable function is a surface in 3-dimensional space. A function of three real variables can be represented by level surfaces. Excepted at certain points, this is the same mathematical situation as before, because of the Implicit Function Theorem. At the beginning of the course, about 70% of our students have problems with surface drawing. A lack of intuition follows, for example concerning the existence of discontinuities. This may incite the student to make successive trials, i.e. to multiply *technical* tasks not always relying on real mathematical *thinking*. Afterwards a symbolic proof is required, and maybe a graphical representation will be needed to give the "final accord".

Graphical features of a Computer Algebra System are used to enhance visual skills of our students, hopefully their manual drawing skills. With higher CAS skills, an animation of level surfaces can help to visualize graphically a 3-variable function. We meet two obstacles:

- The dynamical features of a CAS are somehow limited. In many occurrences, it is possible to program animations, and/or to rotate the plot, but not more.
- A CAS cannot plot the graph of a function in a neighbourhood of a singular point. In this paper we focus on limits and discontinuities. The CAS either does not plot anything near the problematic point (Figure3b) or plots something not so close to the real mathematical situation (Figure 3b: where do these needles come from?). Note that this occurs already with 1-variable functions, but with 2-variable functions the problem is more striking.



**(a) false incomplete plot** **(b) incomplete plot**

**Figure 3: Two Problematic plots for** $f(x,y)=\dfrac{xy}{x^2+y^2-1}$ **.**

# VIRTUAL REALITY

## What's that?

The technology called **Virtual Reality** (VR) is a computer-based physical synthetic environment. It provides the user with an illusion of being inside an environment different from the one he/she is actually. This technology enables the building of a model of a "computerized real world" together with interactive motion inside this world. The VR technology gives the user a feeling that he/she an integral "part of the picture", yielding him/her *Presence*, *Orientation*, and even *Immersion* into the scenario he/she is exposed. After a short time he behaves like it's the real world.

## The goals: VR-concretization and its added value

A CAS is not a cure-all for the lack of mathematical understanding when dealing with discontinuities of multi-variable functions. A more advanced, more dynamical concretization is given by a VR environment. It is an additional support to Mathematics teaching completing the classical computerized environments, beyond the traditional representations (symbolic, tabular-numerical, and graphical). Actually VR provides an integration of computer modes previously separate (Tall 1991):

- Input is not limited to sequential entry of data using a keyboard. Devices such as a joystick are also used.
- A working session and its output mix together the iconic, the graphical and the procedural modes.

When reacting to the student's commands, the VR device computes anew all the parameters of a new view of the situation. The student takes a walk in a landscape which is actually part of the graph of the function he/she studies. At any time, VR simulates only part of the graph, the discontinuity is never reached, but it is possible to get arbitrarily close to it. The VR may provide the student what is missing in his/her 3-dimensional puzzle, by eliminating the white areas appearing in CAS plots, such as Figure 3a. It is intended to provide him/her a real picture of how the function he/she studies behaves.

A VR environment provides compensation to the limitations and the constraints of the imaging devices already in use (CAS and plotters). It presents an image of a real world and gives a direct 3-dimensional perception of this world, as if the user was really located in it. The higher the quality of the VR environment, the more powerful the impression received from this imaginary world's imitation of the real world.

In our starting project, the simulation provided by VR is intended to improve the students' understanding of continuity and discontinuity, and afterwards give also a better understanding of differentiability of a multi-variable function. Among other affordances, the VR simulation cancels problems of discontinuity related to graphs because of its local and dynamical features.

## COGNITIVE CHARACTERISTICS AND SIMULATION FEATURES OF A VR ENVIRONMENT

The final rules may be represented in a concrete fashion by interaction with the environment and by showing to the learner the limitations of the rules, as they appear in a (almost) static environment generally yielded by a CAS. Non graphical representations of functions, such as numerical representations, cannot show continuity and discontinuity. This comes from the discrete nature of these representations, a feature still present in the computerized plots.

The new knowledge afforded by the learner is a consequence of his/her own efforts to explore the situation. His/her ability to change location, to have a walk on the graph, will lead him/her to internalize in a better way the mathematical meaning of continuity and discontinuity. An added value is to help him/her to understand the meaning of changing parameters in the geometric representation. This added value is made possible by the *live experience* of the behaviour of the function, no matter if the transitions are discrete or continuous (according to changes in the variables or in the parameters). The mental ability to feel changes, their sharpness, their acuteness, comes from the immersion into the topography in which the learner moves.

This added value is still more important when the function under study encodes a concrete situation, in Physics, Engineering, Finance, etc. The interactive experience enables the learner to translate the rules to which the function obeys, to find analogues of these rules for other concrete situations. The concrete sensations provided by VR improve the learner's understanding of interpolation and extrapolation, and to translate this understanding into the graphical situation (see also Dana-Picard et al., 2007). The more immersive features of the mathematical knowledge that are incorporated into VR representation for the learner, the faster he/she will find the traces of it whenever it is relevant for further learning. Besides, the more immersive features are incorporated into VR knowledge representation the greater the longevity of preserving the acquired knowledge. This means a slower extinction of it in the memory system (Chen, et.al. 2002).

Interactivity improves the learning experience. Numerous studies show that the more deeply lively experienced the learning process the more internalized its results (Ausburn and Ausburn, 2004; Barnett et al, 2005). The internalization is assessed by an improved conservation of the knowledge, i.e. a slower decrease of the knowledge as a function of elapsed time. Therefore, a Virtual Reality assisted learning process yields a better assimilation of the mathematical notions than with more conventional simulations devices, as it provides this live sensorial experience. This is a more than a realization of the request expressed by a student involved in a research made by Habre (2001); this student wished to be able to rotate surfaces in different directions. A Computer Algebra Systems does this already. VR meets a further requirement of this student, namely to have "a physical model that you can feel in your hands".

According to the brain mapping, the numerical representation of functions is acquired by the left hemisphere of the brain, and the space-live experienced acquisition in a learning process is devoted to the right hemisphere. The transfer from the symbolic rule to a 3D representation and vice-versa requires transfer between two brain lobes with different functionalities. Concerning conceptualization, especially when it must be applied to a concrete domain, there exists a mental difficulty to "move" from one lobe to the other (in terms of longer reacting time, or of completeness of the process). An interactive environment where functional parameter changes are allowed, and where the environment changes can be sensitively experienced, enables a faster building of bridges between the different registers of representation, symbolic, numerical, and graphical.

Finally, the usage of a VR assistant to learning is purely individual. The teacher can show a movie, but it is only an approximation of the requested simulation. The student's senses are involved in the process, the hand on the joystick, the eyes and the ears in the helmet, etc. Therefore the VR device should take in the learning computerized environment a place different from the place of other instruments.

## OUR VR DEVICE AND FUTURE RESEARCH

The digital device described above is now in its final steps of initial development. The user can fly over (or walk along) the terrain, i.e. over the graph of the given function. The details of the graphs, the possible discontinuities, are made more and more visible. This effect is not obtained by regular zooming, as this operation only inflates the size of the cells of the interpolation grid. For new details to appear the data has to be computed anew and only part of the surroundings is displayed.

Furthermore, a VR environment seems to contribute an added value by representing more holistic characteristics of the mathematical knowledge. Among the main contributions are the dynamics or flow traits. A more integrated one is the ability to understand its place in the whole mathematical or physical context it is playing with.

In cognitive terms it means that by VR environment, the teacher should provide to the student a more accurate mental model of the mathematical knowledge, including the applicable images of it (Croasdell et al, 2003).

In particular, the dynamical properties of a VR device and their appeal to various sensitive perceptions (vision, audition, etc.) induce also the need of the integration of the hand into the educative schemes. As Eisenberg (2002) says, the hand is not a peripheral device, but is as important as the brain. He discusses the issue of the importance of physical approximations to purely abstract concepts, rejected by Plato's point of view. Here we use the hand totally coordinated with vision and sensorial perception.

As noted by Artigue (2007), "The increasing interest for the affordances of digital technologies in terms of representations have gone along with the increasing sensitivity paid to the semiotic dimension of mathematical knowledge in mathematics education and to the correlative importance given to the analysis of semiotic mediations". In this perspective, a preliminary double blind research is on its way, with two groups of JCT students. We intend to report on the results in a subsequent paper.

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# DESIGNING A SIMULATOR IN BUILDING TRADES AND USING IT IN VOCATIONAL EDUCATION

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*This paper deals with the design, the production and the uses of a simulator for the activity of marking out on building sites from reading a marked plan. The main design principle of this simulator lies in that it is not meant for reproducing accurately the real context of the activity but it should offer the possibility of posing problems of the work situation through a prior conceptual analysis of the professional activity.*

What is a reading-marking out activity in a building work? Most of building tasks are based on reading plans for marking out on the building site. We call this kind of tasks, reading-marking out tasks. In a building site, setting out elements takes into account what will be set out later. For example, when a floor is to be laid down, the marking out of the floor must leave holes for water pipes and electric cables. Setting out a wall must plan location for windows and doors by marking out their contour. Such marking out is called "boxing out". Generally speaking, a boxing out is a formwork placed in the middle of a structure before casting concrete, used to set aside an area in which additional equipment can be added at a later date. This task of reading information from a plan to mark out contours and boxing out on the building site is usual for workers in building trades.

Two types of controls can be distinguished in the marking out of boxing out:

- controls coming from reading information on the plan
- subsequent and effective controls at the moment of putting the additional elements (pragmatic controls),

the first type of controls being oriented towards the second type of controls.

The first type of controls is the focus of our attention. In absence of pragmatic control, only controls guided by knowledge about space and instruments can take place. The activity of setting out boxing out can allow researchers to observe conceptualisation and help them answer questions such as: what is the nature of knowledge involved in this activity? How is such knowledge organized and what relationship does it have with the artefacts available on the building site?

The observation of students of a vocational school gave evidence of a discrepancy between procedures of students and of professionals in this reading-marking out activity on building site from reading a plan. Two types of analysis were carried out in order to better know this discrepancy and to understand the reasons: an analysis of the geometry in action underlying the students' activity in reading marking out tasks in workshop and an analysis of the transposition of the professional activity in vocational education was needed. The first analysis is presented in Bessot & Laborde (2005). The second analysis focused on the place and status of reading-marking out activities in vocational education, in particular when preparing students to a certification of qualified workers for building trades (in French : Brevet d'Enseignement Professionnel). It was carried out and showed that the reading marking out situation that constitutes an indivisible entity in the professional practice is divided or almost absent from the vocational education institution (Metzler 2006). A simulator is for us a means of designing situations restoring the unity of reading marking out activity in the three teaching places of French vocational education in which knowledge about space is part of the learning aim: in the mathematics teaching (in particular geometry), in the teaching of construction, in the teaching of practice in workshop.

According to a key design choice, the simulator was meant as an *open-ended environment offering the possibility of constructing didactic situations* based on problems previously identified in the analysis of professional situations.

## 1. FONDAMENTAL PROBLEMS INVOLVED IN READING-MARKING OUT PROFESSIONAL SITUATIONS

Previous research on different types of space (Bessot & Vérillon 1993, Brousseau 1983, Berthelot & Salin 1992, Samurcay 1984) as well as the analysis of professional practices (Bessot & Laborde 2005) allowed us to identify three types of problems related to the invariants specific to reading-marking out situation. The two first types are related to mesospace, the third type to the instruments of the building site.

The first type of problems is the problem of locating the local space in which marking out takes place within the mesospace of the building site. Two types of space are involved: the local spaces in which marking out the lines is achieved, and the global space of moves that allows the worker to move from one local working space to another one.

Locating the local space requires coordinating three frames of reference (Samurçay ibid.):

- the frame of reference attached to the subject (egocentric reference frame)
- the frame of reference of the lines marked on the building site (allocentric reference frame) to construct from fixed existing objects of the mesospace that may also be lines already marked on the building site
- the frame of reference of the plan that is the dimension system.

The second problem related to mesospace deals with the coordination of local spaces (Brousseau *ibid.*) that may be distant from each other. This coordination is needed in the process of obtaining the expected global set of marked lines of mesospace.

The third problem is related to the use of instruments: transferring measures requires taking into account the features of the instruments.

## 2. CHOICES FOR SIMULATING MESOSPACE

In order to decouple the problem of local marking out from the one of moving and orienting were created two different windows: the first window allows the worker to have access to various local spaces but never to the entire space; the second one provides access to the visual field of the worker within the global space and his/her move in this global space. In the second window (global space) one can only move, in the first one, one can mark out by means of instruments and one can move without a general view (through the scrolling bars). Here are presented the features of these two windows.

### Window simulating the local space for marking out

This window simulating the visual field of the worker with real dimensions 1,50 m by 1,10 m is the screen of the computer providing a representation of the real visual field on a scale of 1 to 5 (Fig. 3).

One can perform measurement and marking out with the simulated instruments (see below). This window is located within the global space for marking out which is not visually totally accessible. One can move in the global space from one local space to another one by using the scrolling bars of the window (Fig. 1) but with only a partial view at each moment making difficult the linking up of local spaces.

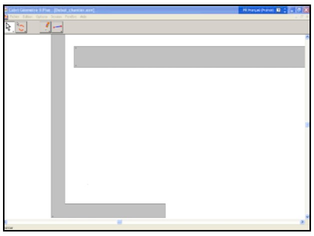


**Fig. 1: Window simulating the marking out local space**

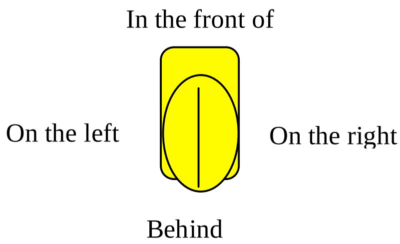
We wanted to simulate the change of viewpoint when the worker is moving away from or closer to the lines marked on the site. Zoom out (Zoom-) and zoom in (Zoom+) possibilities have been set up to simulate these moves, moving away and moving closer. Zoom facilities are limited in order to avoid a global view of the space for marking out. In addition, it is not possible to perform marking out when the zoom tool is active but it is possible to move the instruments. At any time, it is possible to come back to marking out by pressing the key “Zoom 0”. This zooming possibility makes easier an accurate reading of the marks of the measuring tape and the move from one marking out local space to another one at a small distance.

### Window simulating the global space

In order to locate the current marking out local space within the whole space, it is possible at any time to have access to the simulation of the global space by pressing F9 key. The window global space is simulated by a squared vignette with a 7,5 cm long side representing a real squared space with a 5m long side (Fig. 3 et 4).

When opening the window, a yellow hard hat appears that represents the worker with its visual field represented by a rectangle. This rectangle is the image at scale of the screen (marking out local space). When opening the window, the yellow hard hat is always oriented vertically below the rectangle (Fig. 3, 4 et 7).

*It was chosen to simulate the moves of the worker (yellow hard hat) and not its position* (Fig. 6 et 7). Two moves are possible: shifts and rotations which are multiples of a quarter turn. Shifts are performed by directly moving the rectangle through the mouse. Rotations are egocentric and are performed by pressing one of the three buttons « > », « < », «   »: to get the marking out local space on the right of the worker press button « > », on the left of the worker press button « < », behind the worker press button «   ». When back to the local space (Fig. 6), the worker sees the lines oriented as resulting from the move performed in the global space window. In this way the decision of moving and the effect of the move on the visual field are decoupled. If from the marking out local space one comes back to the global view (F9 key), when opening the window, the yellow hard hat is always below the rectangle representing the local space (Fig. 7). Without a fixed frame of reference, the change of position cannot be inferred from the position of the yellow hard hat with respect to the fixed border of the screen.



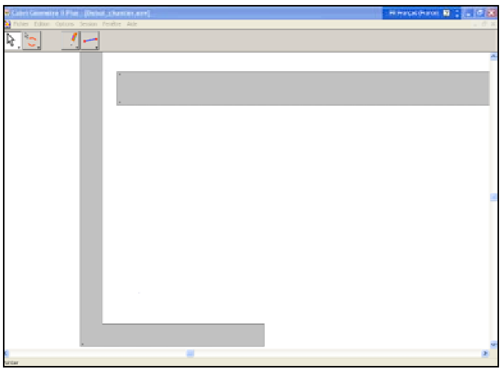



Fig. 2: Window «marking out local space»

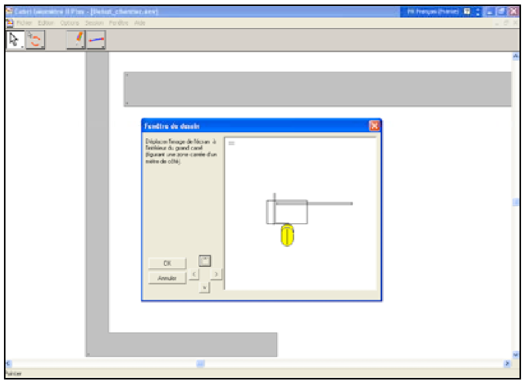


Fig. 3: Window global space in the screen (after pressing F9 key)

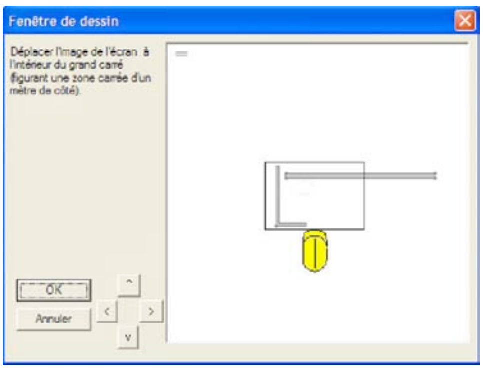



Fig. 4: Local space in the global space window

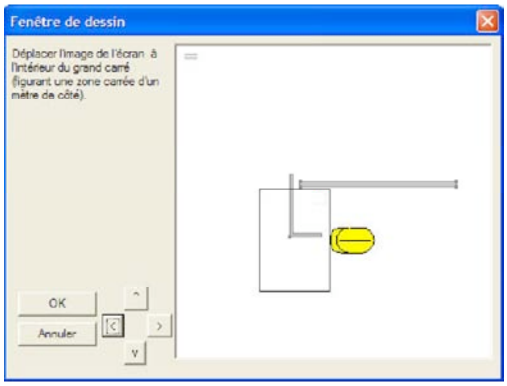



Fig. 5: After pressing button « < »

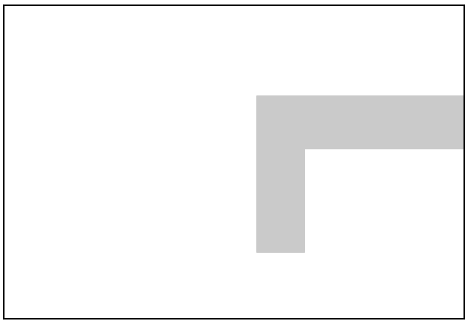


Fig. 6: After pressing « OK » back to local space

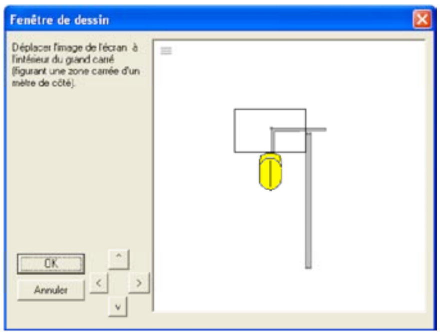



Fig. 7: After pressing « F9 » back to global space

## 3. CHOICES FOR SIMULATING OBJECTS

### Choices for simulating the prefabrication table

The prefabrication table in which the slab is poured, is simulated by three rectangles with same width 0,05m joined in an U shape: the table is 4m long and 2,5m wide. When opening the simulator, the borders of the table may have various directions with respect the borders of the screen: parallel to the screen borders (see Fig. 8) or not (see Fig. 10). The U shape can be oriented in various directions (see Fig. 8 and 9).

Fig. 8: Prefabrication table parallel to the screen borders

Fig. 9: Prefabrication table parallel to the screen borders, in another orientation

Fig. 10: Prefabrication table non parallel to the screen borders

The table is not totally visible in the local space although as fixed object of this space, it can serve as frame of reference of the mesospace for locating lines in coordination with the plan. The table is only totally visible in the global space window (key F9).

### Choices for simulating the use of instruments

The choices for simulating instruments deal with their aspect, their accessibility, their moves and their use. We decided that all instruments should look like real instruments. In particular their dimensions are proportional to real dimensions. The 2,5m long ruler and the 3m long tape even partly unwound stick out beyond the visual field (see Fig. 11 and 12).

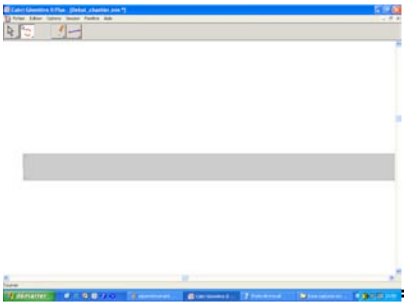


Fig. 11: The ruler cannot be totally seen

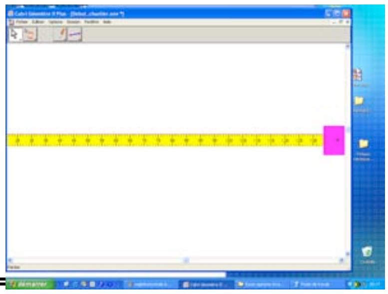


Fig. 12: Apart of the measuring tape

*Marking out instruments,* namely the pen and the blue line are permanently visible as icons at the top of the screen.

*Instruments for measuring and transferring geometric properties read from the plan* (setsquare, ruler and tape) are put at the beginning in three boxes labelled with their names, which are simulated by rectangles located in a corner of the global space accessible by moving in this space. Once an instrument is out by clicking on its box, the worker may have to move to find it again in his/her visual field (resorting to the global space window or to zoom) and to shift it in the screen (local space) to the adequate location in order to perform a marking out.

The materiality of the instruments was not preserved in that simulated instruments can overlap. However seeking to make the edge of an instrument coinciding with the prefabrication table or with the edge of another instrument partly replaces this materiality. However note that the simulated tape is also retractable as in reality in a pink squared case.

## 4. CONCLUSION ABOUT THE DESIGN OF THE SIMULATOR

One of the important contributions of simulators lies in the possibility of being freed of the constraints of reality, like the irreversibility of some actions or the time passage.

It is clear that the simulator transforms the relationships of the worker with space. But what is lost in fidelity can be gained in terms of problems and control. Indeed, in the use of the simulator, separating local and global spaces requires from the subject to make the decision of seeking information in the global space. To this end the subject leaves the local space in order to be and move in the global space, and then must come back in order to perform marking out. These conscious back and forth moves do not occur in reality. As a result of this separation, the subject is certainly faced with a coordination problem of frames of reference of the two spaces.

The additional action of back and forth moves between the two spaces is tedious, it transforms the reading marking out strategies and favours predictions to decrease the number of back and forth moves. But it gives rise to observations for the subject and the educator and consequently can become an object of a reflexive work analysing strategies in real and simulated situations.

Another contribution of the simulator is the possibility of controlled variation offered to the educator. The same simulator can give rise to different uses in vocational education. The educator has the command of the type of use and of tasks given to the students. An example of a didactical situation is briefly presented below.

## 5. EXAMPLE OF A DIDACTICAL SITUATION MAKING USE OF THE SIMULATOR

The situation reported here raises the problem of continuing a marking out already done without transmitting to the worker information on what has been set out. This situation simulates a usual professional problem. Solving this problem requires that the worker identifies the local space within the global space by coordinating various frames of reference including the frame of reference of the plan.

### Instructions

The plan of slab 1 with three boxings out is given (Fig.13) to the students.

1) Open the file "slab 1"
2) As visible, the contour of slab 1 and one boxing out have already been marked.
3) Mark out the two other boxings out of slab 1.

Here below is given the plan of slab 1 provided to the students as well as the windows local space and global space.

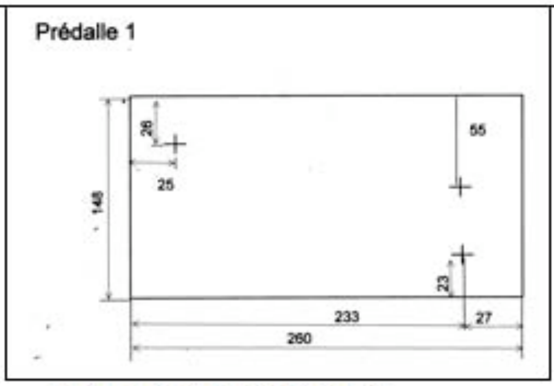


Fig. 13: Plan of slab 1

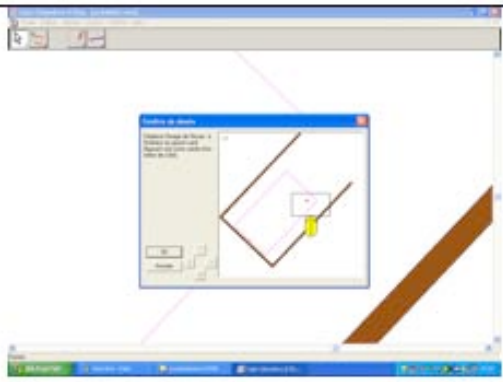


Fig. 14: The two windows

The plan is oriented by the orientation of the writing (from left to right and from top to bottom) and consequently imposes a position for reading. It is represented in this position on Figure 13. When opening the file "slab 1", part of the prefabrication table, part of the lines and the boxing out R(25, 26) are visible in the local space (Fig.15).

In figure 14, it is visible that the slab is rotated through 180° with respect to the frame of reference of the plan.

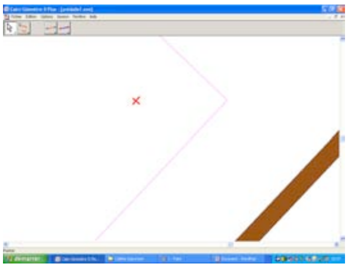


**Fig.15: At the opening of file slab 1**

***A priori* analysis of the situation**

In the marking out activity, the worker's aim is to reproduce in the mesospace the image of the drawing of the fabrication plan. The continuation of the marking out requires interpreting the boxing out already marked in mesospace as the image of a boxing out of the plan.

Two cases are possible:

- Either the plan and its (*unfinished*) image in the working local space have a similar orientation and the boxing out is erroneously considered as R(27,23)
- Or measures are taken in order to identify the already drawn boxing out with a boxing out of the plan.

The choice of the dimensions of boxings out in slab 1 is deliberate. The distances to the border of the two boxings out R(25 ; 26) and R(27 ; 23) are visually close, favouring thus the mistake of the first case in absence of the professional gesture of taking information on what has already set out.

***Incorrect interpretation of the already marked boxing out without measuring : R(27 ; 23)***

Two other boxings out must be marked. Here is only considered the case of boxing out R(27 ; 55) as the only one likely to lead to feedback. Two procedures for marking out R(27 ; 55) are possible:

- Either through an alignment with R(27 ; 23) by resorting to the only measure 55 : *no feedback.*
- Or by resorting to two measures 27 and 55 without making use of the alignment. Once the marking out is done, *the absence of alignment of the two marked boxings out provides feedback that leads to reject the interpretation of the existing boxing out a*s R(27 ; 23). This leads to the second case which is analyzed below.

***Correct interpretation of the already marked boxing out through checking by measuring: R(25 ; 26)***

The coordination between the plan and its unfinished image can be achieved in two ways.

- *Real or mental half turn of the plan of slab 1*

The plan is rotated through 180 ° effectively or in thought to superimpose the image on the screen with the rotated plan: the marking out is performed with a prefabrication table in the position “open on the right, closed on the left”.

- *Move in the mesospace through resorting to the global space window*.

To keep the prefabrication plan in its privileged position and make it coinciding with its image on the screen, it is possible to use F9 key to get access to the global space in order to simulate a half turn in this space: the table is then in the position “open on the left, closed on the right”. When back in the local space, the boxing out already marked is the image of R(25 ; 26). Boxings out can be marked in the same position as they are on the plan.

The situation is aimed to provide multiple opportunities in which checking measures of marked objects in mesospace (prefabrication table, lines) lead to an economy in marking out. Checking is a critical gesture of building trade as claimed by the educators in vocational education.

### *A posteriori* analysis of the situation

As displayed in table 1, only 3 pairs out of 5 resort to measuring on the marking out, in order to identify the boxing out.

| Interpreting the already marked boxing out | without measuring | with measuring | |
|---|---|---|---|
| | R(27,23)<br>*Pairs 1 and 2* | R(20,21) then R(27,23)<br>*Pair 6* | R(25,26)<br>*Pairs 4 and 5* |

**Table 1: Checking procedures of already marked boxing out**

Let us analyze the checking procedures of the three pairs 4, 5 and 6.

Pair 4 made two checks by measuring the dimensions of the slab and the dimension of the already marked boxing out (26 cm) which is sufficient for identifying the boxing out.

Pair 5 checked only one measure (26 cm) and did a half turn of the plan to make the screen matching the plan.

Pair 6 drew surprising conclusions: the already marked boxing out is first considered as not in the plan, then as the erroneous boxing out R (27, 23). Verbal interactions among students V and N of this pair allow us to understand those successive conclusions. As pairs 1 and 2, V immediately identifies the already marked boxing out as R(27,23). But N insists on measuring. Then he measures one of the dimensions of the boxing out and obtains 20 cm as a result of a wrong use of the measuring tape: the distance is measured by making coinciding the centre of the boxing out with the border of the case of the measuring tape (with width 5 cm in real size). He then measures the second dimension in the same way and obtains 21cm. Surprised not to find any boxing out of the plan, he resumes each measuring twice or three times.

V: it fits nothing. It means that it is already marked, then we must mark out the three others. We make one more, that's it.

N doubts that there can exist 4 boxings out and asks questions about the use of the measuring tape to observer O. He admits that he never used a measuring tape!

N: the end of the tape, is it at the black mark (corresponding to the clip of the real tape) or at the other end?

O: it is at the black mark as on a real tape… do you know, don't you?

N: No, I don't know, I never used a tape.

V: Didn't you?

The doubt about correct using of the tape as well as the cost of its use in the simulator lead them to give up checking the correspondence between measures and dimensions on the plan. They come back to the first opinion of V, i.e. identifying the already marked boxing out as R(27,23).

The simulator made possible to face the students with the usual professional problem of continuing a marking out, which is a fundamental issue of the professional activity, as claimed by the teachers. The simulator revealed that even at the end of the vocational training, almost half the students do not resort to checking and among those who checked, the use of instruments may cause difficulties. This checking professional gesture is not available to all students at the end of the school year.

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# COLLABORATIVE DESIGN OF MATHEMATICAL ACTIVITIES FOR LEARNING IN AN OUTDOOR SETTING

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*In recent years, teaching mathematics in an outdoor setting has become popular among teachers, as it seems to offer alternative ways to motivate children's learning. These new learning possibilities pose crucial questions regarding the nature of how mathematical activities should be designed for outdoors settings. In this paper we describe our current work related to the design and implementation of mathematical activities in this particular environment in which a specific mathematical content was used as the central component in the design. We illustrate our collaborative design approach and the results from observations of two activities. Our initial results provide us with valuable insights that can help to better understand how to design and implement this kind of educational activities.*

## INTRODUCTION

A recent trend in Swedish elementary schools is an increasing interest to teach mathematics in an outdoor setting. Teachers believe that this particular approach motivates the children more than solving problems in textbooks, thus offering new ways to introduce and work with mathematical concepts (Lövgren, 2007). Teaching mathematics in an outdoor setting usually refers to school children solving practical problems using whichever forms of mathematics they find appropriate (Molander, Hedberg, Bucht, Wejdmark, Lättman-Mash, 2007). The approach presented in this article is somewhat different. The paper describes our initial efforts with regard to an ongoing project in which a specific mathematical content within the field of geometry was used as the central component in the design of mathematical activities in an outdoor setting.

Our project involves a development team consisting of schoolteachers, university teachers and researchers, who collaborate to develop mathematical activities with the purpose of supporting students' processes of learning. The mathematical activity described in this paper was developed during a period of eight months, counting from the first meeting of the development team and until the completion of the activities involving students. The methodological approach used for developing the mathematical activity will be the central focus of our discussions.

Even if outdoors teaching of mathematics has got an increasing interest among teachers and teacher educators in recent years, we found few published materials with reference to outdoor environments in the research field of mathematics education. For instance, we found no results when searching on *outdoor, outdoors* or *embodied* in titles or keywords in *Educational Studies in Mathematics, Journal for research in Mathematics Education* and *The Journal of Mathematical Behaviour*. When we searched on the term *physical*, some results showed up. However, in a brief check on research methodologies adopted in these studies, no one was centred on an outdoor activity.

Against this background, the current (ongoing) project aims at investigating different possibilities to support students' processes of learning by designing mathematical activities for an outdoor setting. This approach does not aim at replacing traditional mathematics teaching. It should rather be interpreted as a complementary method to be used at the discretion of the mathematics teacher in combination with other teaching methods. In this paper, we particularly aim at discussing our method of design in connection to the principles of Design experiments (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). Throughout the discussions presented in this paper, special attention is paid to the constitution and the working conditions of the development team.

The rest of paper is organized as follows; in the next section we present the mathematical tasks that guided our design and activity while the subsequent section gives a brief overview on the concept of design experiments. The preceding sections illustrate the results from observations of two activities followed by discussions on the notions of group and individual mathematical understanding and practices. The last two sections conclude this article by providing a description of current and coming directions of our work together with a discussion about future challenges.

## DEVELOPMENT OF ACTIVITIES

In this section we describe, both the content of the proposed activities as well as the approach taken while designing the different tasks. The driving force in the design process has been experience-based suggestions from the schoolteachers. Each meeting of the development team has involved four to six teachers and two to three university researchers. The first meeting of the development team focused on identifying mathematical content and learning objectives for an outdoor activity suitable for beginners at lower secondary school. We soon agreed to focus on geometry. Aspects that were discussed dealt with the problems students have on understanding geometrical concepts such as area and perimeter. An early idea was to produce a series of activities showing progression from length to area and then to volume, using physical objects close to the school yard. The university representatives suggested utilizing non-standard measurements (sticks, steps and squares) to be used in relation to triangles, rectangles and polygons defined by trees or within the school soccer field. The school teachers instead suggested to focus on four aspects of the selected domain, namely the following learning objectives; comparison of figures, making own estimates, constructing figures with given measures and, specifically, discovering that a doubling of lengths makes the area four times larger.

It was decided that the university teachers should work on designing a task incorporating as many as possible of the agreed suggestions and present it to the whole group after the summer 2007. The proposed mathematical task, as described in figure 1, aimed at having the students construct the following sequence of figures using ropes and metal hooks to be fastened in the ground.

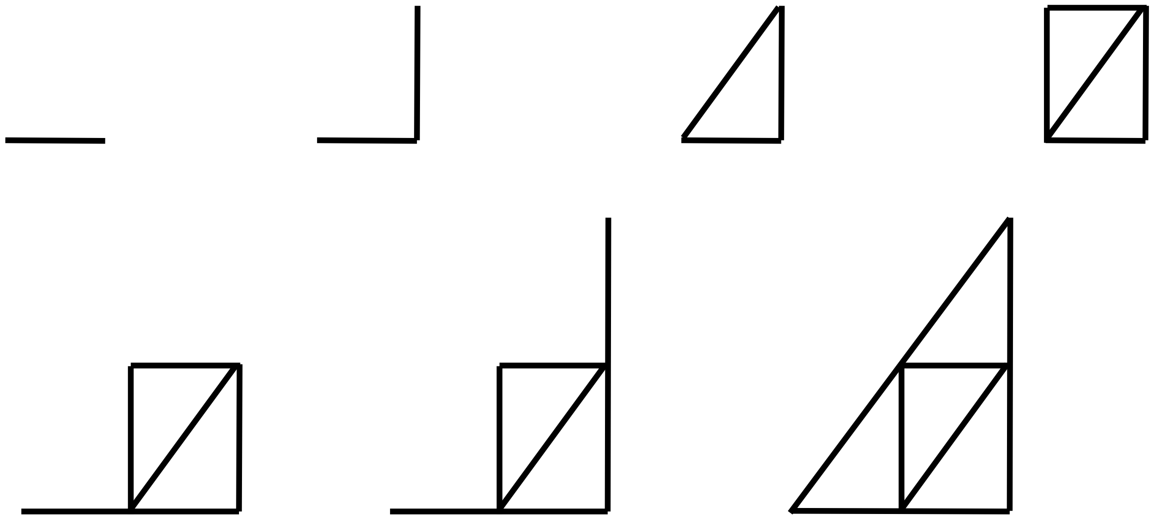

**Figure 1: Intended sequence of figures to be constructed by the students.**

Shortly after the summer, Växjö University hosted Professor Matthias Ludwig from *Pädagogische Hochschule Weingarten* in Germany, who offered to give two one-hour lectures at our department. One of these discussed outdoor geometrical tasks and tools used in connection with the tasks. Inspired by his lecture we decided to suggest construction of two tools; one for producing a right angle and one for measuring arbitrary angles, both based on making judgments by eyesight. The planned right angle tool consisted of a wooden square with markers at the middle of each side, as shown to the left in Figure 2.

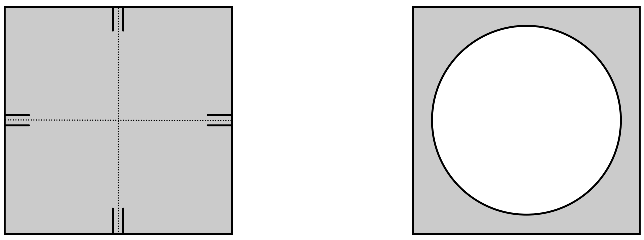

**Figure 2: Ludwig's tool to the left, our tool to the right.**

The woodwork teacher at the school prepared a number of square boards and also prepared a number of round boards intended for use in another activity. The square shaped tool could also be used to represent a square meter since its side was exactly one meter. However, we identified several disadvantages of this tool with respect to the intended task: it could not be used while placed on the ground, it was quite heavy, and the handling required several people operating close to the tool. We later chose the tool shown to the right in the figure above, which was actually what was left over after the round boards had been cut out. This second tool had several advantages. It could be used while placed directly on the ground, it was easy to carry due to the hole in the middle, and could be used at a distance. The right angle was aimed at the sides of the tool.

In the first proposal, the lengths for the catheti (that were to be doubled during the task) were 3 meters and 4 meters. In the construction, metal hooks and flag lines were used. While trying out the task on the (grass-covered) school yard we all agreed that larger measures were needed, to give the students a better overview of the construction and to give them reason to move within the figure. The first suggestion was to double the lengths to 6 meters and 8 meters, but we also agreed to avoid an exact measure for the hypotenuse and ended up choosing 5 meters and 8 meters as lengths for the catheti.

The task was communicated to the students through written instructions on paper. The first page of the instructions described the tools the students were supposed to bring to the school yard (3 flag lines, 6 metal hooks, roll-out length measure, right-angle tool, paper and pen). Three separate tasks were described on the following three pages.

Each task was divided into three subtasks in the same way (construct a figure, determine perimeter, determine area). This was done for several reasons. Since the students were not used to this kind of activity, we wanted to restrict the content in each subtask. We also wanted to encourage the students to discuss their conclusions on each subtask as a group, especially to verify that the construction was made according to the descriptions as we suspected that they otherwise might focus only on calculations. Also, since the written instructions were not supported by figures, we found it reasonable to restrict each subtask in order not to make it too difficult for the students to interpret the task. Our aim was to let the students work on the tasks without the support from the teacher; thereby inviting them to take on different roles and take more own initiatives than they were used to in their usual mathematics classroom. Another important aspect was that the tasks should allow for applying different solution strategies, such as measuring, calculation, and comparison.

## DESIGN EXPERIMENTS

The methodology used in this project is founded on the principles of Design experiments (Cobb et al., 2003). Cobb and colleagues (2003) summarize Design experiments (DE) in five crosscutting features. The first feature, *develop theories*, concerns understanding processes of learning and the means that are designed to support that learning. The second feature, which concerns *control*, may be seen as the focus of the current project: “The intent is to investigate the possibilities for educational improvement by bringing about new forms of learning in order to study them” (Cobb et al., 2003, p. 10). To develop theories about learning processes, and to try to exert control of such processes, implies the need for *prospective* and *reflective* analyses. Prospective and reflective work is the third feature of DE. On the prospective side, our designs have been implemented with a hypothesized learning process in mind. The activity has been carried out with students and the following reflective work has been based on observations of students' actions. The prospective and reflective aspects come together in a fourth characteristic of DE, *iterative design.* Iterations are carried out with the modification and development of explaining learning and the means of supporting learning. The project so far has included only two iterations which have been based on informal observations with a rather weak theoretical base. Our strategy has been to let the preliminary informal observations guide us toward relevant learning theories to support later iterations. The fifth feature refers to the *pragmatic roots* of DE. As school teachers take active part in the design process, we feel confident that the activities are relevant for teachers' practice.

## OBSERVATIONS FROM TWO ACTIVITIES

Two activities involving students have been carried out in the project. The two activities included two different groups of four students (14-15 years old). The activities were neither videotaped nor audiotaped. Instead, two researchers and two teachers observed the activities. The researchers were the same both times.

During the activities, the students were very eager to start working with the lines and hooks. We feel that the division of each task into subtasks made it possible for them to interpret the subtask while arranging lines and hooks. On a few occasions, when they were getting lost in the construction, we had to intervene and ask them to read the instructions again. We also observed that some of the students had problems handling the instruction papers. These problems concern locating and returning to the instructions after they have been left on the ground, as well as documenting answers to the questions.

One specific observation concerned the change in social behaviour. One of the teachers commented on a female student who was busy constructing sides by pulling flag lines:

> Look at her. She seldom takes initiatives in the classroom; she is very quiet and rarely shows interest. Here she is, pulling flag lines, talking to her classmates and really enjoying what she is doing.

Another notable observation can be seen as relating to gender issues. In a group of two boys and two girls, the boys were trying to solve the problem of extending the catheti, seemingly ignoring the girls. As the boys got stuck, one of the girls walked up to the (female) teacher and whispered her solution. The teacher encouraged her to talk to the boys, and the whole group ended up producing the intended construction.

One specific topic of discussion concerning mathematics emerged in our follow-up meetings. To recall, one of our intention with the design was to encourage different solution strategies, such as measuring, calculation and comparison. What was noticed however, was that measuring took a rather dominant role in the activity. Moreover, since the students were not familiar with the Pythagorean Theorem we did not expect them to calculate the hypotenuse of the first triangle, in order to determine its perimeter. However, when the students were asked to determine the perimeter of the larger triangle, i.e. after the catheti of the first triangle being doubled, they also now measured the hypotenuse. None of the students reflected on or argued that also the hypotenuse was doubled. The students did not even reflect on this after the three sides were measured. The data they used for determining the perimeter was the measured data.

During the first activity, the students quickly turned to calculating the area of the larger triangle by the rule; base times altitude divided by two. No attempt was made to compare the larger triangle with the smaller triangle, even if the construction supported looking four smaller triangles within the larger (see Figure 1). In the instructions for the second activity, we therefore explicitly asked the students if they could find out from the constructions any relation between the area of the larger triangle and the smaller triangle. After some discussion and guidance the students at least articulated that the area of the larger triangle was four times the area of the first triangle. However, we were not comfortable that the activity did not by itself provoke the students to involve principles and relations in their discussions.

We observed that the students solved the tasks rather pragmatically and routinely, in terms of measuring and applying rules for calculation. However, we do not have evidence that the students' behaviour depended on conceptual limitations. In the follow-up discussions within the development team we identified possible explanations in terms of the design of the activity and the students' history of being part of a certain educational system. Therefore, to develop the activity and to understand students' actions and potential, we have reached a point where we find it necessary to deepen the theoretical approach of our work, taking into account analytical constructs on several levels of interaction. In the next section we describe principles of the emergent perspective (Cobb et al., 2001), which we find suitable for our purposes.

## CONCEPTUALIZING GROUP AND INDIVIDUAL MATHEMATICAL UNDERSTANDING

In Cobb, Stephan, McClain and Gravemeijer (2001) terms, the evolution of mathematical learning in classrooms constitutes of *social* as well as *psychological* structures of behaviour and reasoning. Within the social structure, they identify three analytical categories: *Classroom social norms*, *Sociomathematical norms* and *Classroom mathematical practices*. Examples of Classroom social norms can be for instance; that students collaborate to solve problems, that meaningful activity is valued more than correct answers, and that partners should reach consensus as they work on activities. With reference to our observations, Classroom social norms may have been in play when the quiet girl had to be encouraged by her teacher to communicate with her team members. *Sociomathematical* norms are defined as social constructs specific to mathematics. These are the norms in play when explanations and justifications are made acceptable (Hershkowitz and Schwarz, 1999). When applying the analytical construct of *classroom mathematical practices* the analytical lens is closer to a certain instructional activities. It concerns regularities of the collective engagement in a specific situation in terms of symbolizing, arguing and validating.

A student may experience a study activity in different ways, as compared to the teacher's and to other students' interpretations (Wistedt, 1987; Iversen and Nilsson, 2007). The psychological perspective concerns the nature of individual students' reasoning. It brings attention to the diversity in students' ways of interpreting and acting in mathematical activities (Cobb et al., 2001).

It is crucial to understand that the relation between the social and the psychological perspective is considered to be reflexive (Cobb et al., 2001): "…neither perspective exists without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective" (p. 122).

An implicit assumption of the current project has been that an unfamiliar teaching arrangement might encourage students to act beyond previously established Classroom social and Sociomathematical norms, with the possibility that these new actions may be more mathematically productive than their correlates of ordinary classrooms. The initial results of our observations, specifically the two separate incidents involving girls, support this assumption.

## THE ORGANIZATION OF MATHEMATICAL PRACTICES

Weber, Maher, Powell, and Lee (2008) summarize some important ways in which discussions may establish opportunities for the learning of mathematics. Discussion can objectify students' experiences, thereby making these experiences the subject of analysis, encourage students to take a more reflective stance on their mathematical reasoning, require students to consolidate their thinking by verbalizing their thoughts, and help students learn to communicate mathematically and participate in a wider range of mathematical argumentation. Weber et al., (2008) also contend that group discussion can facilitate learning by inviting students to be explicit both about the ways in which they make new claims from previously established facts and about the standards they are using in deciding whether an argument is acceptable. Challenges from classmates can encourage students to debate whether a particular method of argumentation is appropriate and provide students with the opportunity either to justify their methods when their reasoning is sound or revise or abandon their methods when their reasoning is flawed.

In the organization of group discussions, Cobb et al., (2001) distinguish between three specific structures: taken-as-shared purposes, taken-as-shared ways of reasoning with tools and symbols, and taken-as-shared forms of mathematical argumentation. A taken-as-shared purpose is what the students and the teachers are trying to achieve together mathematically. The second structure is concerned with the ways in which tools and symbols are used and given taken-as-shared meanings. To account for taken-as-shared forms of argumentation Toulmin's (1969) analytical model of argumentation has proven useful (Cobb et al., 2001). According to Toulmin (1969), an argumentation consists of at least three core components: *the claim, the data*, and *the warrant*. When a speaker makes a claim he or she may be challenged to present evidence or data to support that claim. The data typically consist of facts that lead to the conclusion that is made. If a listener does not understand why the data justify the conclusion that was drawn she may challenge the presenter to clarify why the data led to the conclusion. When this type of challenge is made and a presenter clarifies the role of the data in making her claim the presenter is providing a warrant. A warrant can of course be questioned, thus obligating the presenter backing up the warrant.

## DISCUSSION ON OUR METHOD OF DESIGN

Our choice of method has been influenced by the constitution and working conditions of the development team. The main focus has been on collaborative development of the mathematical activity. The project emphasizes the potential benefits of collaborative development in close interaction with stakeholders. There has been a very open climate of discussion where teachers' knowledge and experiences have been given equal attention as input from the researchers. The teachers have been very active providing ideas and reacting on suggestions from the researchers, both during physical meetings and through e-mail communication. We argue that this way of collaboration differs from the approach usually used by DE practitioners. In DE, theories are usually introduced in early stage of the design process (diSessa & Cobb, 2004). From the observations of two activities, we have been identified a need for supporting theories. The interpretative frameworks outlined above will enable us to strengthen our design and to better understand our observations. However, we have found it fruitful to use an experienced based approach. No theories have been explicitly communicated during the initial work of the development team. Particularly, we believe that introducing abstract theories early in the discussions would have reduced the teachers' interest and possibilities to communicate empirically grounded ideas, thereby limiting the pragmatic root of the project. Our approach may therefore serve as a reasonable model for others, who wish to engage in collaborative activities in order to enhance school teaching. On account of this, we suggest that researchers in collaboration with teachers should take seriously the role of theories, particularly when to introduce them in the project at hand.

We suggest a balance between theories and practice, where practice takes on a rather dominant role in the early work. As the project and iterations proceed, the role of theories may be increased in order to enhance control of the learning activity. The analytical categories argued by Cobb et al., (2001), and Toulmin's (1969) model of argumentation, offer instruments both for supporting the design process and for serving as tools for analysis of observed actions.

Finally, one can question the validity of our approach in relation to the pedagogical implementation and learning outcomes of these activities but the main point here is not assess the effectiveness of the learning materials neither the mathematical content but instead to explore how to design and organize the flow of pedagogical activities in an outdoor learning setting. Our initial impressions indicate that this kind of learning activities seem to encourage discussions and new collaboration patterns, thus promoting deeper understanding among students. Therefore, we believe that a major challenge for the mathematics education community is to create new possibilities for learners to understand complex mathematical concepts, as well as to develop new analytical tools and theories in order to facilitate our understanding on how learning takes place under these new circumstances.

## FUTURE EFFORTS

Based on the discussions presented in this paper, the following suggestions appear to be relevant for the design of the next iteration. The design of the next activity should take into consideration how:

- collective understanding can be provoked by encouraging students to make claims and be explicit about the warrants on which the claims rest,
- collective discussion can capitalize on individual variations (implying that the activity should encourage a variation in reasoning and solution strategies),
- norms and structures of mathematical practices may support or limit students' behaviour.

The last aspect specifically refers to the observation of how measuring took on a rather dominant role in the activities, narrowing the students' conceptual structures. On account of these guidelines we suggest to follow up the described activity with a second activity, where the students are not allowed to use a measuring tool. Instead they start with a triangle with given perimeter and given area and whose sides are not known. The triangle will be marked with flag lines and the students will be asked to continue the construction of the same pattern as in the previous construction and will be asked to determine the perimeter and the area of the larger triangle. We conjecture that such a setup will provoke the students to reflect on conceptual aspects, by comparing features of the triangles. Another suggestion is to let the students choose their own measures and construct a triangle which will be extended to a rectangle, with the aim that they discover the connection between the areas of the two figures.

An obvious next step of the project is to investigate how the described outdoor activity can be followed up in the regular classroom. Earlier mentioned shortcomings concerning students' documentation may be overcome by using mobile technologies. According to Spikol and Milrad (2008), mobile technologies offer the potential for a new phase in the evolution of technology-enhanced learning, marked by a continuity of the learning experience across different learning contexts. In particular, we propose to let students use mobile technology in order both to communicate the tasks and to support the documentation of their solutions. Moreover, offering the students possibilities to videotape and taking pictures during the activity will support them in recalling and sharing experiences when they return to their regular classroom. We believe moreover that interesting applications may be developed in additional fields such as arithmetic and statistics, and even in algebra and functions. Our ambition is to invite students from the teacher training program at our university, so they can participate in widening our design approach to the above mentioned fields.

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# STUDENT DEVELOPMENT PROCESS OF DESIGNING AND IMPLEMENTING EXPLORATORY AND LEARNING OBJECTS

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*In 2001 a core undergraduate program, called* Mathematics Integrated with Computers and Applications (MICA) *was introduced in the Department of Mathematics at Brock University, Canada. In this program that integrates evolving technologies, students complete major projects that require the design and implementation of 'Exploratory and Learning Objects' (ELO). In this paper, we propose schematic representations and descriptions of the student development process as s/he completes an ELO project. We highlight the important role that ELO interfaces play in this development process.*

*Keywords: Exploratory and Learning Objects (ELO); student development process; students designing and implementing ELO; university mathematics education.*

## INTRODUCTION

There have been a number of publications (Muller, 1991, 2001; Muller & Buteau, 2006; Buteau & Muller, 2006; Pead et al, 2007; Muller et al., forthcoming) about the long-term implementation of evolving technology use in undergraduate mathematics education at Brock University (Canada) that started in the early 80s. The most recent development is the 2001 implementation of the core undergraduate mathematics program called *Mathematics Integrated with Computers and Applications* (MICA). Two of the program aims are to (1) develop mathematical concepts hand in hand with computers and applications; and (2) encourage student creativity and intellectual independence (Brock Teaching, 2001). Three innovative core courses, called *MICA I, II, III*, were implemented in addition to a review of all traditional courses to incorporate the MICA program aims. Results of a 2006 MICA student survey and an enrolment analysis covering the years 2001 to 2006 are reported in Ben-El-Mechaiekh et al. (2007). Highlights include

> Students overall rated the use of technology in their mathematics courses as positively beneficial (77.74% of responses; 79.36% when restricted to mathematics majors). (p.10)

and, furthermore,

> ... students overwhelmingly rated the use of technology in [MICA] courses as [positively] beneficial (91.13% of responses) (p.9)

In this paper proposal, we focus on one of the major student activities in the MICA courses, namely their designing, implementing (VB.net, Maple, C++), and using of interactive and dynamic computer-based environments, called Exploratory and

Learning Objects (ELO). By Exploratory Object (EO) and Learning Object (LO), we mean the following.

> *An Exploratory Object is an interactive and dynamic computer-based model or tool that capitalizes on visualization and is developed to explore a mathematical concept or conjecture, or a real-world situation*
>
> and,
>
> *A Learning Object is an interactive and dynamic computer-based environment that engages a learner through a game or activity and that guides him/her in a stepwise development towards an understanding of a mathematical concept.* (Muller et al., forthcoming, p.5)

To illustrate these objects, we provide without comment three examples of original student ELO projects that can be accessed at (MICA Student Projects website, n.d.): (1) *Structure of the Hailstone Sequences* EO by first-year student Colin Phipps for the investigation of a mathematical conjecture; (2) *Running in the Rain* EO by second-year students Matthew Lillie and Kylie Maheu for the investigation of a real-world situation; and (3) *Exploring the Pythagorean Theorem* LO by first-year student Lindsay Claes for the learning of a school mathematical concept.

In previous publications, we have elaborated how the MICA I course is designed to progressively bring the students to acquire the skills and understanding required for the development of ELOs (Muller & Buteau, forthcoming). In brief, as the course progresses, our students are guided through each step in the development process of ELOs that we describe in the next section of this paper. We have also explained that this requires a significant change in the teaching paradigm of faculty involved in these courses, and motivates a change in attitude in the students about learning and doing mathematics with technology at the university level (Muller et al., forthcoming). And also, we have argued that learning activities in the MICA program accelerates students' growth towards independence in doing mathematics (Buteau & Muller, 2006).

In this paper we propose a first attempt at defining a structure for the student development process in their activity of designing, implementing, and using an ELO. These final MICA projects are completed individually or in pairs selecting a topic of their own choice. We also briefly discuss the role of interfaces in the student development of an ELO. As in the past, we, as mathematicians in a mathematics department, look forward to receiving constructive feedback from mathematics educators. We hope that the presentation of our innovative student learning activities, as part of the systemic integration of technology in our university mathematics curriculum, will instigate educational research questions on learning mathematics with use of technology in tertiary education.

## STUDENT DEVELOPMENT PROCESS OF EXPLORATORY AND LEARNING OBJECTS

In what follows, we suggest schematic representations of the development process for ELOs. Even though the schematic representations are worded generally, in their descriptions we focus on students in MICA courses.

### Development Process of an Exploratory Object to Investigate a Conjecture

We propose the following diagram (Figure 1) to illustrate this development process.

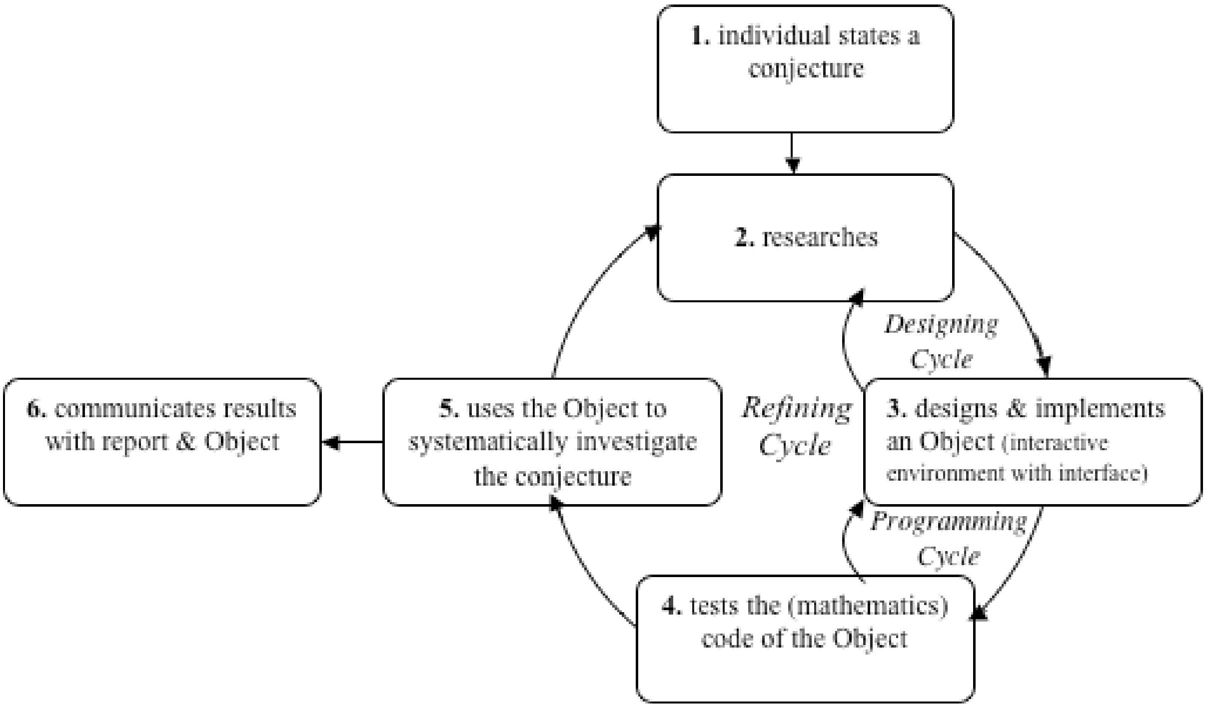


**Figure 1. Development process of an Exploratory Object for the purpose of investigating a conjecture.**

Here is a description of each step in the diagram.

**1.** Student states a conjecture, and may discuss it with the instructor; some of the more independent students wait until step 3 to discuss their project.

**2.** Student researches the conjecture using library and Internet resources, and may refine his/her conjecture. In conjunction with step 3, student identifies the mathematics, such as variables, parameters, etc., and is involved in a *Designing Cycle*.

**3.** With his/her understanding of the conjecture, student starts designing and implementing (i.e., coding) an interactive environment (i.e., program with interface) with a view to testing the conjecture. Student organizes the interface to make parameters accessible and to display diverse representations of results. As the interface plays such an important role in EO, we discuss it further in the next section.

**4.** Student selects, in a step-wise fashion, simple and more complex cases to test that the mathematics is correctly encoded and that the interface is fully functional. Together with step 3, the code testing and revising involve the student in a *Programming Cycle*.

**5.** At this step, student now returns to focus on his/her conjecture and uses the Object to systematically investigate it. Following the results of the investigation, the student may decide to refine the Object, e.g., introducing new parameters, etc., and be involved in *a Refining Cycle* (with steps 2, 3, and 4).

**6.** Student produces a report of his/her results and submits it with the Object. The report includes a statement of the conjecture, the mathematical background (from step 2), results of the exploration including an interpretation of the data and graphs (from step 5), a discussion, and a conclusion. This is somewhat similar to a science laboratory report. Building on this analogy, the Object is the laboratory itself. In other words, student submits his/her self-designed 'virtual laboratory' for the investigation of a self-stated conjecture together with his/her laboratory report.

**Development Process of an Exploratory Object to Investigate a Real-World Situation**

We propose the following diagram (Figure 2) to illustrate this development process.

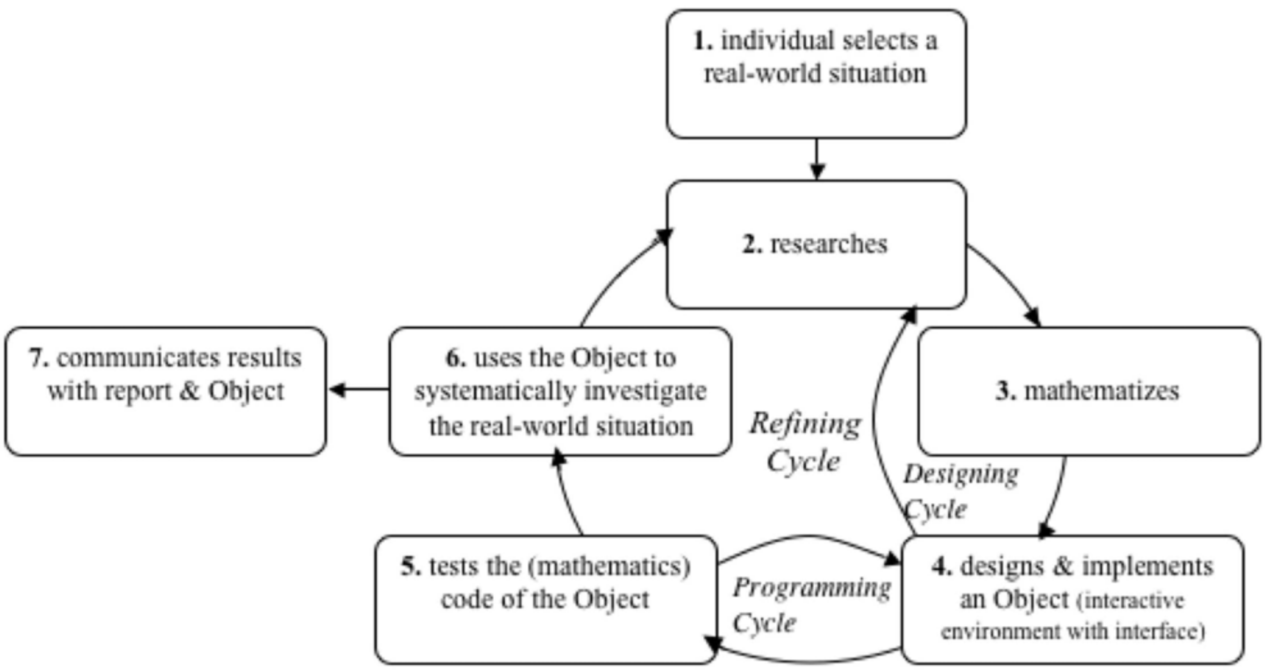


**Figure 2: Development process of an Exploratory Object for the purpose of investigating a real-world situation.**

Here is a description of each step in the diagram.

**1.** Student selects a real-world situation of particular interest, and may discuss it with the instructor; some of the more independent students wait to discuss their project until step 3 or 4.

**2.** Student researches the real-world situation using library and Internet resources, and may restrict or modify the scope of the real-world situation. In conjunction with steps 3 and 4, student identifies the mathematics, such as variables, parameters, etc., and is involved in a *Designing Cycle*.

**3.** Student develops a mathematical model of the real-world situation using the variables and parameters selected in step 2 and in the majority of cases, consults the instructor.

**4.** With his/her understanding of the model, student starts designing and implementing (i.e., coding) an interactive environment (i.e., program with interface) with a view to investigating the real-world situation. Student organizes the interface to make the model parameters accessible and to display diverse representations of solutions. As the interface plays such an important role in EO, we discuss it further in the next section.

**5.** Student selects, in a step-wise fashion, simple and more complex cases to test that the mathematical model is correctly encoded and that the interface is fully functional. Together with step 4, the code testing and revising involve the student in a *Programming Cycle*.

**6.** At this step, student now returns to focus on his/her real-world situation and uses the Object to systematically investigate it. Following the results of the investigation, the student may decide to refine the model and the Object, e.g., introducing or deleting, new parameters and variables, new conditions, etc., and may be involved in a *Refining Cycle* (with steps 2, 3, 4 and 5).

**7.** Student produces a report of his/her results and submits it with the Object. The report includes a description of the real-world situation, a development of the mathematical model (from step 3), results of the exploration (from step 6) including an interpretation of the data and graphs, a discussion, and a conclusion. This is somewhat similar to a science laboratory report. Building on this analogy, the Object is the laboratory itself. In other words, student submits his/her self-designed 'virtual laboratory' for the investigation of a self-selected real-world situation together with his/her laboratory report.

### Development Process of a Learning Object of a Mathematical Concept

We propose the following diagram (Figure 3) to illustrate this development process.

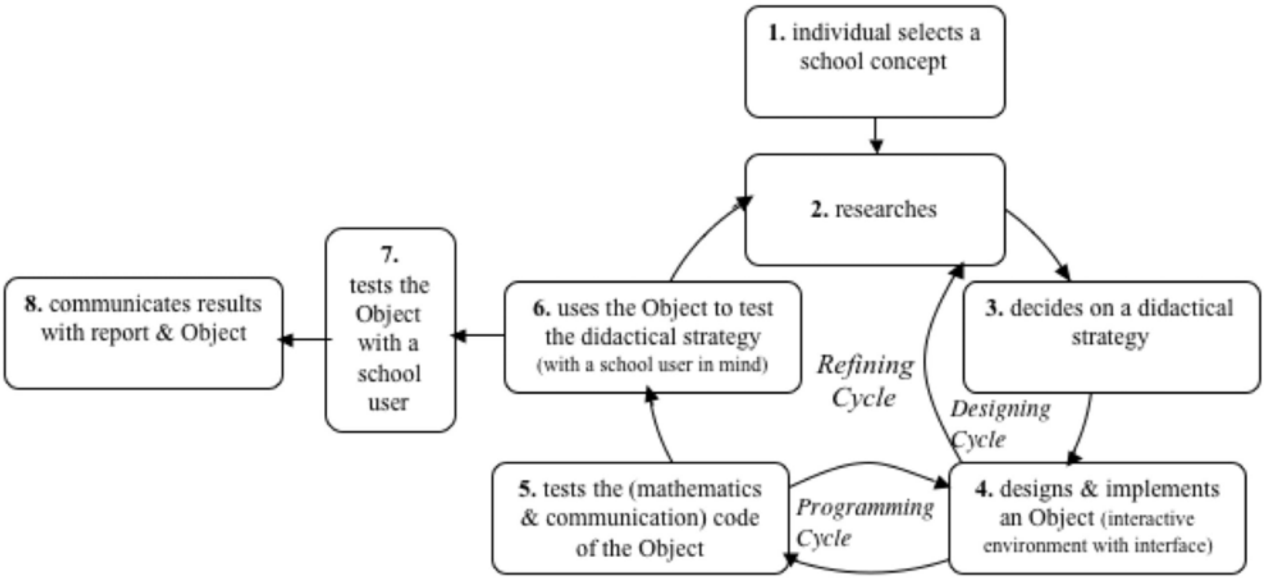


**Figure 3: Development process of a Learning Object of a mathematical concept.**

Here is a description of each step in the diagram.

**1.** Student selects a school concept.

**2.** Using library and Internet, student looks at resources about the concept and its teaching. In particular, student identifies when in the school curriculum the concept is taught, reviewed and expanded, what previous mathematical understanding, general knowledge and reading capabilities can be assumed, etc. In conjunction with steps 3 and 4, student identifies and develops the mathematics didactical features that could be used for his/her Object, and is involved in a *Designing Cycle*.

**3.** Based on the information gathered in step 2, student selects a didactical strategy for a fictive school pupil learning of the concept that may include developing a game or activity to engage the learner, breaking down the concept, setting up a testing procedure, etc. Student may discuss the strategy with the instructor or wait until the next step.

**4.** Student starts designing and implementing (i.e., coding) an interactive environment (i.e., program with interface) with a view to implement the didactical strategy. Student structures a self-contained interface realizing that the fictive school pupil will be using the LO independently. As the interface plays such an important role in LO, we discuss it further in the next section.

**5.** Student tests that the interface (communication, navigation, etc.) is fully functional and tests with simple and more complex cases that the mathematics is correctly encoded. Together with step 4, the code testing and revising involve the student in a *Programming Cycle*.

**6.** At this step, student now returns to focus on his/her didactical strategy and works through the Object with a school pupil in mind. Following the results of this investigation, the student may decide to refine the Object, e.g., changing the sequence of activities, improving the clarity of communication, etc., and may be involved in a *Refining Cycle* (with steps 3, 4, and 5).

**7.** Student tests his/her Object by observing a school pupil, at appropriate grade level, working with the Object. In some cases, student returns to the refining cycle and revises the Object.

**8.** Student produces a report of his/her results and submits it with the Object. The report includes the didactical purpose, the target audience, the mathematical background of the target audience, a brief account of the school pupil experience (step 7), and a discussion. This report is somewhat similar to a lesson plan, including a post-lesson reflection, though without a description of the lesson. Building on this analogy, the Object is the lesson itself. Thus, student submits his/her lesson plan of a self-selected mathematical concept in which the written description of the lesson is replaced by an 'interactive self-directed lesson (with a virtual learner)', i.e., by the Object.

## ROLE OF THE INTERFACE IN THE DEVELOPMENT PROCESS OF EXPLORATORY AND LEARNING OBJECTS

The interface provides interactivity and (dynamic) visualization. In the Development Process of ELOs (Figures 1, 2, and 3), the student creates an interface in the Designing Cycle with the aim of using it for his/her mathematical or didactical investigation (step 5 in Figure 1 and step 6 in Figures 2 and 3).

During the Designing Cycle of an Exploratory Object, the potentiality of interactivity encourages the student to make explicit the parameters that could play a role in the investigation of his/her conjecture or real-world situation in such a way that they are accessible from the interface. The potentiality of visualization urges the student to decide on the representations to be displayed in his/her interface so as to best support his/her investigation.

At the step in the Development Process when the student uses the Object for his/her investigation (step 5 in Figure 1 and step 6 in Figure 2), both interactivity and visualization aspects of the interface play a role in the student's systematic investigation. The latter can be seen as a dialogue between the student and the computer, though the discussion is fully controlled by the student. During the systematic investigation, the student sets a question by fixing values to parameters (interactivity), the computer answers the question (visualization), and the dialogue continues in that way unless the student concludes that the answers are not satisfactory to meet his/her goal and decides to refine the Object (Refining Cycle). In other words, the student is in an *intelligent partnership* (Jones, 1996) with technology.

The interface plays a central role in Learning Objects but which is different than in the Exploratory Objects. A Learning Object is designed for other users to use by themselves, i.e., without the Object designer who is the student in our case. Thus the navigation in the interface should be very clear and easy. The interface should also provide, at any time, motivation for the intended users to go to a next step in the Object. As such, the visual presentation and the wording should be adapted to the intended users:

> For Learning Objects students [are] reminded constantly that they are designing interfaces for people who are not experts and that they need to take into account such issues as the user's age, educational level, gender, cultural background, experience with computers, motivation, disabilities, etc. (Muller et al., forthcoming, p.12)

Also, students should

> ... break away from the linearity of the written tradition in order to take full advantage of the technological paradigm. (Muller et al., forthcoming, p.12)

In step 8 of the Development Process of the LO (Figure 3), we introduced an analogy where the Object is a 'lesson with a virtual learner'. Using this analogy, the interface's potentiality of interactivity encourages the student during the Designing Cycle to develop an active 'lesson', i.e., a lesson that is interactive, with the intended fictive pupil. The interface's potentiality of visualization facilitates the development of transparent communication of the 'lesson' flow and makes it possible for the student to test his/her 'lesson' (steps 6 and 7 in Figure 3). In other words, we suggest that these two potentialities allow the student to develop a 'guided intelligent partnership' between a fictive pupil and technology.

## REFLECTIONS

Diagrams shown in Figures 1, 2, and 3 clearly indicate our view that the student mathematics learning experience through the designing and implementing of an ELO is richer than what is experienced through activities of only programming mathematics. The interface plays a major role through its interactivity and visualization potentialities as it provides students with an opportunity to be involved in an 'intelligent or guided intelligent partnership' with the technology.

In a recent collaborative project between a local elementary school, *École Nouvel Horizon*, and our Department of Mathematics, MICA student Sarah Camilleri was involved as part of her Honour's project in the development of *Fractions Fantastiques/Fantasy Fractions* Learning Objects (Camilleri, 2007; Buteau et al., 2008a and b; MICA Student Project website, n.d.). In this development, she worked with a Grade 5 class, the teacher, and the school principal. It is worthwhile to explore the ways in which individuals took different roles and responsibilities in the Development Process (Figure 3).

Sarah and the teacher selected the fraction concept (step 1), and Sarah researched it (step 2). The teacher taught fractions to the class and presented the collaborative project. In the *Designing Cycle*, guided by the teacher and the principal, the Grade 5 pupils developed the dynamic mathematics lessons, interactive mathematics games, story line of the Object, its characters, etc., and provided drawings and written materials to communicate their ideas to Sarah who had to select and adapt some of them for programming purposes. The pupil design work was achieved in class discussions and in smaller groups of two or three. Within the *Programming Cycle* Sarah took the responsibility of faithfully implementing the pupils' design which also involved the digitizing of the pupils' drawings. The *Refining Cycle* involved Sarah and the teacher for testing the functionality of the Learning Object and checking the faithful integration of the pupils' ideas. *Fractions Fantastiques* Learning Object was presented by Sarah to the Grade 5 class and each pupil received a CD-ROM copy of *their* Learning Object (step 8).

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# HOW CAN DIGITAL ARTEFACTS ENHANCE MATHEMATICAL ANALYSIS TEACHING AND LEARNING

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*Digital technologies seem to be still very promising to fruitfully support the construction of mathematical knowledge. Far more interesting is the way to incorporate them into the design of a learning environment framed by certain institutional constraints. Through this study we present some reflections and ideas arising from the dialectic interplay between the environment and the students in their effort to formulate a calculus theorem and construct its proof. Related teaching and learning phenomena providing information on instrumental genesis processes are primarily discussed.*

## INTRODUCTION

Elementary pre-calculus is at the heart of the syllabus at secondary level mathematics education and the entry-point to undergraduate mathematics as well. Many research studies witnessing students' problems to attain a satisfactory level of conceptualisation have been held on this field (for example, see Artigue, 1999). This fact is related to mathematically superficial strategies (Lithner, 2004) implemented by traditional procedure-oriented teaching practices. We claim that these practices are generated by both teachers' attitudes and institutional constraints implicitly or explicitly imposed by textbooks and curricular objectives (Ferrini-Mundy & Graham, 1991). Even at the university level, this situation results in detecting serious difficulties on behalf of the students when faced with non-algorithmic type demands which entail reasoning and conceptual understanding (Gonzales-Martin & Camacho, 2004).

On the other hand, the development of mathematics has always been dependent upon the material and symbolic tools available for mathematical computations (Artigue, 2002). Current research on mathematics education regarding the relationships between curriculum, classroom practices and software applications (Lagrange, 2005) offers the ground to address and develop questions concerning technology's fitting into learners' actual social and material environments, the problems users have that technology can remedy, and, furthermore, ways of conceptualizing the design of innovative learning tools as emergent from dialectics between designers and learners-users of those tools.

The learning environment is supported by a Dynamic Geometry software (DGS) enhanced by a function-graphing editor to help Mathematical Analysis teaching at the level of $12^{th}$ grade.

The produced didactic sequence covers the introduction of global and local extrema definitions, Fermat's theorem (stationary points) with its proof, the mean value theorem, monotonicity definitions and the derivative sign/monotonicity theorem along with the proof and its applications. Selection of the exact targeted mathematical material on the field of differential calculus, as well as further elaboration of the activities, were attempted with the intention to form a rational succession of concepts to a coherent local unity of mathematical knowledge, mainly including introduction of definitions, formulation of theorems and construction of proofs. From this still on-going research, we present here some elements derived only from an activity concerning the teaching and learning of Fermat's theorem formulation and proof on the field of differential calculus.

## THEORETICAL FRAMEWORK

Complexity and close interweaving of cognitive, institutional, operational and instrumental aspects obliged us to adopt a multidimensional approach (Lagrange et al, 2003) in order to design the learning environment and study the teaching/learning phenomena produced.

According to Duval (2002), construction of mathematical knowledge is strongly attached to the manipulation of different semiotic representations. This term refers to productions made up of the use of signs and formed within a semiotic register which has its own constraints of meaning and function. More specifically he defines a *"register of semiotic representation"* as a system of representations by signs that allows the three fundamental activities tied to the processes of using signs: the formation of a representation, its treatment within the same register, its conversion to another register. Interaction between different registers is considered to be of great importance and necessity to achieve understanding of a mathematical concept. Under this aspect, our tools were designed with the intention to mobilise and flexibly articulate semiotic representations within the numerical, the algebraic and the graphical register, so that to generate mathematical conjectures.

Very special and idiomorphic conditions existing within the local educational culture of Greek 12$^{th}$ grade students obliged us to take into consideration the notion of didactical transposition (Chevallard, 1991). At this level, a huge amount of institutional pressure results in the development of an "exam-oriented mentality" on behalf of the students as well as their families, which promotes a procedure-oriented attitude towards the mathematical knowledge in context. Candidates' needs to be prepared for a final national university-entrance examination at the end of the year result, finally, in an implicit (or even sometimes explicit!) meta-didactical attitude leading them to ignore or reject conceptual approaches not strongly attached to exam demands. Through this perspective, we were obliged to take into account and reinforce both the epistemic and the pragmatic value (Artigue, 2002) of the mathematical knowledge to be taught without any decrease or discount of any of them, in the economy of the available didactical time. Relating this idea to the tools' design, we considered the possibility to teach basic mathematical concepts within a reasonable amount of learning time, and in ways compatible to both its institutional dimension and the transition to advanced mathematical thinking.

The theory of didactic situations (Brousseau, 1998) helped us conceive the whole learning environment (milieu) as a source of contradictions, difficulties, and disequilibria stimulating the student (on his own responsibility to control it) to learn by means of adaptations to this environment. At this point, we took also into account activity theory (originated in socio-cultural approaches and mediation theories rooted in Vygotski, 1934) to assign to the environment a character sometimes antagonistic to the subject (as pointed by TDS) but also sometimes cooperative and oriented to an educational aim, guided by distinctive didactical intentions.

In order to best incorporate digital artefacts in our didactical engineering, we considered the potential technology offers for linking semiotic registers within the frame introduced by the instrumental approach (Rabardel, 1995, Artigue, 2002, Trouche, 2004). A cultural tool or artefact, designed to mediate mathematical activity and communication within a socio-cultural context, differs from the corresponding instrument into which this artefact can be transformed. The artefact, as the final result, encompasses a psychological component; a construction by the subject, in a community of practice, on the basis of the given artifact by means of social schemes. This transformation is developed through an instrumentation process directed towards and shaping the subject's conceptual work within the constraints of the artifact and an instrumentalisation process directed towards and shaping the artifact itself. Both constitute a bidirectional dialectic and sometimes unexpectedly complex process called instrumental genesis (Artigue, 2002). Concerning tool design, we tried to keep simplicity and friendliness to the user, in the sense that their implementation demands, as far as possible, a short process of appropriation by the user and an easy way to be transformed into mathematical instruments to be utilised in the context of the activities. The necessity of any technical support by the teacher was also minimised as far as possible.

The crucial question to answer through our research is whether a design philosophy under the norms mentioned above has the potential to determine a set of effective digital learning tools, pre-constructed on the dynamic software, which can be easily transformed to learning instruments successfully integrated into the teaching of important calculus concepts at the level of theorem formulating and proof. By the term *successfully integrated* we mean that, firstly, they can make visible phenomena previously invisible, secondly, they can potentially generate innovative approaches to important mathematical concepts, and, thirdly, they shape and better our understanding of some productive or problematic dimensions of the computer transposition regarding the mathematical knowledge accessed through the instrument's mediation.

## METHODOLOGY

The activity (of total duration 90 min) was developed in two different schools in groups of 12th grade students (10 in one group and 5 in the other) during the month of February, 2008. The main differences between the students of the different schools were identified on the socio-cultural and financial background of the corresponding families (we did not address any comparison issue in our research goals) and as well to the fact that comparatively more students belonging to a certain school had a facility for using mathematics software, being exposed several times in the past at different kinds of technology enhanced approaches. For the latter we did not find enough evidence to support the idea that different software cultures of the students have great impact on their attitude and capabilities of manipulating the pre-constructed software tools induced by our activities.

The informatics laboratory of every school was used and the pupils were at couples situated in a PC-environment. This time the researcher played the role of the teacher as an orchestrator of in-class situations. A Teacher-Analysis sheet has also been developed to provide necessary details so that other teachers can handle the in-class orchestration.

At the beginning, a worksheet was given to the students to work with and at the end of the session they received a corresponding post-assessment sheet including several T/F type questions of mathematical nature, which they returned back completed next day. The whole didactic sequence (consisting of four Sessions) was recorded by a voice-recorder and, the whole sequence being completed, a post-questionnaire was passed to the students in order to collect and save some of the instrumental marks being left on them through the entire approach. Finally, four students (two for each group) were interviewed to explicitly clarify their answers at this questionnaire concerning the instrumented actions performed and the students' attitude towards mathematics teaching before and after the whole experience.

The way of obtaining results-serving the a posteriori analysis-from the raw input data has to be explained here. The whole content referring to the 2nd Activity (Fermat's Theorem: Stationary Points) has been divided (according to the conceptual meaning development) into 12 *Episodes* and each one of them potentially to one up to four *Phases*. Next, for every one of the 24 *Phases* produced, we used the transcribed outcomes of the recorded class discourse, along with the written notes and answers of the students on the worksheet, to produce some discrete entities of information we called *Events*. An *Event* in this terminology is characterised and differentiated by components of mathematical or didactical or instrumental nature which can probably coexist. The study and analysis of these *Events* provided our a posteriori analysis with the material to compare the results composed up to this point to the analysis of the students' answers to the corresponding post-assessment sheet-being sorted out and analysed separately. Finally, we took into consideration the students' answers on the final post-questionnaire as well as the transcribed explicitation interviews in order to enhance our vision and come up to some final conclusions.

## LEARNING ENVIRONMENT

Concerning the tools' design (and being sensitive to the complexity of instrumental genesis processes), we tried to reduce, at least, the complexity of the interface. We tried also to keep tools' implementation strongly attached to the mathematical needs emerging within the predefined context. The learning environment regarding the whole activity was, thus, perceived with the intentions to:

a) Mobilise students' interest to estimate local extrema departing from a real problem, b) Make up a link with the students' previous knowledge on the subject of local extrema and the limit concept, c) Stimulate the students to construct the targeted mathematical knowledge by mobilising different registers of representation (graphic, numerical, symbolic, and verbal) for the same concept and favouring representational interconnections between them, d) Use the in-class discourse to generate an activity space favouring students' effective instrumental processes, e) Support conjecturing, conceptualisation, and institutionalization, f) Insert certain examples or counter-examples when necessary (Gonzales-Martin & Camacho, 2004).

We focus especially on the activity designed to introduce the concept of Fermat's theorem. As far as the students were concerned, our specific didactical aims were: to conjecture the theorem, construct its formal statement and proof realising the absolute necessity of its presuppositions and its application range, to perceive that the reverse form of the theorem is not valid, and, finally, to apply it in calculating the local extrema of the function-given by a formula-induced by the problem.

In the following we describe and analyse some selected *Events* drawn out of two different *Episodes*. The material that will be presented is coming from a blend of actual events produced by both groups of students, whose comments and actions have been complementing each other over the flow of the activity.

*Remark*: The term S-Tools refers to the specific on-Screen pre-constructed tools on the software.

Episode A: Introduction to the *Line y*=k, *IntersectionPoints*, *Magnification* S-Tools and applications in approximating local extrema positions on the function graph.

Tool Description: The students were prompted to open *Line y=k* and *Intersection Points* S-Tools. The first one draws a horizontal parametric line, whose position can be controlled by the active parameter *k* (a number in yellow background on the screen that can be modified by the user, see Image 1). If this line has some common points with the function graph then the second S-Tool *IntersectionPoints* draws these intersection points and provides their $x$-coordinates. Furthermore, a technique permitting the students to change the decimal length and the digits of any active parameter was explained to them by the teacher.

The following question given by the corresponding worksheet came to stimulate students to S-Tools utilisation:

| |
|---|
| Q1: Could you find or estimate points of local extrema for function *P*? |

Aim Description: The main intention of the constructed situation was to encourage students to explore and use the S-Tools in order to estimate several intervals of $x$-axe that could enclose positions of internal local extrema and to get approximate values for these positions by shortening the length of the corresponding intervals. Moreover, they had to identify the kind of local extremum (maximum or minimum) and perceive which of them are internal to the interval.

Events: The teacher asked the students to change the active parameter $k$ and see what happens. Some of them could not understand the changes on the counters of intersection points coordinates and that was clarified by the related discussion in the class community. Then, the students were asked to use these tools to numerically estimate the local extrema positions (Question Q1) on the graph the better they could (Image 1). Some students could not cope with changing the decimal length and the values of several digits so they were given additional technical instruction for that. The teacher asked them to find an interval including the abscissa of a local maximum (this was done very easily) and then to try to shorten this interval by means of the tool. This was not so easily done by every pupil but remarks made by several students and on-screen indications gave good results.

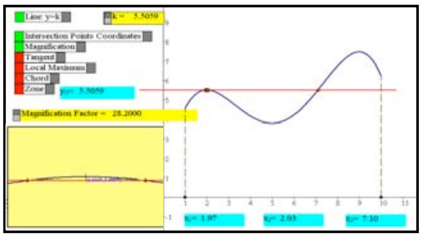

**Image 1**

Interesting events identified on behalf of the students were:
-During exploring with decimal digits many students observed two intersection points approaching each other and, finally, coinciding to only one but the indications on the corresponding counters were different.
-Six of them noticed that they could see intersection points on the screen but the indications on the coordinate counters did not attest such an existence.
Concerning these two events, the teacher's proposition was to use the *Magnification* S-Tool.

Tool Description: This S-Tool could be used to magnify a selected region around a point which can be displaced anywhere on the graph and is controlled by the *Point-Abscissa* and the *Magnification Factor*.

Subsequently, the students were asked to use the same process to estimate the values of every local extremum they could perceive on the graph.

Remarks: Students' written answers on the worksheet revealed that the whole class succeeded at the qualitative level (number of local extrema, approximate position and characterisation). However, at the numerical level, only a small part (26% of them) tried to test in the extreme the instrument's potentialities and even less (6,6%) achieved at exhausting them-providing the values asked at 3rd or 4th decimal digit accuracy as we had anticipated. A technical weakness versus time disposal has been estimated as a possible reason for that.

Results: This first contact with the notion of approximation opened up the ground for a further in-class discussion. The discourse came up to the point that the tool is able to provide visual images of a certain validity only as an indication generator (which in certain cases can be of great importance for the mathematical knowledge) but not always to produce an arithmetic value in absolute accuracy. The teacher reinforced this situation by asking what would happen if the extremum in search had the real value $2/3$ or $\sqrt{2}$. This fact conducted the discussion to bring into light the inherent inadequacy of every computing system to represent infinite decimal numbers in a complete way. So the students realised that, through this attempt, and also in general, they could obtain only relative accuracy for the local extrema values. The necessity of devising new mathematical tools that could probably provide absolute accuracy for these values came in the discourse.

Episode B: Introduction to the tangent: Relating line y=k when passing through an internal local extremum to the function graph – Derivability

Next Question Q2 had the intention to sensitise students' attention and make them focus to what is going on locally at the area near an internal local extremum point.

| Q2: When line *y=k* is passing through an internal local extremum point on the graph, how is this line related to the curve at an area near this point? |
|---|

Description: Within this *Episode* the students were asked to express their thoughts regarding the visual relation between the line y=k when passing through an internal local extremum point on the graph and the curve itself near the extremum point. The first attempt was made on normal view and the second by means of the *Magnification* S-Tool (Image 2). Subsequently, at the third phase of the *Episode* a new subroutine program file was invoked, where the students could alternatively observe under magnification the behaviour of functions $y = x^2$ and $y = abs(x)$ in the neighbourhood of $x = 0$ (Image 3). This was done by changing only the function formula through a menu of the file. The necessary technique was shortly explained by the teacher.

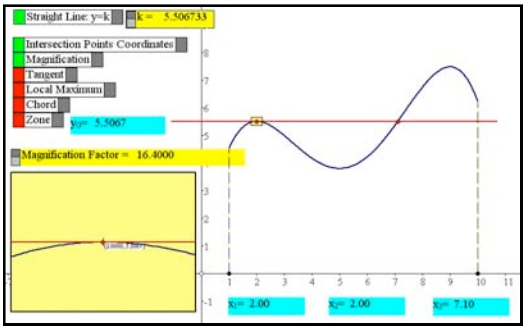


**Image 2**

Events: The class discourse developed at this *Phase* helped many of the students to communicate their thoughts and formulate them in an intelligible way. They came up with the visualisation of the inequality relations $f(x) \le k$ or $f(x) \ge k$ near the local extremum. Relating this fact to the image produced by the function graph and the horizontal line, they could easily conjecture that this line when passing through a local extremum point on the graph *"leaves the whole curve on one side"* or *"does not cut it"* at the area near this point.

Remarks: Analysis of students' written answers on the worksheet showed that the big majority of them (80%) succeeded in perceiving the visual relation between the curve and the line and, moreover, 26,6% of them were able to connect it with the corresponding symbol relation. 26,6% of the students proceeded to conjecture that, at this case, this horizontal line should be a tangent of the graph, whereas even fewer (13,3%) mentioned that there was only one common point of the line and the curve at the area near the local extremum.

To the question of the teacher if these two conditions (namely existing of a single common point and "not cutting" in the area near a local extremum) are able to assure the existence of a local extremum, confusion arose and the community could not provide a clear answer. This event, along with the term tangent mentioned earlier, was used as a bridge to the discussion of next question:

| | |
|---|---|
| Q3: | At the area near the extremum point, can you observe any additional relation between the curve and the line $y=k$ when the latter is passing through this point? |

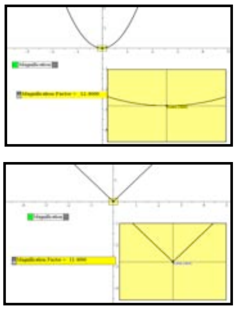

**Image 3**

Remarks: Class discourse concerning this question resulted in the assertion on behalf of the students that under magnification the curve tends to become a horizontal line or to coincide with it. Moreover, there were some more students stating in a clear way the conjecture that the horizontal line when passing through a local extremum point on the graph *keeps the position of a tangent of the graph at this point*. This conjecture provided the bridge through which the teacher introduced the issue of the existence of the tangent at such a point. Additionally, as a natural consequence of the previous discussion, the subroutine file was used to support students' exploring and help them visualise the difference between the function graphs of $y=x^2$ and $y=abs(x)$ on point $x=0$ under magnification (Image 3) and relate it to the derivability of the function at this point. Most of the students' expressions were for example *"Oh, there's an angle there!"* or *"… in this case we have a peak point …"* etc.

## DISCUSSION AND PRELIMINARY RESULTS

In this paper, we tried to describe a few situations concerning only the instrumental dimension of our research. The *Episodes* presented above contribute, as a first step, to Fermat's theorem construction departing from an intuitive approach. This goal is achieved by exploring and visualizing the local extrema positions related to the premises of the theorem.

As it has been pointed by Guin and Trouche (1999), students' answers were strongly dependent on the environment:

At a first attempt, many students tried to configure the artefact regarding the needs of the specific work: screen view adaptation by transposition of toolboxes and active parameters configuration (i.e. changing the decimal length and the values of certain digits of parameter *k*). These facts confirm, on their behalf, an effort to adapt the artefact to the demands of the specific task induced by the first question Q1 (Estimation of local extrema values) and we consider that as a step to the direction of instrumentalisation in the evolution of instrumental genesis processes (Rabardel, 1995, Trouche, 2004). As instrumentation processes had intently been designed and anticipated not to be very complex, soon after, we could observe automaticity towards certain instrumented action schemes to the execution of necessary tasks (i.e. utilising active parameters).

We point to an internal constraint (Trouche, 2004) of the instrument, which is related to computer's inherent deficiency in providing absolute preciseness through computations, regarding infinite decimal numbers. This issue was discussed with the students during several activities and, finally, was used as an entry to the discussion concerning the notion of approximation. Additionally, a common feeling was developed pointing out that computers will not solve all the mathematical questions inserted. This fact was also used to encourage students to develop their knowledge so as to overcome these limitations.

Students' answering to questions of the post-assessment sheet regarding the statement of Fermat's theorem or its applications within only the graphic register showed that the great majority of them (86,6%) could cope very good at this level. However more complex questions relating this register to the algebraic one have been far too difficult for the students, proving that more work is necessary to be done at this level.

Analysis of students' answers to the final post-questionnaire testified a generally positive attitude towards *"this way of teaching"*. For example, to the question: "*Could you identify any positive or negative points through this series of activities you have been attending?*", some of their answers were: *"We could discover and see by ourselves most of the things on the screen…"* or *"By the aid of the computer we could really see and work on the staff we treat usually in the class"*, or *"It was easy-going because, we first, … we didn't realize that we had made the proof of the theorem and only at the end we got the typical statement"* or *"It was very helpful to recollect the images on the screen, but the problem was that we didn't solve many exercises!"* etc. Of course, more work and analysis need to be done on this issue in order to obtain some reliable results.

Due to the lack of space, we did not address issues concerning the ways through which the rest of our theoretical perspectives shape our research. However, some results seem to deepen our reflection. They show the potential of such a learning environment design to produce didactical phenomena giving an illumination to both problematic and productive aspects of the mathematical knowledge developed through the educational use of digital technologies.

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# A LEARNING ENVIRONMENT TO SUPPORT MATHEMATICAL GENERALISATION IN THE CLASSROOM

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*This paper discusses classroom dynamics and pedagogical strategies that support teaching mathematical generalisation through activities embedding a specially-designed microworld. A prototype of our microworld was used during several one-to-one and classroom studies. The preliminary analysis of the data have allowed us to see the implications of designing and evaluating this specific technological tool in the classroom as well as the teachers' and the students' requirements. These studies feed into the design of the intelligent support that we envisage the system will be able to offer to all students and the teacher. In particular, they helped us identify which aspects of teachers' interventions could be delegated to our system and what types of information would be useful for supporting teachers.*

**KEYWORDS:** Mathematical Generalisation, Microworlds, Classroom Practices, Teachers, Intelligent Support

## INTRODUCTION

It seems that there is a growing diversity of computer-assisted material and tools for mathematics classrooms. Even though this proliferation of digital tools and new technologies has broadened the instructional material available for teachers, they are still rather insignificant to classroom practice and their use is far from regular (Artigue, 2002, Mullis et al., 2004, Ruthven, 2008). This suggests a challenge for mathematics educators to develop complete, consistent and coherent systems that not only assist students, but also support teachers' practice in the classroom.

The aim of the MiGen[1] project is to design and implement a system with teachers that meets their as well as students' requirements. We are developing an intelligent exploratory learning environment for supporting students in making mathematical generalisations. In more detail, our focus has been on the difficulties, first students face in their efforts to generalise and second teachers face in their efforts to support students appropriately during lessons with 20-30 students. For our initial investigations, we restricted the domain of mathematical generalisation to the generation and analysis of patterns. Activities with patterns often appear in the UK mathematics curriculum and have been identified as motivating for students (see Moss & Beatty, 2006). They also comprise a good domain for generalisation, since they allow students to come up with different constructions for the same pattern, find the corresponding rules and realise their equivalence.

Our aim is to develop a system that provides the means to understand the idea of generalisation, but also the vocabulary to express it, while supporting rather than supplementing the teacher. The system is intended to provide feedback to the teacher about their students' progress and, where the system's 'intelligence' is unable to help students, to prioritise the students in critical need of the teacher's assistance.

The core of our system[2] is a microworld, called the eXpresser (described briefly in the next section), in which students can construct and analyse general patterns using a carefully designed interface. In order to build the microworld, our team[3] started with a first prototype (Pearce et. al, 2008). Using an iterative design process, and in order to investigate the effectiveness of our approach,  we carried out a number of studies with individual students or pairs of students, each time using the feedback we obtained to build the next prototype. This process resulted in the evolution of the prototype and its subsequent evaluation in classroom.

This paper, after a brief discussion of our methodology, presents the preliminary data analysis of the classroom studies that not only support the next version of the microworld, but also feed into the design of the intelligent support that we envisage the system will be able to provide. Our focus here is on the teachers' pedagogical strategies and the students' needs for support and assistance during their interactions with the microworld.  This analysis is followed by a discussion of the teachers' interventions that could be delegated to the 'intelligent' system and what types of information would be useful for supporting teachers and therefore necessary for the development of the intelligent support components of our and other similar systems.

## A microworld for patterns – the eXpresser

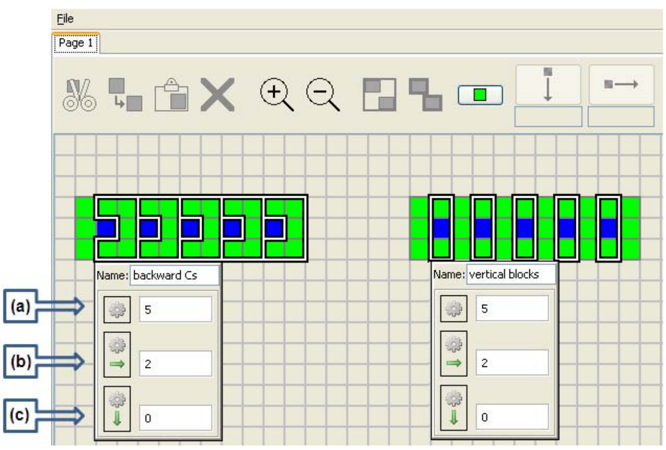


**Figure 1. The interface of eXpresser with two different constructions of the same pattern. The left one is made out of a vertical block of 3 squares and 5 'backward C-s' and the right one of alternating vertical blocks.**

First, we present briefly the main features of the eXpresser. We emphasise that at the stage of the study, attention was focused largely on the features key to our research goals. So, the following description of the system is by no means complete. In addition, its design has evolved significantly through studies such as the ones described in this paper. The interested reader is referred to Noss et al. (2008), where the system's rationale and design principles are described in detail.

In eXpresser, students can construct patterns based on a 'unit of repetition' that consists of square tiles. These patterns can be combined to form complex patterns, i.e. a group of patterns. A pattern's property box (depicted in Figure 1) shows three numeric attributes that characterise the pattern[4]. The first specifies the *element count* (number of repetitions) of this pattern (a). The icon with the right arrow (b) specifies *how far* *to the right* each shape should be from its predecessor and, similarly, the icon with the down arrow (c) specifies *how far down* a shape should be.

A requirement of our constructivist approach was to allow students to construct patterns in a variety of ways (Figure 1). Additionally, an important design feature is the ability to 'build with n' (see Noss et al., 2008), i.e. to use independent variables of the task to create relationships between patterns.

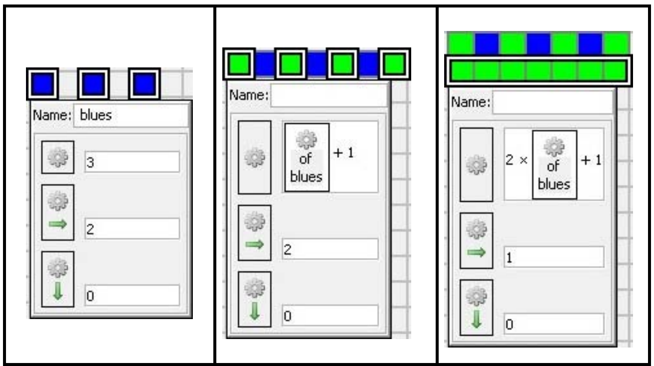


**Figure 2. Another way to construct the pattern in Figure 1. To relate the middle row with the first pattern (named "blues"), the number of repetitions should be one more than the number of repetitions of "blues". For the bottom row it should be twice more plus one. These relationships are specified iconically.**

This feature not only provides students additional ways to construct patterns but we hypothesised that it enables students to realise what are the independent variables and use them to express relationships. To overcome difficulties that students face with symbolic variables the microworld employs what we call 'icon-variables', which are pictorial representations of an attribute of their construction. We have illustrated in previous work (Geraniou et al., 2008), that these 'icon-variables' provide a way to identify a general concept that is easier for young learners to comprehend. An example of expressing such relationships is depicted in Figure 2.

## METHODOLOGY

Our own previous work and studies by Underwood et al. (1996) and Pelgrum (2001), for example, concerning the adoption of educational software in classrooms emphasise the importance of teachers' involvement in the whole design process of computer-based environments. Therefore, several meetings with the teacher were held before each classroom session so that they were familiarised with the prototype, agreed and made input to the lesson plans and in order to clearly state the teacher's, the students' as well as the researchers' objectives.

The overall methodological approach is that of 'design experiment', as described by Cobb et al. (2003). One of our goals during these sessions was to inform our system's design and evaluate the effectiveness of our pedagogical and technical approach. We aimed at investigating the classroom dynamics by looking at individual students' interactions with the microworld, the collaboration among pairs or groups of students as well as the teachers and researchers' intervention strategies.

We investigated the use of eXpresser in several one-to-one and classroom sessions with year 7 students (aged 11-12 years old). Particularly for the classroom sessions, two researchers played the role of teaching assistants and another was observing and keeping detailed notes regarding the researchers' and the teacher's interventions. The sessions were recorded on video and later analysed and annotated with the help of the written observations. Based on these, we were able to get information regarding the time and duration of the interventions, the type of feedback given, the students' reactions and immediate progress after the interventions. Therefore, our goals in the study reported in this paper were to identify not only the students' ability to collaborate successfully and articulate the rules underpinning their generalisation of the patterns but particularly when and how the teacher or the researchers intervened.

However, to maintain the essence of exploratory learning, research suggests a teacher's role should be that of a 'technical assistant', a 'collaborator' (Heid et al., 1990), a 'competent guide' (Leron, 1985) or a 'facilitator' (Hoyles & Sutherland, 1989). Our aim was to achieve the right balance between students' autonomy and responsibility over their mathematical work and teachers' and researchers' efforts to scaffold and support their interactions. The teacher and the researchers set out to adopt this role by following a specific intervention philosophy that adhered to our framework of interventions (Mavrikis et al., 2008), which was based on our previous work with Logo and dynamic geometry environments. This framework was extended after the analysis of the data and is presented in the 'Classroom Dynamics' section. Our aim was to avoid imposing our (or the teacher's) views or ways of thinking, but instead allowing students to express their viewpoints and assist them by demonstrating the tools they could use: for example, by directing their attention, organising their working space and monitoring their work.

## CLASSROOM SCENARIO

We illustrate here a classroom scenario carried out with a year 7 class with 18 high-attaining students. Students were introduced to the microworld through a familiarisation process, during which the teacher introduced all the key features to construct a simple pattern and students followed his actions on their laptops.

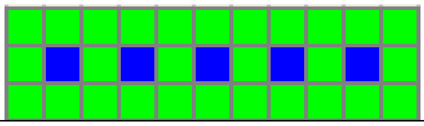

**Figure 3. The activity: Find a rule for calculating the number of green (light) tiles for any chosen number of blue (dark) ones.**

Students were then presented with the task in Figure 3. The pattern was shown dynamically on the whiteboard; its size changed randomly showing a different instance of the pattern each time. This made it impossible for students to count the number of tiles while allowing them to 'see' variant and invariant parts of the pattern. We hypothesised that a dynamically presented task would discourage 'pattern-spotting', which focuses on the numeric aspect of specific instances of the pattern, and counting, which encourages constructing specific cases of the pattern. It also provided a rationale for the need of a general rule that provides the number of tiles for *any* instance of the pattern.

Students were given the freedom to construct the pattern in their own way, using the system's features they had been shown earlier. They were asked to write on a hand-out how they constructed the given pattern and then discuss in pairs their constructions and the methods they followed. They also worked collaboratively to find a rule that gives the number of green tiles for any chosen number of blue ones. Students' next challenge was to find different ways to replicate the pattern and describe them on the hand-out explicitly, so as their partner could understand it. After discussing with their partner, if they had come up with the same constructions, they were expected to try to see whether there were any other ways and find all the rules that represented their constructions and write them down. Finally, the teacher initiated a discussion, where students were asked to present their rules to the rest of the class. Rich arguments were developed and students challenged each other to justify the generality of their construction and the rules they have developed.

During this classroom study many interesting issues regarding the classroom dynamics were identified that informed our further design of the microworld and the overall system and the next phase of the research.

## CLASSROOM-DYNAMICS

As expected, to ensure the success and effectiveness of students' interactions with the eXpresser, there was a need for significant support from the teacher and the researchers. As discussed already, we had agreed a specific intervention philosophy with the teacher. The analysis of the data (video recordings and written observations) revealed further strategies and extended our previous framework of interventions (Mavrikis et al., 2008). The revised framework is presented in Table 1.

- Reminding students of the microworld's affordances
- Supporting processes of mathematical exploration
  - *Supporting students to work towards explicit goals*
  - *Helping students to organise their working environment*
  - *Directing students' attention*
  - *Provoking cognitive conflict*
  - *Providing additional challenges*
- Supporting collaboration
  - *Students as 'teaching assistants'*
  - *Group allocations*
  - *Encourage productive discussion (group or classroom)*
- Ensuring task-engagement and promoting motivation

**Table 1. Types of interventions observed during our studies**

Below we pull out some illustrative episodes under each category.

### Reminding students of the microworld's affordances

As facilitators the teacher and the researchers (referred to as 'facilitators' for the rest of the paper) managed to support students' interactions and explorations by reminding them of various features of the system that assisted students' immediate goals. This intervention acted sometimes as a prompt and other times as an offer of assistance. If the facilitator sensed a student was working towards a direction where they could be assisted by a specific tool, they would point it out to their students. This teaching strategy might have proved rather common as for some students the one lesson spent on familiarisation with the system seemed not enough.

**Supporting processes of mathematical exploration**

We often needed to support the students' problem-solving strategies. For example, we noted that students tended to forget their overall goal. Students seemed to get lost in details and got carried away with various constructions ('drawings'), which, even though offering students more experience of the system's features and affordances, it sometimes led them in the wrong direction. One of the downsides of any microworld is that students' actions can become disconnected from the mathematical aspects under exploration. Even though, the system's affordances were carefully designed to support students' thinking processes, they were not always naturally adopted by them. Therefore, when needed, we provided a reminder of their goals or helped them re-establish them by asking questions like "What are you trying to do?" or "What will you do next?" (*supporting students' work towards explicit goals*).

Another aspect of problem-solving skills (particularly when working in microworlds) that some students seemed to lack was being able to come up with *an organised working environment*. We occasionally advised students to delete shapes that were irrelevant to the solution or change the location of a shape so that they could concentrate on ones that could prove useful. It was evident that students who worked effectively and reached their goals were the ones that organised their working space and therefore supported their perception of the task in hand.

*Directing students' attention* was a necessary pedagogic strategy. We prompted students to notice invariants or other details which are important for their investigations without giving away the answer. For example, we asked questions such as "Did you notice what happened when you increased the length of this pattern?" or "when you changed this property of your pattern?". These pointed out certain facts that students might have missed out or ignored, but also exposed possible misconceptions and misinterpretations. If students were focusing on or manipulating unnecessary elements of their construction, the facilitators provided hints towards more constructive aspects. If students' responses revealed any misconceptions, then such a prompt acted as an intervention for *provoking cognitive conflict*. There were cases where the cognitive conflict was not obvious to the students directly and further explanations were required from the facilitators. These normally involved giving counter-examples to provoke students' understanding and challenge their thinking processes. Besides this intervention we used another strategy, referred to as "messing-up", used in our previous work in dynamic geometry (Healy et al., 1994). This strategy challenged students to construct a pattern that is impervious to changes of values to the various parameters of the tasks. Students tended to construct patterns with specific values and had their constructions ‘messed-up’ when the facilitators suggested: “What happens when you change this to say 7 (a different value to the student’s chosen one)?”. This strategy gave a rationale for students to make their constructions general by encouraging them to think beyond the specific case. In other cases where students seemed to have reached a satisfactory general construction, the facilitators intervened by *providing additional challenges*. For example, “Could you find another way of constructing the pattern?”.

### Supporting collaboration

Students who achieved a seemingly general construction and found a rule (general or not, representing their construction or not), often failed to find different ways of constructing the pattern. Our approach in these circumstances was to introduce them to the collaborative aspect of the activity, in which they had to discuss, justify and defend the generality of their constructions and their rules to their partners. We envisaged that learners’ general ways of thinking would be enhanced by the sharing of their different perspectives. Accompanied by the facilitators’ or fellow students’ assistance, students could appreciate the equivalence of their approaches and possibly adopt a more flexible way of thinking. In this study, the rationale behind collaboration was to give students an incentive to enrich their perception and understanding of the given pattern, to find more ways of constructing it and begin to appreciate their equivalence mathematically. The allocation of students to groups aimed at ensuring the best possible collaboration (*group allocations*). Ensuring though that discussions carried out within the groups were fruitful was not an easy task. The first step towards this goal was grouping the students in a way that promoted participation from all members of the group while discouraging students from dominating a discussion (*encourage productive discussion*).

On some occasions, the facilitators, particularly the teacher who has better insights into his students’ competence, encouraged students to take the role of a *‘teaching assistant’* and help others who were less successful in their constructions. This intervention boosted students’ confidence, but also gave them an opportunity to reflect upon their actions and an incentive to explain their perspective.

### Ensuring engagement and promoting motivation

Finally, although the activities and the system affordances were designed to assure engagement as well as promote students’ motivation, there were various occasions (e.g. being stuck or ‘playing’ by drawing random shapes) when the facilitators’ intervention was required. Our vision was to give the right rationale for students to solve the task and praise their efforts. These studies supported our view that avoiding tedious activities that were pointless in the students’ eyes, not only reduces the risk of off-task behaviour, but also sustains a productive atmosphere for students.

## TOWARDS AN INTELLIGENT SYSTEM IN THE CLASSROOM

The interventions that were discussed above require an intensive one-to-one interaction with the students who require help. However, it is unrealistic to expect teachers in classrooms to be able to adhere to the demanding role of facilitators, keeping track of all students' actions while allowing them to explore and have the freedom to choose their immediate goals. As mentioned above, there are multiple ways of constructing a pattern and therefore multiple ways of expressing general solutions for such activities. It is at this point that the value of a system that can provide information to the teacher becomes apparent.

One of the most practical issues regarding students' interactions in such environments is that despite the familiarisation process, there is a need to remind students of certain features or even prompt them to use those which could prove useful for their chosen strategy. Therefore, it should be possible to identify (based on students' actions) which tasks of the familiarisation activity they should repeat. An intelligent system could highlight tools relevant to their current actions or offer a quick demonstration directly taken from their familiarisation activity. Furthermore, it could repeat their previous successful interactions relevant to the current activity.

In terms of the teachers' responsibility to attend to and help all the students in a classroom our studies highlighted the difficulty to prioritise which student to help. It is inevitable, therefore, sometimes to offer support to students who do not need it as much as others or even leave some students unattended due to the time constraints of a lesson. Moreover, it is possible for students to misunderstand certain concepts and leave a lesson with a false sense of achievement. Of course, it is difficult for an intelligent system to detect this accurately. However, it is possible to draw the teacher's attention to students potentially in need. By providing therefore information regarding students' progress at various times during a lesson as well as alerting them of likely misconceptions, it becomes possible for the teacher to spend their time and effort efficiently.

Besides these teachers' difficulties, there are situations when, despite having carefully-planned lessons, teachers are required to take immediate and effective decisions during lessons to accommodate their students' needs. For example, noticing when students are having difficulty with certain tasks or providing extension work are interventions which could be delegated to our system, allowing more time for teachers to provide essential help. Moreover, the collaborative component of an activity could be supported by the system by recommending effective groupings of students and allowing them to co-construct patterns whilst reducing dominance and promoting successful collaboration. The system could inform the teacher about the dynamics of different groups and alert them of possible concerns regarding the groups' progress as well as suggest more productive groupings (e.g. group students with different constructions but equivalent general expressions).

In addition, although we acknowledge the strong dependency between motivation, engagement and the design of the activities, it was evident that some students were at points disengaged. Even if off-task behaviour can sometimes lead to fruitful outcomes and intrigue students' thinking processes towards a direction, there is a need in automatically detecting such behaviour and informing the teacher. It then becomes the teacher's responsibility to decide how and whether to intervene.

The aforementioned suggestions for intelligent support could ease the use of an exploratory environment like the eXpresser in the classroom. It is often the case that such systems end up being used as a tool just to demonstrate certain mathematical concepts because of similar difficulties faced in classroom as those we reported here. Moreover, although quite a few 'intelligent' tutoring systems have been designed to provide support and personalised feedback to students and are starting to be integrated in classroom (Forbus et al., 2001), they usually scaffold the students with predetermined solution methods and by definition restrict students' reaching their own generalisations. Our team's challenge is to build a system that provides students the freedom to explore, make mistakes, get immediate feedback on their actions while assisting teachers in their difficult role in the classroom and therefore enable the successful teaching and learning of the idea of mathematical generalisation.

## NOTES

1. See http://www.migen.org/ for details. Funded by the TLRP, e-Learning Phase-II; Award no: RES-139-25-0381.

2. Our system comprises of two additional components, the eGeneraliser, which aims to provide students with personalised feedback and support during their interactions with the microworld, and the eCollaborator, which aims to foster an online learning community that supports teachers in offering their students constructions and analyses to view, compare, critique and build on.

3. We would like to acknowledge the rest of our research team and particularly Sergio Gutierrez, Ken Kahn and Darren Pearce who are working on the development of the MiGen system.

4. Each attribute has an associated icon tentatively depicted as cogs "to indicate the inner machinery of a pattern". As the design of eXpresser is evolving our team is evaluating the appropriateness of these icons.

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# ESTABLISHING A LONGITUDINAL EFFICACY STUDY USING SIMCALC MATHWORLDS®

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*We describe the construct of a 4-year longitudinal efficacy study implementing dynamic mathematics software and wireless networks in Algebra 1 and 2 classrooms. We focus on student learning and motivation over time, and issues of effective implementation in establishing a longitudinal study.*

## INTRODUCTION: BACKROUND TO DYNAMIC MATHEMATICS

New forms of mathematics technology (e.g., dynamic geometry) can provide *executable representations*—representations that transform the mathematics made by students into a more tangible and exciting phenomenon (Moreno-Armella, Hegedus & Kaput, 2008). In particular, we have designed and used SimCalc MathWorlds® to transform students' mathematical constructs into fascinating motion phenomena. Second, networks can intimately and rapidly *link private cognitive efforts to public social displays*. Consequently, students can each be assigned a specific mathematical goal (e.g., playing the part of a single moving character by making a graph with certain mathematical characteristics), which instantly links to public social display (e.g., the parade constituted by all characters moving simultaneously). This approach shifts the types of critical thinking that are possible in mathematics classrooms and transforms the role of instructional technology by integrating it into the social and cognitive dimensions of the classroom.

Our connected approach to classroom learning highlights the potential of classroom response systems to achieve a transformation of the classroom-learning environment. Similarly other investigators have expanded their approaches to include devices that allow aggregation of mathematical objects submitted by students. (Stroup, Ares & Humford, 2005).

## SITUATED NEED

Our proposed work addresses three essential needs: (i) the Algebra Problem (RAND, 2002), (ii) the related problem of student motivation and alienation in the nation's schools, especially urban secondary schools (National Research Council, 2003), and (iii) the widely acknowledged unfulfilled promise of technology in education, especially mathematics education (e.g., Cuban, 2001).

An important analysis by the National Academies Institute of Medicine (National Research Council, 2003) of student motivation at the high school level reveals in painful detail what most high school teachers (and parents) know only too well: that student motivation in high schools, and even more acutely in urban high schools, is an urgent and complex national problem. The report also recommends that high school courses and instructional methods need to be redesigned in ways that will increase adolescent engagement and learning.

Ethnographical studies of high school students (Davidson & Phelan, 1999; Phelan, Davidson, & Yu, 1998) reveal a world of alienation with strongly negative responses to standard practices (Meece, 1991) and strong sensitivity to interactions with teachers and their strategies (Davidson, 1999; Johnson, Crosnoe & Elder, 2001; Skinner & Belmont, 1993; Turner, Thorpe, & Meyer, 1998). Negative responses, particularly as they are intimately connected with self image and sense of personal efficacy, can be deeply debilitating, both in terms of performance variables (Abu-Hilal, 2000) as well as in the ability to use help when it is available (Harter, 1992; Newman & Goldin, 1990; Ryan & Pintrich, 1997). See the comprehensive reviews by Brophy (1998), Newmann (1992), Pintrich & Schunk (1996), and Stipek (2002). On the other hand, students exhibit consistently positive responses to alternative modes of instruction and content (Ames, 1992; Boaler, 2002; Mitchell, 1993), particularly those that build upon intrinsic instead of external motivation (Linnenbrink & Pintrich, 2000).

The literature on motivation in education and social situations in general has focused on intrinsic and extrinsic motivation with a great deal of debate (Sansone & Harackiewicz, 2000). Intrinsic motivation reflects the propensity for humans to engage in activities that interest them. Extrinsic motivation, such as rewards, can have an undermining effect and decrease intrinsic motivation, i.e., the reason why the person chose to want to do the activity in the first place (Deci, 1971). Yet both intrinsic and extrinsic motivation, as a key feature of participation in mathematics classrooms, have appeared to be an orthogonal field of inquiry to the development and instruction of content, with motivation hesitantly intersecting with education in the form of "motivational strategies," incentivizing students to learn mathematics because it is "fun" or "applicable" to *their* life, through relevant contexts, e.g., sports or vocations.

Relevance, unfortunately, is a somewhat indirect means to link motivation and mathematics—the link between immediate cognitive effort and later applications that may seem improbable to students. There is a more direct alternative. Students can become motivated because they want to participate more fully in what their classroom is doing now. The alternative, thus, is to link motivation and mathematics through *participation*.

We advocate two radically new forms of participatory activity in technology-enhanced environments:

1. Mathematical Performances. These activities emphasize individual student creations, small group constructions, or constructions that involve coordinated

interactions across groups that are then uploaded and displayed, with some narration by the originator(s).

2. Participatory Aggregation to a Common Public Display. These activities involve systematic variation, either within small groups, across groups, or both, where students produce functions that are uploaded and then systematically displayed and discussed to reveal patterns, elicit generalizations, expose or contextualize special cases, and help raise student attention from individual objects to families of objects.

These activities aim at enhancing mathematical literacy, debate and coherent argumentation—all fundamental mathematical skills. The central point is that each requires and rewards students for cognitive engagement in producing tangible phenomena that are simultaneously phenomenologically exciting and mathematically enlightening. This happens not at some future time when mathematics can be applied to a career or personal goal; instead these activities draw students in and sustain their interest because they are exciting and enlightening in the moment, in the classroom. These activities create an intrinsic motivation context with a socio-cultural view to “motivation in context” (Hickey, 2003) that is also intrinsically mathematical, accomplishing a much more intimate intertwining of motivation and mathematics that can be typically accomplished in existing classrooms.

## PRIOR WORK

SimCalc MathWorlds® creates an environment where students can be part of a *family of functions*, and their work contributes to the mathematical variation across this mathematical object. Consider this simple activity, which exemplifies a wider set of activity structures. Students are in numbered groups. Students must create a motion (algebraically or graphically) that goes at a speed equal to their group number for 6 seconds. So, Group 1 creates the same function, Y=(1)X, Group 2, Y=(2)X, etc. When the functions are aggregated across the network via our software, students’ work becomes contextualized into a family of functions described algebraically by Y=MX (see Figure 1 below). Students are creating a variation of slope and in doing so this can help each student focus on their own personal contribution within a set of functions.

At the heart of SimCalc is a pedagogical tool to manage classroom flow. This tool allows teachers to control who is connected to the teacher computer using a simple user interface, and choose when to “freeze” the network and aggregate students’ work or allow students to send a number of tries via the TI-Navigator™. In addition, teachers have control over which set of contributions (e.g., Group 1’s functions) and which representational perspectives (e.g., tables, graphs, motions) to show or hide. Thus, the management tool encapsulates a significant set of pedagogical strategies supported by question types in existing curriculum materials to satisfy a variety of pedagogical needs, focus students' attention depending on their progress, and promote discussion, reasoning and generalization in a progressive way at the public level.

In our prior research, students build meaning about the overall shape of the graphs and have demonstrated gestures and metaphorical responses in front of the class when working on this activity. For example, in two entirely different schools, students have raised their hand with fingers stretched out (see Figure 1 below), and said it would look like a "fan." In this socially-rich context, students appear to develop meaning through verbal and physical expressions, which we observe as a highly powerful way of students engaging and developing mathematical understanding at a whole group level. Various forms of formative assessment can said to be evident as each student's work emerges in a public display, and representations can be "executed" (Moreno-Armella & Block, 2002) to test, confirm or refute ideas. These forms of reflection, enabled through particular question-types and classroom dialogue focused on the dynamic representations, can be attributed to students learning and resonate with established research on formative assessment (Black & William, 1998; Boston, 2002).

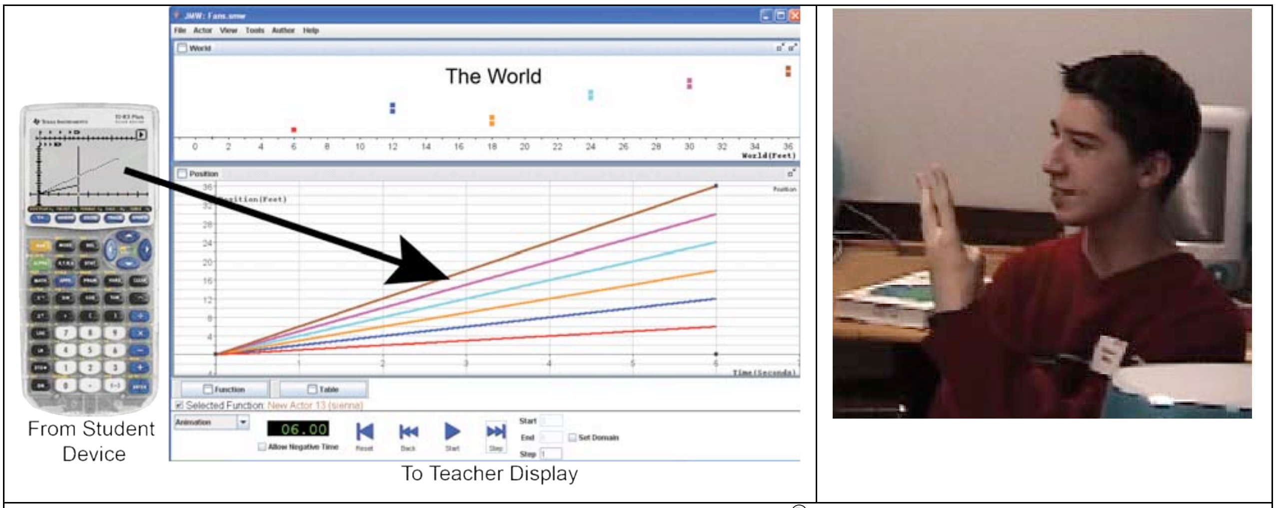


*Figure 1. Sample Function in SimCalc MathWorlds®*

Over the past ten years, over the course of three consecutive research and development projects (NSF ROLE: REC-0087771; REC-0337710; REC-9619102) and related projects at TERC (NSF REC-9353507), the SimCalc project has examined the integration of the Mathematics of Change and Variation (MCV) as a core approach to algebra-intensive learning. This work has led to a Goal 3 IERI-funded study (NSF REC-0437861), led by SRI International, focusing directly on large-scale implementability and teacher professional development in TX, and a recently funded IES Goal 2 project in the high school grades (IES Goal 2 # R305B070430) focusing on longitudinal impact of our curriculum and software products distributed by Texas Instruments on their popular graphing calculators in combination with a commercially available wireless network (TI-Navigator™ Learning system).

The Scale-Up pilot work employed a set of SimCalc resources in a delayed-treatment design. Teachers were initially randomly assigned to one of two groups. An ANOVA of difference scores (again teacher nested within condition) was significant [$F(1,282)=178.0$, $p<0.0001$]. The effect size for the gain in the group that used SimCalc is 1.08. In our main study, which is a randomized controlled trial in which 95 7th-grade mathematics teachers were randomly assigned to implement a 3-week SimCalc curriculum unit following training, our analyses show an effect size of 0.84 (Roschelle, Tatar, Shectman et al., 2007).

Prior work has documented statistically significant evidence for impact of SimCalc materials in connected "networked" environments with computers and calculators (Hegedus & Kaput, 2004) under multiple quasi-experimental interventions across grades 8-10 and college students demonstrating statistically significant increases ($p<0.001$) in student mean scores (effect=1.6) but with an even higher effect on the at-risk 9th grade population (effect=1.9). A major finding of our work was that critically important skills such as graphical interpretation were improved, i.e., cognitive transfer was evident. Recent studies show similar statistically significant results in terms of student learning and shifting attitudes towards learning mathematics in connected environments (Hegedus, Kaput, Dalton et al., 2007). We have also analyzed the changing participation structures using frameworks from linguistic anthropology (Duranti, 1997; Goffman, 1981). Our work has described new categories of participation in terms of gesture and language (Hegedus, Dalton, Cambridge et al., 2006) new forms of identity (Hegedus & Penuel, 2008), and theoretical advances in dynamic media and wireless networks (Hegedus & Moreno-Armella, 2008; Moreno-Armella et al., 2008).

## DESIGN ASPECTS OF EFFICACY WORK

In this context, our research program (funded by the US Department of Education, IES Goal 2 # R305B070430) builds on prior work to examine this problem. It is focused on outcomes in terms of both grade-level learning gains and longitudinal measures that relate to students' progress and motivation in mathematics across the grades in Algebra 1 and 2 classrooms.

SimCalc combines two innovative technological ingredients to address core mathematical ideas: Software that addresses content issues through dynamic representations and, wireless networks that enhance student participation in the classroom. We have begun to develop materials that fuse these two important ingredients in mathematically meaningful ways and developed new curriculum materials to replace core mathematical units in Algebra 1 (8-12 weeks) and Algebra 2 (4-8 weeks) at high school. We are measuring the impact of implementing these materials on student learning (high-stakes State examinations in Massachusetts (MA), USA) and investigating whether one or multiple involvements in this type of learning environment over the course of their high school years affects their motivation to continue studying mathematics effectively and enter STEM-career trajectories.

Our work is conducted in eight school districts in MA offering a wide variety of settings in terms of performance on State exams and Socio-Economic Status (SES). Our treatment interventions are in $9^{th}$ and $11^{th}$ grade classrooms (Algebra 1 then 2) but we will also track some students when they are in $10^{th}$ and $12^{th}$ grade collecting simple questionnaire data. Our study is a small-scale cluster randomized experiment where we cluster at the classroom level, randomly assigning two classrooms in each school to treatment in our main studies (total of 28 classrooms and a. 500 students in each main study).

We are using two instruments comprised of standardized test items to measure student's mathematical ability and problem-solving skills before and after each intervention. We are also collecting survey data on student's attitude before during and after the intervention. We are administering these tests and surveys at similar times (with respect to curriculum topics covered) in treatment and control classrooms. Video data from periodic classroom visits are being analyzed using participation frameworks from prior work and triangulated with variations in student survey data on attitude.

We are using suitable statistical methods to assess gain relative to the control groups, and between-cluster variation using mixed-Hierarchical Linear Modeling. We are also collecting survey and classroom observation data to assess changes in attitudes and participation, and daily logs by teachers to monitor fidelity of implementation.

We have completed our first year of 4 years work with our first cohort of students that we will track for the duration of their high school career and will present initial findings from our pilot study and challenges we have addressed in sampling and establishing a longitudinal program of research. We focus on results from factor analyses of our survey instruments on student and teacher attitude and correlations with student learning. Following a minimal effect size in our pilot study, we aim to present findings for improving effective implementation from analyses of teacher daily logs and classroom video.

Such methodologies build a comprehensive program for evaluating how prior findings (briefly highlighted above) can scale to larger implementations whilst being cognizant of issues of fidelity. Our ongoing work and preliminary analyses report of the potential effect on outcome measures such as student learning and motivation.

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# INTEROPERABLE INTERACTIVE GEOMETRY FOR EUROPE – FIRST TECHNOLOGICAL AND EDUCATIONAL RESULTS AND FUTURE CHALLENGES OF THE INTERGEO PROJECT

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*In this overview article we describe the manifold achievements and challenges of Intergeo[1], a project co-funded within the eContent*plus *programme[2] of the European Union.*

## THE INTERGEO PROJECT

The Intergeo project started in October 2007 and will be funded until September 2010. Its main concern is the propagation of Interactive or Dynamic Geometry Software.

### Goals

Interactive Geometry is a way to improve mathematics education by using computers and Dynamic Geometry Software (DGS) and there are many advantages in comparison to “classical” geometry without DGS. Figures can e.g. be easily manipulated [see e.g. Roth 2008] and thus virtually be brought to life, comparable to what movies mean to images or to what interactive computer games mean to motion pictures.

It is therefore not amazing that Interactive Geometry obtains more and more attention in many educational institutions. Around 25 per cent of the countries within the EU refer explicitly to DGS in their national curricula or guidelines and roughly 40 per cent refer to ICT in general. And although the remaining countries do not mention ICT, some of them recommend the use of DGS in schools [Hendriks et al. 2008].

Still, the adoption of DGS at school is often difficult. Despite the fact that a lot of DGS class material exists, Interactive Geometry is still not used in classrooms regularly. Many teachers do not seem to know about the new possibilities, or they do not have access to the software and/or resources.

The Intergeo Project has identified the three following major barriers, that have a negative impact on the use of Interactive Geometry in classrooms [Intergeo Project 2007]:

[1] http://inter2geo.eu
[2] http://ec.europa.eu/information_society/activities/econtentplus/index_en.htm

- Missing search facilities
  Though many resources exist, there remains the problem of finding and accessing them. If the files were put on the internet by their developers, they are virtually scattered all over the web and it is extremely hard to retrieve them by using search engines like Google.

- Lack of interoperability
  There are many different programmes for Interactive Geometry on the market and each software has its own proprietary file format. Thus, finding a file does not automatically mean that it can be used – it must be a file for the specific software that is used.

- Missing quality information
  And even if a teacher finds a file and the file works with her DGS, it may still be unsuitable for the use in class due to a lack of quality. Lacking quality can be software-sided in the way the figures are constructed or missing (or even wrong) mathematical background.

The aims of Intergeo are to dispose of the problems stated. In other words, Intergeo will

- enable users to easily find the resources they are looking for,
- provide the materials in a format that can be used with different DGS systems, and
- ensure classroom quality.

All three facets will be dealt with in the following chapters in extenso.

Furthermore, Intergeo attends to a topic that is mostly neglected but of high importance nonetheless: the question of copyright.

**Consortium**

The Intergeo Consortium, the founding partners of the Project, assembles software producers, mathematicians, and mathematics educators: Pädagogische Hochschule Schwäbisch Gmünd (D), Université Montpellier II (F). Deutsches Forschungszentrum für künstliche Intelligenz DFKI (D), Cabrilog S.A.S. (F), Universität Bayreuth (D), Université du Luxembourg (LUX), Universidad de Cantabria (ES), TU Eindhoven (NL), Maths for More (ES), and Jihočeská Univerzita v Českých Budějovicích (CZ). As the common interest of all partners is the propagation of sensible use of Interactive Geometry in the classroom, it was possible to collect both commercial, semi-commercial and free software packages. This is one of the key ingredients of the project: By building upon the joint knowledge and expertise of all parties, we hope to be able to address the needs of the teaching community.

## Participation of External Partners

The participation of External Partners, as Associate Partners, Country Representatives, and User Representatives justifies the basis for assuring the sustainability of the projects' goals as mentioned above. Furthermore, gathering partners, as software developers, teachers, and persons at school administration level enables the development of a Europe-wide network that is indispensable for obtaining the projects' major achievements.

Since the project start in October 2007, several key actors in interactive geometry throughout Europe, including software producers, mathematics educators, governmental bodies, and innovative users that can provide additional content or serve as test users for the first content iterations were acquired.

*Associate Partners*

The role of Associate Partners implicates a variety of tasks and expectations, as the adoption of the common file format for their software, the provision of significant content to the Project, the development of ontologies, and the conduction of classroom tests. The project could successfully find several important Associate Partners, see [Intergeo Project 2008] and the following table.

Table: List of Associate Partners

| Nr. | Country | Name | Nr. | Country | Name |
|---|---|---|---|---|---|
| **1** | Austria / USA | Markus Hohenwarter (GeoGebra) | **15** | Germany | Andreas Göbel (Archimedes Geo3D) |
| **2** | Brazil | Leônidas de Oliveira Brandão (iGeom) | **16** | Germany | Reinhard Oldenburg |
| **3** | Canada / Spain | Philippe R. Richard, Josep Maria Fortuny (geogebraTUTOR) | **17** | Germany | Andreas Meier |
| **4** | Canada | Jérémie Farret (3D Geom) | **18** | Germany | Roland Mechling (DynaGeo) |
| **5** | Croatia | Sime Suljic (Normala) | **19** | Italy | Giovanni Artico (CRDM) |
| **6** | France | Cyrille Desmoulins | **20** | Luxembourg | Daniel Weiler |
| **7** | France | Odile Bénassy (OFSET) | **21** | México | Julio Prado Saavedra (GeoDin) |
| **8** | France | François Pirsch (JMath3D) | **22** | Portugal | Arsélio Martins |
| **9** | France | The Sesamath association | **23** | Portugal | José Francisco Rodrigues (CMAF) |
| **10** | France | EducTice - INRP / Luc Trouche | **24** | Slovakia | Dusan Vallo |
| **11** | France | IUFM - Jacques Gressier (Geometrix) | **25** | United Kingdom | Albert Baeumel |
| **12** | Germany | Jürgen Roth (Universität Würzburg) | **26** | United Kingdom | Nicolas van Labeke (Calques 3D) |
| **13** | Germany | Heinz Schumann | **27** | United States | Joshua Marks (Curriki) |
| **14** | Germany | René Grothmann (C.a.R. / Z.u.L.) | | | |

*Country Representatives*

For each EU country a Country Representative serves as a contact person in their respective country. They come from ministries of education, preferably, and enable the Project to easily contact the relevant persons at school administration level. Based on these contacts, the project develops ways to map curricula into the ontology for geometry that suits all countries of the EU. The project could successfully find several Country Representatives, and a list is available at [Intergeo Project 2008].

*User Representatives*

User Representatives, as teachers and software partners, build the basis for the sustainability of the project. They are a contact point with their associations, in order to support the relationship with potential Intergeo-users [Intergeo Project 2008].

- Selected teachers ease experimentations in the classroom of educational content gathered by the project, promote the use of the Intergeo-platform and the philosophy of resource sharing and quality control.
- Selected Software-partners promote the uploading of content to the Intergeo-platform.

Among others, the selection of external partners will be performed at several local user meetings during the project period. The local user meetings have a central role in gathering the community of practice. They intend to help providing a complete European coverage:

- The Local User Meetings present Intergeo to the users: The need of a common file format for interoperability, the need of a web platform to share resources, the need of the ontology and the curriculum mapping to share resources across all European countries.
- The Local User Meetings are a good way to reach power users and engage them into the project to improve the projects' dissemination.
- Local User Meetings identify suitable schools for the Quality Assessment.

## MAJOR ACHIEVEMENTS

### Content Collection

The consortium promised to offer a significant amount of content for use in the database. Before the project started in Oct. 2007 we identified more than 3000 interactive resources to be used. All these and more[3] have been collected through the Intergeo platform by September 2008, first as *traces*, and now being converted to real assets that are searchable and tagged with meta-data. The available content ranges through all ages and educational levels, and also mathematical topics and competences. See http://i2geo.net to access and use the content.

### Copyright/Licence issues

A major issue with content re-use and exchange is the handling of intellectual property rights. This affects not only the copying of resources, but also the modification and the classroom use. Without being able to process the data, it is also impossible to offer the added value of cross-curriculum search, for example.

Thus, all content that is added to the Intergeo portal has a clear license, usually of the creative commons type allowing for modification and free (non-commercial) use. See http://creativecommons.org for details.

[3] On September 30th, 2008, there was a total amount of 3525 traces available.

## Theoretical Foundation For Cross-Curriculum Categorization and Search

Interactive geometry has one quality that makes it very particular among learning resources: it is often multilingual. This led us naturally to propose a search tool for interactive geometry resources that is not just a textual search engine but a *cross curriculum* search engine.

A simple scenario can explain the objective of cross-curriculum search: a teacher in Spain contributes a Cabri construction which is about the intercepting lines theorem (the *Teorema de Tales*) and measuring segment lengths; a teacher in Scotland looks for a construction which speaks about the *enlargement* transformation, *segment lengths*, and the competency to *recognize proportionalities*. They should match: the Scottish teacher should find the Cabri construction of the Spanish teacher (and be able to convert it to his preferred geometry system). No current retrieval system can afford such a matching process: there is no common word between the annotation and the query.

For cross-curriculum matching to work, a language of annotations is needed that encompasses the concepts of all curriculum standards and that relates them. Careful observation of the current curriculum standards (see [Laborde et al. 2008]) has shown that topics, expressed as a hierarchy, and competencies are the two main type of ingredients that are needed. To this end the Intergeo project has built an ontology of topics, competencies, and educational levels called GeoSkills. This OWL ontology [McGuinness et al. 2004] has been structured and is now being populated by a systematic walk through the national curriculum standards; a report of this encoding is at [Laborde et al. 2008]; completeness for several school-years has been reached in French, English, and Spanish curriculum standards. Because the edition of an ontology using a generic tool can be difficult, a dedicated web-based tool is under work which will make it possible for the complete German, Spanish, Czech, and Dutch curriculum standards to be encoded by the Intergeo partners and its associates.

For the match to happen, the input of topics or competencies has to be cared for. We use the auto-completion paradigm for this purpose: the (textual) names of each topic and competency are searched for in this process and the user can thus choose the appropriate node with sufficient evidence, maybe browsing a presentation of the topics and competencies. An alternative approach proposed is to browse curriculum standards, being

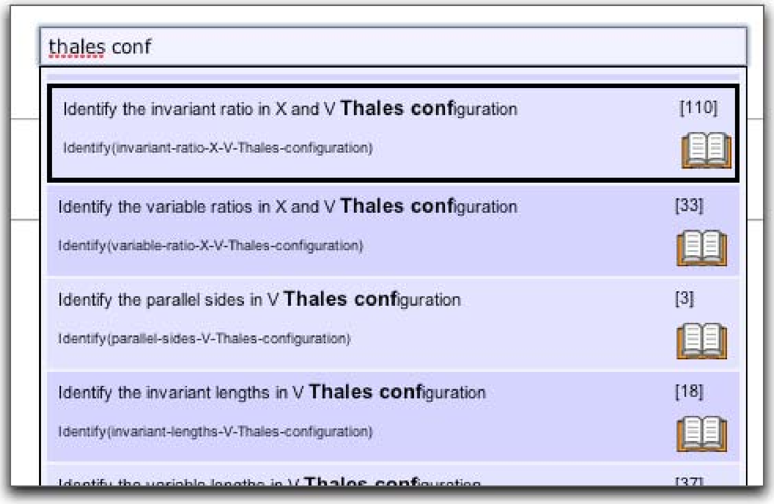


Figure 1: The skills textbox

documents that teachers potentially know well, in order to click a paragraph to choose the underlying topics and competencies.

**Quality Assessment Framework**

A Quality Assessment Framework for the Intergeo project was set up based on a questionnaire filled freely by the teachers themselves [Mercat et al. 2008]. This assessment has two different aims:

- To rank the resources so that, in response to a query, "good" resources are ranked before "bad" resources, at equal relevance with respect to the query.
- To help improve resources by identifying criteria to work upon in order for the author to revise his resource according to the user's input.

The questionnaire is both easy and deep; it can provide a light 2 minutes assessment as well as a deep pedagogical insight of the content. This is achieved by a top-down approach: The quick way just asks for 8 broad statements that can be answered on a scale from "I agree" to "I disagree":

- I found easily the resource, the audience, competencies and themes are adequate
- The figure is technically sound and easy to use
- The content is mathematically sound and usable in the classroom
- Interactivity is coherent and valid
- Interactive geometry adds value to the learning experience
- This activity helps me teach mathematics
- I know how to implement this activity
- I found easily a way to use this activity in my curriculum progression

These broad questions can be opened up by the reviewer to give more detailed feedback on issues of interest for him, such as "Dragging around, you can illustrate, identify or conjecture invariant properties" in the "Interactive geometry adds value to the learning experience" section.

Of course a thorough questionnaire is weighted more than a quick reply in the averaging of the different answers. The questionnaire is to be taken twice, as an a priori evaluation, before the actual course, and as an a posteriori evaluation, after the teaching has taken place. This second variant is being more weighted than the first one.

Different users are weighted differently as well: seasoned teachers with a lot of good activity, or recognised pedagogical experts, will have a high weight: their reviews are taken into account more than the average new user. Negative behaviour like steady bashing or eulogy will, on the contrary, lower user's weight. We are thinking as well about a social weight: teachers could flag some of their colleagues as "leaders", users whose past choices they liked, because they are teaching at the same level for example, and the weight of these leaders would increase.

## The I2Geo Platform

The central place of exchange of interactive geometry constructions is a web-platform; the i2geo.net platform is becoming a server where anyone with interest to interactive geometry can come to search for it and to share it.

The i2geo.net platform is based on Curriki, an XWiki-extension tuned for the purpose of sharing learning resources: strong metadata scheme, quality monitoring system and self-regulated groups. Being based on a wiki platform, Curriki offers an online editing and inclusion facility and thus also makes collaborative content construction possible.

The i2geo platform has three major adaptations compared to the tools provided by Curriki: the search and annotation tools, the review system, and the support for interactive geometry media.

The i2geo search and annotation tool uses the GeoSkills ontology described above: this allows the trained topics and competencies, the required ones, and the educational levels to be all entered using the input methods described above (auto-completion and pick-from-document).

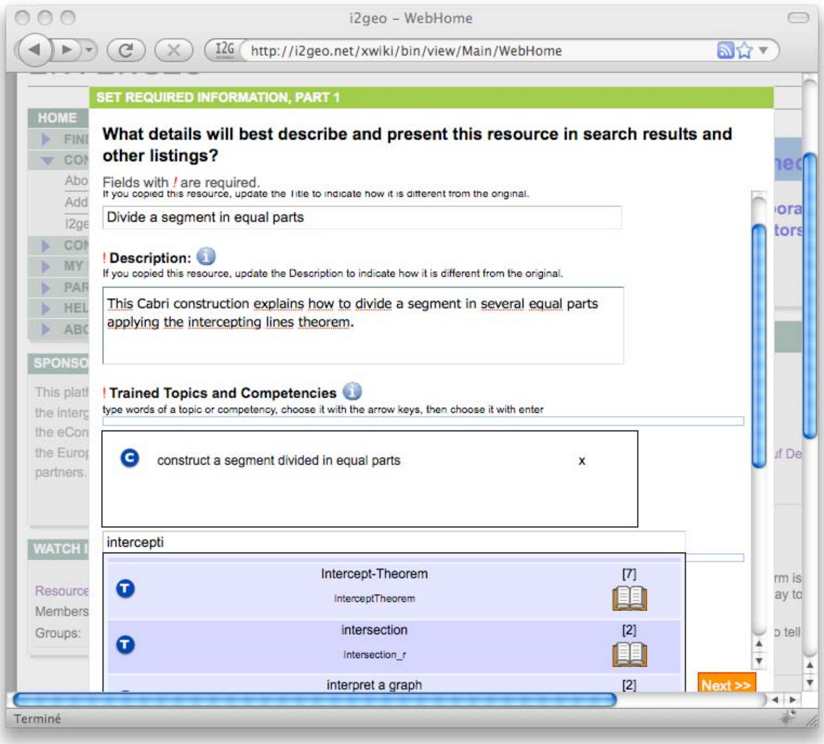


Figure 2: Editing metadata

Such elaborate methods are needed if one wants to honour the rich set of educational levels in Europe and the diversity of curriculum standards sketched in [Laborde et al. 2008].

The i2geo search tool uses the GeoSkills ontology as well: queries for any concept are generalized to neighbouring concepts which thus allows the match of the intercepting-lines-theorem when queried for the concept of enlargement.

The i2geo platform is under active development and can be experimented with on http://i2geo.net. Its current development focus is the input of metadata annotated resources and the review system described in the previous sections. The services specialized to the geometry resources, enabling easy upload, preview, and embedding of interactive geometry resources will be provided later.

### A Common File Format

A wide variety of Dynamic Geometric Systems (DGS) exist nowadays. Before this project, each system used incompatible proprietary file formats to store its data. Thus, most of the DGS makers have joined the project to provide a common file format that will be adopted either in the core of the systems or just as a way to interchange content.

The Intergeo file format aims to be the convergence of the common features of the current DGS together with the vision of future developments and the opinion of external experts. Its final version based on modern technologies and planed to be extensible – to capture the flavour of the different DGS – could serve as a standard in the DGS industry.

The specification of the first version of the Intergeo file format has been released by the end of July as deliverable D3.3 [Hendricks et al. 2008] after intensive collaboration between DGS software developers and experts. At present, the file format is restricted to the geometry in the plane, although it does not seem difficult to extend it, in the future, to the space. Besides it specifies only a restricted subset of possible geometric elements, which however lead to an agreement on the structure and basic composition of the format.

The general framework was clear from the outset: to design a semantically rich format that could be interpreted by at least all DGS in the consortium. One main design decision in this respect consists of the choice of constructions, as opposed to constraints, because in general, it is very difficult to give any particular solution for a set of constraints. Besides constraints of a strictly classical geometric nature do not say anything about the dynamic behaviour of a figure. A natural way to shed light on both of these problems is a more precise specification of how the objects depend on each other, stipulating first which objects are free and then proceeding step by step. Such a specification is called a construction. This decision implies less interoperability with constraint-based systems, since some of their resources will not be encodable into this format. But it ensures that construction-based DGS – the majority of the existing systems – will be able to interpret the resources.

As stated in the Description of Work, OpenMath Content Dictionaries are used to specify the symbols – the main ingredients used to describe a construction – of the file format. The XML schema can be generated automatically with some knowledge of how the atoms are expressed in XML. The complete list of official symbols defined so far can be found at http://svn.activemath.org/intergeo/Drafts/Format/.

As soon as version 1 of the file format got more concrete, some software developers started to investigate its practical usage by integrating it (partially) into their software. It was possible to move simple content between several of the packages in the project. For more information on the file format we refer to [Hendriks et. al 2008], which also lists the relevant URLS to see the progress.

## NEXT STEPS AND CHALLENGES

### Metadata Collection

With the arrival of the first curriculum-aware beta version of the i2geo.net platform we are now able to attach metadata to the existing content. This includes information about the authors, but also about the intended audience for a resource, the skills and competences that can be acquired through the resource, the prerequisites, and, of course, the topic – categorized according to the ontology.

While some of this information can be extracted automatically, there is still need for a lot of manual intervention. At the same time, the curricula available on the platform have to be revised and extended to accommodate all the content.

### Quality Testing

The partners in the Quality Assurance work package will conduct small-scale experimentations in the classroom during the period January-April 2009. Teachers, whether alone or in homogeneous teams, will

- Use the platform in order to identify content suitable for their course,
- First fill an a priori questionnaire,
- Teach the resource in the classroom,
- And finally report on its use by updating the a posteriori questionnaire.

We will have to agree on a modus operandi, recruit volunteers, especially among the teachers that were contacted during the users meetings, instruct them and have them conduct the experimentations.

Then these assessments will be analyzed. The analysis will be used to iteratively improve the quality assessment framework according to the users' feedback on usability and relevance of the different items and of the online platform.

It is a primary concern that all resources receive at least basic testing. Thus, we will check the overall coverage in the project and, if necessary, identify resources to be tested.

As the quality assessment primarily aims to make it possible to improve ranking and quality of the resources, we can use this as a performance indicator. For this, the changes in ranking due to the quality evaluation will be measured. Additionally, selected examples will be analysed in order to understand whether authors can infer improvements of their resources.

Via interviews with selected authors we try to understand how they perceived quality assessment and how we can improve its perception as positive, constructive and scientific more than negative, useless and personal.

In the final year of the project, mass scale experimentations will take place. More countries and more parts of the curriculum shall be covered.

### File format

As for version 1 of the file format some decisions that should be made with the help of other developers of DGS have been postponed, those experts are invited to join the discussion and propose solutions or give remarks, see [Hendriks 2008]. Thus, substantial modifications of this specification are expected to solve all practical issues that might arise.

### Better Visibility

The ultimate goal and a measure of success is the visibility of the Intergeo platform in Europe as a whole. After the first year was devoted to setting up the technical prerequisites and administrative processes, as well as clearly describing how we can measure and improve the standards for successful interactive resources, we can now offer a *usable* platform with substantial content. We now have to make the platform more visible and raise interest within the didactical community, the teachers, and the governments throughout Europe.

Today, the websites of the individual software packages from the project still have much more visits a day than the i2geo.net portal. So a first step will be to announce the portal on the websites of the software packages and on the websites of (associate) partners using banners and an i2g-compliance badge that shows the compatibility of the software with the i2g file format.

## CONCLUSIONS AND CALL FOR PARTICIPATION

In this article, we can only highlight the basic structure of the project. We invite everybody to visit the project website at http://inter2geo.eu, submit their own content on http://i2geo.net, join as an Associate Partner or become a User or Country Representative.

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# QUALITY PROCESS FOR DYNAMIC GEOMETRY RESOURCES: THE INTERGEO PROJECT

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*In this contribution, we present the European project Intergeo whose aims are first to develop a common language for a description of geometric figures that will ensure interoperability of the main existing dynamic geometry systems, and second, to gather and to make available pedagogical resources of a good quality. This text focuses on the quality process for dynamic geometry resources aiming at their perpetual improvement.*

***Keywords****: pedagogical resource, quality of a resource, dynamic geometry, teacher training*

## INTRODUCTION

This contribution concerns the issue of integration of ICT tools into teachers' practices and the means of supporting it. One of the keys is to provide teachers with pedagogical resources helping them to develop new activities for their pupils. However, we now know that the availability of resources is not sufficient. On the one hand, the abundance of resources makes difficult to find appropriate and quality resources (Guin and Trouche 2008, Mahé and Noël 2006). On the other hand, the availability of resources does not solve the problem of their appropriation by the teachers, which requires an evolution of teachers' competencies and their conceptions about the role of technology in teaching and learning mathematics (Chaachoua 2004).

This leads to consider the issue of teachers training. Numerous research works pointed out the efficiency of training based on co-design of pedagogical resources (Krainer 2003, Miyakawa and Winsløw 2007). Various training actions have been developed in France based on this principle, e.g., SFODEM and Pairform@nce (Gueudet et al. 2008). In Brazil, AProvaME project aimed to study the effects of a collaborative design of resources involving ICT tools by the teachers on their conceptions about the notion of proof and its teaching, as well as about the role of technology in mathematics learning (Jahn et al. 2007).

## THE INTERGEO PROJECT

Despite the availability and accessibility of ICT tools, and despite the recommendations in the curricula to use technology in France and in Brazil, teachers are reluctant to use these technologies (Artigue 2002). In the case of dynamic geometry systems (DGS) several reasons explain this resistance. The most important is certainly the shift in considering mathematical activity and teacher profession caused by the introduction of ICT into mathematics classroom (Lagrange and Hoyles 2006). However, other obstacles to using DGS by the teachers can not be neglected. First, the complexity of choice of a reliable and easy to use DGS among a number of existing systems, and the resulting constraints on the choice of resources that must match the chosen DGS. Next, it is hard to find pedagogical resources appropriate to a specific educational context. This can be attributed to a great amount of resources available on the Internet, but mostly to the lack of metadata, providing an accurate description of the resource content. Moreover, available resources do not often have the required quality to be used in a classroom. The difficulty for a teacher to evaluate quality and adequacy of a resource to her/his specific context is an obstacle to the ICT integration. For this reason, tools for indexing resources, as well as evaluating their quality appear essential.

These considerations lead to 3 goals of Intergeo project (www.inter2geo.eu/fr): (1) interoperability of the main existing DGS, (2) sharing pedagogical resources, and (3) quality assessment process of resources discussed in this paper.

## THEORETICAL BACKGROUND

### Notion of pedagogical resource

First, it is important to clarify what we mean by pedagogical resource. Indeed, Noël (2007) points out that the issue of resource evaluation relies on the definition of what is a pedagogical resource. Nevertheless, according to the author, in spite of numerous efforts, the definition of pedagogical resource remains vague and rather broad in its scope. The most often used one is drawn from LOM standards (2002): "… *any entity, digital or non-digital, that may be used for learning, education or training*" (p.5). Flamand (2004) specifies that in order to enhance learning, a Learning Object has to possess intrinsically a pedagogical intention. Thus, for the purposes of Intergeo project, we will consider as resources those "entities" (dynamic geometry figures, texts…) for which pedagogical intention is specified.

In addition, we share Trouche and Guin's (2006) point of view, which, referring to the instrumental approach (Rabardel 1995), considers a pedagogical resource as an artefact that needs to be transformed into an instrument by a teacher in the process of its use in her/his class. For the authors, usage of a resource is a condition for its existence. Resources are therefore living entities in evolution through their usages. In this perspective, the quality assessment process of Intergeo DG resources aims at enabling their perpetual improvement.

### Quality assessment process

The quality of a resource depends on its intrinsic characteristics, as well as on its adequacy to the context in which it will be used. A given resource can be "good" in one context and "poor" in another. Thus clarifying its educational goals and the school context in which its use is intended is also essential in determining and improving the quality of the resource.

Mahé and Noël (2006) constituted an evaluation typology based on a detailed analysis of evaluation means set up by various web sites offering pedagogical resources: a priori evaluation by the adherence institution; validation of resource conformity to a deposited content; peer-review by expert teachers; user evaluation;

cross-evaluation both by peers and users. The quality assessment in Intergeo project regarding DG resources consists of an evaluation by users and a peer review of a number of resources by a group of teachers supervised by math education researchers based on a priori analysis, use in a class, and a posteriori analysis of the resources. This process corresponds to the 5$^{th}$ type of evaluation mentioned above, rarely encountered according to the authors.

Mahé and Noël (ibid.) bring to light critical aspects of a resource to take into account in the evaluation process: technical aspect, content, design aspect and metadata. Criteria we have set up for the quality assessment process of DG resources draw from these categories, as well as from theoretical frameworks suitable for resource analysis: (1) *didactic theories*, namely Brousseau' theory of didactic situations offering tools for analysing pupil's activity and teacher's role, and Chevallard's anthropological theory allowing to address issues of resource adequacy to institutional expectations, and (2) *instrumental approach* (Rabardel 1995) providing a framework for instrumented activity analysis.

## USER EVALUATION OF THE QUALITY OF A RESOURCE

Our elaboration of a questionnaire for DG resource quality evaluation by users started by listing characteristics or elements of a resource related to its mathematical, didactical and pedagogical quality. We attempted to obtain a list as complete as possible. These characteristics were classified into 9 classes considered as relevant indicators of the resource quality: metadata, technical aspect, mathematical dimension of the content, instrumental dimension of the content, potentialities of DG, didactical implementation, pedagogical implementation, integration of the resource into a teaching sequence, usage reports. In what follows, we give an overview of criteria related to four classes referring to mathematical and didactical value of a resource.

### Mathematical dimension of the content of a resource

There is no doubt that, for a resource to be usable in a school context, its content has to be mathematically correct. Adequacy of the content with the curricula allows the evaluation of the resource utility. Finally, mathematical activities need to be in adequacy with the declared educational goals.

| *Criterion* | *Question* |
|---|---|
| Validity | Are the activities in the resource correct from a mathematical point of view? |
| Adequacy to the curriculum | Are the activities in adequacy with curricular and institutional constraints? |
| Adequacy to declared goals | Are the activities in adequacy with the declared educational goals? |

**Table 1. Mathematical dimension of the content of a DG resource**

### Instrumental dimension of the content of a resource

When a resource includes a DG file, it is necessary to check the coherence between the proposed activity and the geometric figure. In addition, the figure should behave as expected. Particular attention should be paid to the handling of limit cases and of numerical values such as measures of lengths and angles. Indeed, the dynamic diagram should behave according to mathematical theories and didactical goals. If special functionalities, such as macro-constructions, are used, a description of their operating mode will make easier the appropriation of the resource by a teacher.

| *Criterion* | *Question* |
|---|---|
| Adequacy of diagrams | Do the dynamic diagrams correspond to the proposed activities? |
| Behaviour of diagrams | Do the dynamic diagrams behave as expected in the activity? |
| Management of limit cases | Is the management of limit cases in the dynamic diagrams acceptable from the mathematical point of view? |
| Management of numerical values | Is the management of numerical values acceptable in the sense that it does not hinder mathematical aims of the activity? |
| Special functionalities | If the diagrams rely on special functionalities (e.g., macro-construction), is their operating mode clearly described? |

**Table 2. Instrumental dimension of the content of a DG resource**

## Potentialities of dynamic geometry

Numerous researches on DG put forward its potentialities and their contribution to the learning of geometry (Laborde 2002, Lins 2003, Tapan 2006). Criteria in this class aim first at evaluating how these potentialities are exploited in the resource, and more specifically to what extent DG contributes to improve learning activities comparing to paper and pencil environment. Second, its contribution to the achievement of educational goals is also analysed. This class comprises two criteria: (1) specific features of DG offering an added value to the resource, (2) role and use of drag mode, drawing on diversity of DG potentialities highlighted by research works (Laborde 2002, Healy 2000, Mariotti 2000). Even if a resource cannot benefit from each of them, we consider a resource that does not take any advantage of DG is of poor quality. Our hypothesis is that teachers perceive DG mainly as enabling to drag points to make pupils observing invariant properties (Tapan 2006).

| *Criterion* | *Question* |
|---|---|
| Elements contributing to the added value of DG in the resource | Is DG a visual amplifier improving graphical quality and accuracy of diagrams? |
| | Is DG used to obtain easily and quickly many cases of a same figure? |
| | Does DG provide an experimental field for the learner's activity? |
| | Do the feedbacks enable students validate their constructions by themselves? |
| | DG offers a possibility to articulate different representations of a same mathematical problem. Is this possibility used in the resource? |
| | Does DG allow students to overcome the spatio-graphical characteristics of a diagram to focus on its geometrical properties? |
| | Is the activity specific to DG, i.e., it would be meaningless without it? |
| | Does the use of DG in the activity contribute to achieve the educational goals? |
| Use and role of the drag mode in the resource | Is dragging used to illustrate a geometrical property, i.e., students are encouraged to drag elements and observe a given property that is invariant while dragging? |
| | Is dragging used to conjecture geometrical relationships, i.e. the point is to observe whether a supposed property is invariant while dragging elements? |
| | Is dragging used to study different cases of the diagram? |
| | Is dragging used to obtain a specific configuration satisfying given conditions? |

| | |
|---|---|
| | Is dragging used to identify dependencies between objects? |
| | Is dragging used to illustrate link between hypotheses and conclusion in a theorem, i.e., the point is to momentarily satisfy hypotheses by dragging elements (soft construction) and consider obtained properties as necessary consequences? |
| | Is dragging used to explore trajectories of geometrical elements (locus, trace)? |
| | Is the use of dragging explicitly mentioned in the instructions for students? |

**Table 3. Potentialities of dynamic geometry**

## Didactical implementation of the resource

Trouche (2005) points out that a successful integration of ICT requires a specific organization of pupil-computer interactions, which he calls "class orchestration". The author emphasises the importance of instrumental processes management in relation with learning mathematics. For this reason, we are convinced that a quality resource should provide a kind of assistance related to the class orchestration by means of elements concerning mathematics learning management with technology, which would help the teacher organize favourable learning conditions. We propose the criteria and questions, reported in table 4, addressing the issue of didactical implementation of a resource.

| *Criterion* | *Question* |
|---|---|
| Mathematical learning management | Do the students get involved easily in the proposed activity? |
| | Does the activity let enough initiative to students to choose their strategies? |
| | Does the resource describe students' possible strategies and answers? |
| | Does the resource provide information about teacher reactions to students' errors? |
| | Does the resource provide information about the teacher interventions at the beginning of the activity with the students? |
| | Does the resource provide information about the teacher interventions making the students' strategies evolve? |
| | Does the resource provide information about the teacher interventions during the phase of synthesis? |
| | Does the resource provide information about the validation phases? |
| | Does the resource discuss main characteristics of the activity, their effects on students' behaviours and other possible choices? |
| Instrumented activities management | Does the resource provide information about feedback from the software? |
| | Do the dynamic diagrams provide feedback enabling the student to progress in solving the given tasks? |
| | Does the resource provide information about the possible teacher interventions regarding instrumental aspects of the activity? |

**Table 4. Didactical implementation of a resource**

The resulting questionnaire comprises 9 classes with 59 questions altogether. It deals with a great variety of aspects of a quality DG resource and should be comprehensive. However, the questions are not homogenous from the point of view of expertise required to understand and to be able to provide a sound answer to each question. It can be expected that all users will not evaluate all aspects of a resource, but they will rather focus at those that correspond to their own expertise and their own representation of what is a quality resource. Nevertheless, the quality of a resource will take account of all evaluators; therefore we expect that each aspect will be evaluated by some of the users.

Given the length of the questionnaire, it seemed necessary to start by proposing a lighter version to users focusing on a few large questions (one per class) addressing globally each aspect of the resource. At the same time, the user will have the possibility to deepen her/his answer by answering more precise questions related to aspects s/he will wish to analyse further, according to her/his expertise. Moreover, s/he will be given opportunity to go back to the evaluation repeatedly. Note that the process of resource ranking (under development) will take account of the user's declared expertise and assign a weight to each provided answer accordingly.

Since the end-users of the questionnaire are teachers, we wished to test relevance and clarity of the questions. For this purpose, we organized a pilot experimentation with a group of teachers using a simplified version of the questionnaire. The experiment and some results are described in what follows.

## EXPERIMENTATION

Some elements of the initial questionnaire available in (Mercat et al. 2008) have been tested in Brazil, within an in-service teacher training "Geometry" module. Our goal was to analyse the relevance of evaluation criteria we defined, as well as to understand what a quality resource is for the teachers. A few more open questions were added aiming at identifying elements of a resource the teachers consider as helpful in order to appropriate and use the resource in their classes. A DG resource has also been designed to control some of its aspects for the experiment purposes and to be relevant for a teacher training.

### Presentation of the resource and of the questionnaire

The resource addresses the "quadrilaterals" topic and makes use of Cabri-geometry. It is constituted of a student worksheet, a teacher document and three DG files: two dynamic figures (cf. Fig. 1) and one macro-construction.

The teacher document provides a description of the resource: topic, school level, educational goals, prerequisites and required material. It also provides a brief presentation of the suggested organization of the sessions: classroom setting and roles of teacher and students.

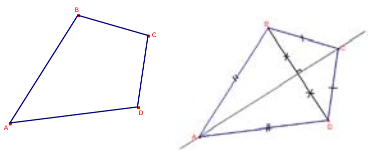

**Figure 1. Dynamic figures composing the resource**

The first mathematical activity, whose aim is to introduce a special type of a quadrilateral, an isosceles kite, draws from the idea of a "black box" specific to DG environments. It consists in reproducing a geometrical figure that behaves in the same way as a given model. Students are expected to explore the model in order to identify relationships between its elements, then to reconstruct the kite and validate their construction by using the macro-construction. In the resource, the exploration phase is partly guided to lead the students to characterize a kite by means of a maximum of its properties (related to its sides, angles and diagonals). Indeed, the activities are intended for 12-14 year old students and the instructors consider inappropriate to let them completely responsible of exploring the figure and identifying properties and relationships linking its elements. In the second activity, the students are invited to explore the figure and to conjecture a possibility to obtain other types of quadrilaterals (square, rhombus, non squared rectangle) from the kite. In both activities, the drag mode is essential to explore given dynamic diagrams.

For the purpose of the experiment, we selected and adapted several questions from the Intergeo questionnaire (cf. Fig. 2), namely those concerned with mathematical and instrumental quality of the resource, potentialities of DG and didactical implementation of the resource. The questions regarding DG are intentionally open aiming at highlighting which elements the teachers spontaneously mention as contributing to the added-value of DG in the resource.

Regarding the **pupil's worksheet**:

1. Are the texts of the activities:
   a. Clearly formulated?
   b. Mathematically correct?
2. Are mathematics tasks proposed to pupils easily identifiable?
3. Are DG figures provided with the resource:
   a. Easily accessible?
   b. In adequacy with the proposed activities?
   c. Do the figures behave as expected?
4. Would you use this kind of resource with your pupils? With or without modifications? Which ones?

Regarding the **dynamic geometry**:

1. What do you think about the role of dynamic geometry in the resource? Do you think this sequence could be proposed in paper-and-pencil environment?
2. If not, what aspects justify the use of the software?
3. Does the use of drag mode contribute to the achievement of the declared educational goals?

Regarding the **teacher document**:

1. What elements of the teacher documents do you consider as essential and the most relevant for a pedagogical use of the resource?
2. Do you think this document is sufficient for an easy appropriation of the proposed teaching sequence? Do you miss some information? Which?
3. Would you like to have other types of material or documents? Which ones?
4. Suppose that some of your colleagues have tested the resource in their class. What information do you consider important to be shared their experience?

**Figure 2. Questionnaire for resource evaluation used in the experiment**

Written answers provided by the teachers were one kind of data we gathered. These were completed by field notes of an observer recording relevant elements of exchanges among teachers.

## Experimentation and first results

The experimentation consisted in one 2h30 training session for 22 secondary mathematics teachers, who had, in average, six years of experience in teaching and most were “beginners” in DG. The training session was organized in three phases: solving activities from the student worksheet, a priori analysis of these activities, and analysis of the resource guided by the questionnaire (cf. Fig. 2). In what follows, we describe the phase 3 and present the first results.

In the teacher document, the participants particularly appreciated the brief description of the sequence considered as a kind of the resource "*visit card*", as well as the synthetic description of the sequence organisation: "*very well like that, one gets directly every essential information*"; "*one understands immediately how to organise the sequence*".

As regards the student worksheet, the teachers have found the tasks easily identifiable, mathematically correct and clearly formulated. A special attention was paid to the vocabulary with the intention to make the wording of activities accessible to pupils. The teachers used these worksheets also to understand the sequence organisation and its progression: "*student sheets allow us to understand well the whole sequence and to spot contents and objectives*"; "*Student sheets are very well designed. […], one sees clearly the sequence progression: observation of sides, symmetry between vertices and angles. Then, the construction is proposed and finally the study of some cases […]*".

Regarding elements helpful for resource appropriation but missing in the resource, the teachers expressed a need to understand how the macro had been constructed and how it works. They would also have liked to have more information about the teacher's role: what interventions and when, particularly during the institutionalisation phases; how to assist students' work. Some teachers pointed out that a document with reports of use, containing expected solutions and answers, but also possible students' difficulties accompanied with advices how to cope with them (e.g., student worksheet with commentaries for a teacher) would be helpful for a better appropriation of the resource.

Regarding DG, all teachers find unquestionable its contribution in the resource: "*activities specific to Cabri*"; "*the software is essential*"; "*impossible without Cabri*". This is not surprising since the resource was designed for. The teachers state more precisely that "*the software favours checking of properties*"; "*without drag mode and possibility to modify diagrams, properties wouldn't be visualized*". They spontaneously mention that dragging enables manipulating the figure and thus identifying its properties; checking properties; obtaining easily many different cases of a same figure; constructing figures easily, quickly and more precisely; making conjectures.

It is important to note that the teachers formulated all these criteria spontaneously, but they admitted that they would not have been able to do it without the framework of the questionnaire and without having done previously an a priori analysis of the resource. The questionnaire helped them focus on important aspects of the resource and they were able to provide a deeper analysis than expected. Thereof, the criteria set up for the evaluation questionnaire seem to be understandable by teachers, but what's more, they helped them analyse the quality of the resource. Thus, the questionnaire is not only a tool for characterizing the quality of a resource and for highlighting aspects to be improved, but it can also be used to train users' awareness of positive and negative aspects of a resource and in this way develop their professional skills enabling them to use it efficiently with their pupils.

## CONCLUSION

The results from the experimentation show the importance of training teachers to resource analysis. Indeed, the questionnaire helped the teachers focus on important aspects of the resource to look. These aspects were rarely taken into account before the training session. Among those, there is the teacher document containing information about the implementation of the resource and the added value of DG, in particular the role of drag mode.

On the other hand, the quality assessment process will lead to an improvement of a quality of resources, both at the metadata level highlighting information allowing an easier spotting of relevant and quality resources and at the level of the resource itself. Indeed, the quality criteria may be considered as a grid allowing to improve certain aspects of resources or to design new resources satisfying these criteria from the very beginning. Thus, this process can eventually give rise to a model that would act as a guide for resource designers by pointing necessary elements and helping make them explicit in an understandable and accessible way for potential users.

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# NEW DIDACTICAL PHENOMENA PROMPTED BY TI-NSPIRE SPECIFICITIES – THE MATHEMATICAL COMPONENT OF THE INSTRUMENTATION PROCESS

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*Relying on the collective work carried out in the e-CoLab project concerning the experimentation of the new calculator TI-nspire, we address the issue of the relationships between the development of mathematical knowledge and instrumental genesis. By analyzing the design of some resources, we first show the importance given to these relationships by the teachers involved in the project. We then approach the same issue from the student's perspective, using some illustrative examples of the intertwining of these two developments framed by the teachers' didactical choices.*

## INTRODUCTION

Educational research focusing on the way digital technologies impact, could or should impact on learning and teaching processes in mathematics has accumulated over the last two decades as attested for instance by the on-going ICMI Study on this theme. Questions and approaches have moved as far as research understood better the ways in which the computer transposition of knowledge (Balacheff, 1994) affects mathematical objects and the possible interaction with these, the changes introduced by digital technologies in the semiotic systems involved in mathematical activities and their functioning, and the influence of such characteristics on learning processes (Arzarello, 2007). They have also moved due to the technological evolution itself, such as the increased potential offered by technology to access mathematical objects through a network of inter-connected and interactive representations, or to develop collaborative work (Borba & Villareal, 2004). Increased technological power, nevertheless, generally goes along with increased complexity and distance from usual teaching and learning environments, and researchers have become more and more sensitive to the processes of instrumentalization and instrumentation that drive the transformation of a given digital artefact into an instrument of the mathematical work (Guin, Ruthven, & Trouche, 2004). They have revealed their underestimated complexity, and the diversity of the facets of such instrumental genesis both on the student and teacher side (Vandebrouck, 2008).

This contribution situates within this global perspective. It emerges from a national project of experimentation of the new TI-*n*spire in which we are involved. This artefact is quite innovative but also rather complex and distant from standard calculators, even from the symbolic ones. This makes the didactical phenomena and issues associated with its instrumentalization and instrumentation especially problematic and visible. In this contribution, we pay particular attention to the interaction between the development of mathematical knowledge and of instrumental genesis, analyzing how the teachers involved in the project manage it and how students experience it. Through a few illustrative examples, we point out some phenomena which seem insightful from this point of view, before concluding with more general considerations.

## PRELIMINARY CONSIDERATIONS

Let us first briefly present the TI-*n*spire and its main innovative characteristics, then the French project e-CoLab and also the theoretical frame and methodology of the study.

### A new tool

TI-*n*spire CAS (Computer Algebra System) is the latest symbolic ‘calculator’ from Texas Instruments. At first sight it undoubtedly looks like a highly refined calculator, but also just a calculator. However, it is a very novel machine for several reasons:

- Its nature: the calculator exists as a “nomad” unit of the TI-*n*spire CAS software which can be installed on any computer station;
- Its directory, file organiser activities and page structure, each file consisting of one or more activities containing one or more pages. Each page is linked to a workspace corresponding to an application: Calculator, Graphs & Geometry, Lists & Spreadsheet, Mathematics Editor, Data and Statistics;
- The selection and navigation system allowing a directory to be reorganised, pages to be copied and/or removed and to be transferred from one activity to another, moving between pages during the work on a given problem;
- Connection between the graphical and geometrical environments via the Graphs & Geometry application, the ability to animate points on geometrical objects and graphical representations, to move lines and parabolae and deform parabolae;
- The dynamic connection between the Graphs & Geometry and Lists & Spreadsheet applications through the creation of variables and data capture and the ability to use the variables created in any of the pages and applications of an activity.

When presented with the TI-*n*spire, we assumed that these developments could offer new possibilities for students’ learning as well as teachers’ actions. They could foster increased interactions between mathematical areas and/or semiotic representations. They could also enrich the experimentation and simulation methods, and enable storage of far more usable records of pupils’ mathematics activity. However, we also hypothesized that the profoundly new nature of this calculator and its complexity would raise significant and partially new instrumentation problems both for students and teachers and that making use of the new potentials on offer would require specific constructions, and not simply an adaptation of the strategies which have been successful with other calculators.

Excerpts both from students’ interviews and teachers’ questionnaires carried out/ handed out at the end of the first year of experiment support our hypotheses:

> "At first it was difficult, honestly, I couldn't use it… now it's OK, but at first it was hard to understand… the teacher, other students helped us and the sheet we got helped us out… how to save, use the spreadsheet, things like that…" Student's interview

> "In my opinion the richness of mathematical activities thanks to the connection between the several registers is the key benefit […] The difficulty will be the teacher's workload to prepare such activities so to render students autonomous." Teacher's questionnaire

> "There are still a few students for whom mathematics poses a big problem and for whom the apprenticeship of the calculator still remains arduous. These students find it hard to dissociate things and tend to think that the obstacles they face are inherent to the tool rather than to the mathematics themselves." Teacher's questionnaire

## Context of the research

This study took place in the frame of a two-year French project: e-CoLab (Collaborative mathematics Laboratory experiment) [1]. It was based on a partnership between the INRP and three IREM: Lyon, Montpellier and Paris. It involved six 10$^{th}$ grade classes, all of the pupils of which were provided with the TI-*n*spire CAS calculator. The students kept their calculators throughout the whole school year and were allowed to take them home. The groups on the 3 sites were composed of the pilot class teachers, IREM facilitators and university researchers. They met regularly on site although the exchange also continued distantly through a common workspace on the *EducMath* site, which allowed work memories to be shared and common tools (questionnaires, resources, etc.) to be designed.

All pilot teachers had a strong mathematical background but the expertise in using ICT varied from one to another. In the 1$^{st}$ year of the project, teachers and students were equipped with a prototype of the TI-*n*spire they had never worked with before. However, the willingness to articulate mathematical with instrumental knowledge was shared by all teachers, despite the work they later on admitted it required:

> "We have to devote an important amount of time to the instrumentation. This requires teachers to invest quite some time in order to design the activities, especially if they want to associate the teaching of mathematical concepts." Teacher's questionnaire

## Theoretical framework

Two theoretical streams guide our analyses. The first one is related to the *instrumental approach* introduced by Rabardel (1997). For Rabardel, the human being plays a key role in the process of conceiving, creating, modifying and using instruments. Throughout this process, he also personally evolves as he acclimatises to the instruments, both with regard to his behaviour as well as to his knowledge. In this sense, an instrument does not emerge spontaneously; it is rather the outcome of a twofold process involved when one "meets" an instrument: the instrumentation and the instrumentalization. Rabardel's ideas have been widely used in mathematics education in the last decade, first in the context of CAS (cf. Guin, Ruthven & Trouche, 2004 for a first synthesis) then extended to other technologies as spreadsheets and dynamic geometry software, and more recently on-line resources. Recent works such as the French GUPTEn project have also used the concept of *instrumental genesis* for making sense of the teachers' uses of ICT (Bueno-Ravel & Gueudet (2008).

We are also sensitive to the semiotic aspects of students' activities. Not only are we taking into account Duval's theory of semiotic representation (Duval, 1995) and the notions attached to it (semiotic registers of representation and conversion between registers), but more globally the diversity of highly intertwined semiotic systems involved in mathematical activity including gestures, glances, speech and signs, *i.e*. the "semiotic bundle" (Arzarello, 2007). In particular, when examining students' activity, we pay specific attention to the embodied and kinesthetic dimension of it (Nemirovsky & Borba, 2004) via the pointer movement or students' gestures.

### Methodology

We are interested in the students' instrumental genesis of the TI-*n*spire and in particular in considering the role mathematical knowledge plays in this genesis. Such analysis cannot be done without taking into account the characteristics of the tasks proposed to students and the underlying didactical intentions. Our methodology thus combines the analysis of task design as it appears in the resources produced by the e-CoLab group, and the unfolding of students' activity.

The analysis of students' activity relies on screen captures of students' activities made with the software Hypercam. HyperCam, already used in other research involving the study of students' use of computer technology (see for e.g. Casyopée, Gélis & Lagrange (2007)), enables us to capture the action from a Windows screen (e.g. 10 frames/sec) and saves it to an AVI movie file. Sound from a system microphone has also been recorded and some of the activities have been video-taped.

When relevant, we also back up our analysis by relying on students' or teachers' interviews/questionnaires carried out independently from the activities.

## TEACHERS' INSTRUMENTATION – DIDACTICAL INTENTIONS

### Didactical intentions

The pilot teachers involved in the experiment cannot be said to be "ordinary teachers". All of them have been involved, in one way or another, in the IREM's network, thus they were all somehow sensitive to didactical considerations and shared a fairly common pedagogical background. The relative success of the project was in part due to this familiarity, as one teacher acknowledged: "It is easier to communalize if we share the same pedagogical principles."

In particular, the willingness of intertwining mathematical content with instrumental knowledge was commonly held and despite the hard work that it meant, the joint work was perceived as a true added value as teachers seemed to work in harmony:

"We have to carry the instrumentalization and the mathematical learning in parallel. Activities are not evident to think of and take time to design. The help from others make us gain time and provide us with new ideas." Teacher's questionnaire

## Imprint on resources

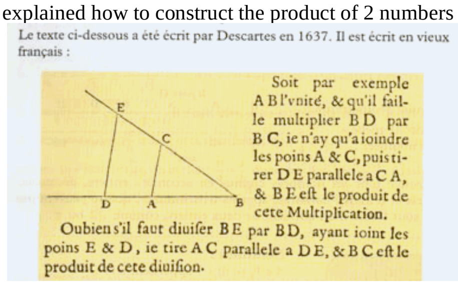
Around 25 resources were designed during the two years of the project. There are two kinds of resources: those created essentially to familiarize pupils with the new technological instrument (presentation of the artifact and introduction of some of its potentials), and those constructed around (and we should add "for") the mathematics activity itself [2]. In what follows, we mainly focus on the resources that support the teaching/learning of mathematical concepts and examine how teachers managed to articulate mathematical concepts with instrumental constituents.

The didactical intentions previously mentioned are clearly visible when examining the resources teachers designed, showing that these were built *from* the mathematical component yet at the same time *planning* a progressive instrumentation.

The *Descartes* resource is very enlightening in this sense. Teachers who have designed it acknowledged it appeared to be useful as an introduction into the dynamic geometry of the calculator, articulated with an application of the main geometrical notions and theorems introduced in Junior High School. It also offered the advantage of linking the work which had just been performed on numbers and geometry.

In this resource, several geometrical constructions are involved, enabling products and quotients of lengths to be produced and also the square root of a given length to be constructed. For the first construction proposed, the geometrical figure is given to the pupils together with displays of the measurements required to confirm experimentally that it does provide the stated product (fig. 1). The pupils simply had to use the pointer to move the mobile points and test the validity of the construction. Secondly, for the quotient, the figure provided only contained the support for the rays [BD) and [BE). The pupils were required to complete the construction and were guided stepwise in the successive use of basic tools as "point on", "segment", "intersection point", "measurement" and "calculation". Thirdly, they were asked to adapt the construction to calculate the inverse of a length. Finally for the square root they had the Descartes figure and were required to organise the construction themselves. Instructions were simply given for the two new tools: "midpoint" and "circle".

In his treatise on Geometry, Descartes explained how to construct the product of 2 numbers


Le texte ci-dessous a été écrit par Descartes en 1637. Il est écrit en vieux français :

Soit par exemple A B l'vnité, & qu'il faille multiplier B D par B C, ie n'ay qu'a ioindre les poins A & C, puis tirer D E parallele a C A, & B E eſt le produit de cete Multiplication.

Oubien s'il faut diuiſer B E par B D, ayant ioint les poins E & D, ie tire A C parallele a D E, & B C eſt le produit de cete diuiſion.

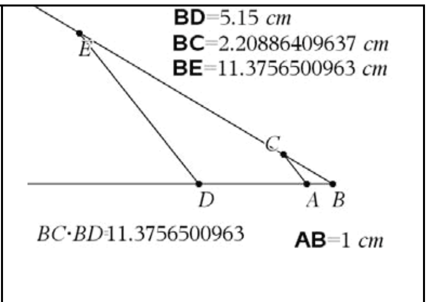



**Figure 1.** *First part of the Descartes resource (extracted from the pupil sheet and the associated tns file)*

In what concerns the resource *Equal areas*, the mathematical support is an algebraic problem with geometrical roots; it consists in finding a length OM such that two given areas are equal (fig. 2). The expressions of the two areas as functions of OM are 1st and 2nd degree polynomials and the problem has a single solution with an irrational value. This therefore falls outside the scope of the equations which the observed students are able to solve independently. In the first version of the resource, their work was guided by a sheet with the following stages: geometrical exploration and 1st estimate of this solution, refining the exploration with a spreadsheet to give the required value within a tolerance of 0.005, the use of CAS to obtain an exact solution, and finally the production of the corresponding algebraic proof by paper/pencil.

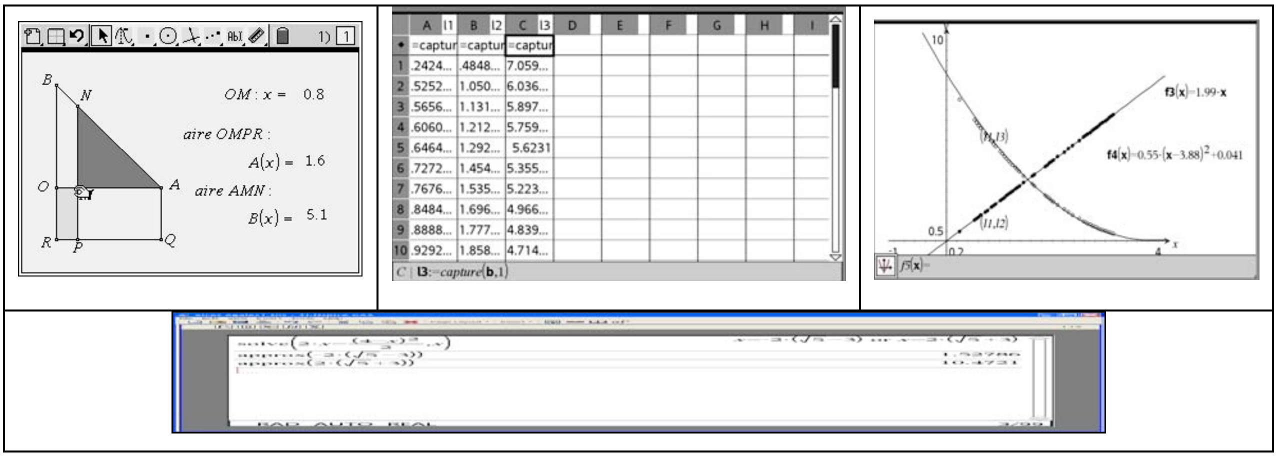


**Figure 2.** *Exploring progressively the problem of Equal Areas using different applications*

Experimentations led to the development of successive scenarios where more and more autonomy was given to the students in the solving of this problem, yet still requiring the use of several applications, discussing the exact or approximate nature of the solutions obtained, and the global coherence of the work.

## MERGING MATHEMATICS AND INSTRUMENT – STUDENTS' VIEWPOINT

Our analysis will rely on the experimentation of two particular resources already mentioned (*Descartes* and *Equal areas*) for the following reasons: they have been designed with an evident attention to both mathematical and instrumental concerns, but take place at different moments of students' learning trajectory and have different mathematical and instrumental aims. *Descartes* has been proposed early in the school year; it aims at introducing the dynamic geometry of TI-*n*spire while revisiting some main geometrical notions of junior high school, and connecting these with numbers and operations. *Equal areas* was given to students several months later, at the end of the teaching of generalities about functions. It aims at the solving of a functional problem from diverse perspectives, and at discussing the coherence and complementarities of the results that these perspectives provide. It also aims at informing us about the state of students' instrumental genesis after 6 months of use of the TI-*n*spire.

### Students and the *Descartes* resource

Two sessions and some homework were associated with this resource in the experimentation, and an interesting contrast was observed between the two sessions. The smooth running of the first session evidenced that a first level of instrumentalization of the dynamic geometry of the TI-*n*spire was easily achieved in this precise context. The successive difficulties met in the second session illustrated both the limits of this first instrumentalization and the tight interaction existing between mathematics and instrumentation. In what concerns the instrumentalization, we could mention students who inadvertently created a point that could superimpose on the points of the construction and invalidate measurements; the fact that they could not handle short segments on the calculator, or that they had not understood how to "seize" length variables in the geometry window for computing with them…

Regarding the interaction between mathematics and instrumentation, one difficulty appears to be especially visible in this situation: measures and computations in the geometry application are dealt with in approximate mode. Thus, when testing the validity of the construction proposed by Descartes for the quotient for instance, the students did not get exactly what they expected and were puzzled. Very interesting classroom discussions emerged from this situation which attest the intertwining of mathematical and instrumental issues. Students had limited familiarity with the tool, and had to understand that exact calculations are restricted to the Calculation application. The problem nevertheless was not solved just by giving this technical information, showing that this was not enough for making sense of such information, rather related to the idea of number itself, the distinction between a number and its diverse possible representations, the notions of exact and approximate calculations.

### Students and the *Equal area* resource

As already explained, this resource is quite different from the previous one and students had been using the TI-*n*spire for more than 6 months. It has been experimented several times with different scenarios, and the analysis of the data collected is still ongoing. Some instrumentalization difficulties were still observed, even when students worked with an improved version of the artifact. These often concerned the spreadsheet application, less frequently used, but the main difficulties involved tightly intertwined mathematics and instrumental issues as in the previous example. We will illustrate this point by the use of a spreadsheet for finding and refining intervals including the solution.

Students used the spreadsheet application after a geometrical exploration of the problem. This convinced them of the existence and uniqueness of the solution, provided its approximate value and showed that the geometrical application could not provide exactly equal values for the two areas. The use of the spreadsheet application generally raised a lot of difficulties linked to the syntax for defining the content of the successive columns, for refining the step taking into account the existing limitation in the number of lines available. Students often tried to refer to spreadsheet files used in previous problems to solve them. Some could be helpful (another functional problem), some were problematic (a probabilistic situation recently studied). Choosing an appropriate file required an ability to see the similarities and differences between the mathematical problems at stake. Benefiting from an adequate file required the matching of the two mathematical situations, establishing correspondences between the data and variables involved, and understanding how these reflected in the syntax of the commands. The use of the generated tables, once obtained, also raised many difficulties. Students tried to get the same values for the two areas or to find the closest ones. This was not at all easy, and very few of them were spontaneously able to create a new column for the difference. Moreover, when asked to find an interval for the solution, they were unable to exploit the table in a successful way. The idea that the solution of the problem corresponded to an inversion in the order of the two areas, and that they had thus to look at the two successive lines showing this inversion for getting the limits of the interval asked for was not a natural idea. The screen copies and discussions between students or/and with the teacher of this episode clearly illustrates to what extent mathematics and instrumentation are intertwined.

In these two examples, we have focused on the mathematical/instrumental connection through the analysis of students' difficulties but the observations also show episodes where an original mathematical/instrumental synergy is at stake, made possible by the students' joint mathematical and instrumental progression. We will illustrate this by examining students' activity when working on the previous problem, but with greater autonomy. A group of two students had begun with a geometrical exploration, then defined the two functions expressing the areas and moved to a graphical exploration, selecting an appropriate window for the problem ($0 \leq x \leq 4$). They carried out this exploration cleverly, created the intersection point of the two curves to get its coordinates and found numerical values with only 6 decimals. This fact associated with the visual evidence of the intersection point convinced them that they had got the exact solution. They came back to the geometry page and checked that this solution was coherent with the approximate value with 2 decimals they had already got. They then moved to the calculation application (exact mode) and asked for the solution of the equation. They obtained 2 irrational values and were puzzled. The screen captures show several quick shifts between the graphic and calculation pages, before one of the boys decided to ask for an approximate value of the two solutions. Once obtained, they came back to the graph page, changed the window so to visualize the 2nd intersection point, seemed satisfied, went back to the geometry page and discarded the 2nd solution as non relevant. Once more, we cannot enter into more details, but the productive interplay here is evident. Let us just add that there has been an interesting collective discussion about the conviction of obtaining an exact solution in the graph page and the rationale underlying it. Linked with a deep mathematics discussion, the way TI-*n*spire manages approximations in the different applications and the way the user can fix the number of decimals was clarified.

For making sense of such synergies and instrumented practices, there is no doubt in our opinion that a semiotic approach limited to the identification of treatments inside a given semiotic register of representation or conversions between such registers is not fully adequate. What we observe indeed is a sophisticated interplay between different instruments belonging to the students' mathematical working space and a swing between these certainly supported by technological practices developed out of school. These are efficiently put at the service of mathematical activity and part of their efficiency also results from their kinesthetic characteristics.

Beyond that, there is no doubt that the work performed by the students in this task, through the diversity of perspectives developed around the same mathematical problem, and the small group and collective discussion raised about the potential and limits of these different perspectives and their global coherence, corresponds to a quality of mathematical activity hardly observed in most grade 10 classes.

## CONCLUSION AND PERSPECTIVES

Due to its specific features which distinguish TI-*n*spire from other calculators and as it had been envisaged *a priori*, the introduction of this new tool was not without difficulty and required considerable initial work on the part of the teachers, both to allow rapid familiarisation on their part and those of the pupils but also to actualize the potentials offered by this new tool in mathematics activities. When examining both the design of the resources created by the pilot teachers and the work performed by students, as we have tried to show in this contribution, we grasp how delicate and somehow frail the harmony between the mathematical and instrumental activity is, and how the semiotic games underlying it are complex. We also see the impact of new kinds of instrumental distances (Haspekian & Artigue, 2007) and closeness that shaped teachers' and students' activities: on the one side, distance from more familiar mathematical tools and especially graphic and even symbolic calculators, on the other side closeness with technological artifacts on offer out of school (computers, IPods, etc…). These characteristics affect teachers and students differently, and individuals belonging to the same category differently, according to their personal characteristics and experience. They can have both positive and negative influences on teaching and learning processes and need to be better understood. For that purpose, beyond the theoretical constructs we have used in this study, we consider it interesting to extend the tool/object dialectics (Douady, 1986) to the instrumental component of the activities. By choosing to closely articulate mathematical and instrumental knowledge, the latter is inevitably introduced within a specific mathematical context. Reinvesting instrumental knowledge also requires students, even implicitly, to decontextualise and to a certain extent generalize what has been acquired.

## NOTES

1. A more general overview of the project as well as other findings can be found elsewhere (see Aldon et al., 2008).

2. Some resources can be found at: http://educmath.inrp.fr/Educmath/partenariat/partenariat-inrp-07-08/e-colab/

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# ISSUES IN INTEGRATING CAS IN POST-SECONDARY EDUCATION: A LITERATURE REVIEW

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*We discuss preliminary results of a literature review pilot study regarding the use of CAS in higher education. Several issues surrounding technology integration emerged from our review and are described in detail in this paper. The brief report on the type of analysis and the integration scope in curriculum suggest that the multi-dimensional theoretical framework proposed by Lagrange et al. (2003) needs to be adapted for our focus on systemic technology integration in tertiary education.*

## INTRODUCTION

A growing number of international studies have shown that Computer Algebra System (CAS-based) instruction has the potential to positively affect the teaching and learning of mathematics at various levels of the education system, even though this has not been widely realized in schools and institutions (Artigue, 2002; Lavicza, 2006; Pierce & Stacey, 2004). In contrast to the large body of research focusing on technology usage that exists at the secondary school level, there is a definite lack of parallel research at the tertiary level. However, Lavicza (2008) highlights that university mathematicians use technology at least as much as school teachers, and that the innovative teaching practices involving technology that are already being implemented by mathematicians in their courses should be researched and documented. Further, Lavicza (2008) found that within the research literature there existed only a small number of papers dealing with mathematicians and university-level, technology-assisted teaching. In addition, most of these papers are concerned with innovative teaching practices, whereas few deal with educational research on teaching with technology. These findings coincide with school-focused technology studies conducted by Lagrange et al. (2003) and Laborde (2008).

We aim to point out that it is particularly important to pay more attention to university-level teaching, because universities face new challenges such as increased student enrollment in higher education, decline in students' mathematical preparedness, decreased interest toward STEM subjects, and the emergence of new technologies (Lavicza, 2008). Mathematicians must cope with these challenges on a daily basis and only a few studies have offered systematic review and developed recommendations in this area. Our project aims at both documenting university teaching practices involving technology, and formulating recommendations for individual and departmental change. Our research program also aims at raising the amount of attention paid to tertiary mathematics teaching from a research point of view and, from a more practical side, elaborating on specific issues and strategies for the systemic integration of technology in university mathematics courses.

## METHOD DESIGN AND IMPLEMENTATION

Based on the above-mentioned Lavicza (2008) findings and recommendations, we designed a mixed methods research study which involves a systematic review of existing literature regarding CAS use at the tertiary level. The theoretical framework developed by Lagrange *et al.* (2003) involved several stages. They first reviewed a large number of papers in relevant journals and then categorized these papers into five "types." Based on these types, they then selected a sub-corpus of papers dealing specifically with educational research papers focusing on technology use mainly in the secondary school. Through the careful analysis of this sub-corpus of papers, they further developed seven dimensions, each with key indicators, and then proceeded to identify and further analyze papers that best described each of these dimensions.
The theoretical framework of Lagrange et al. (2003) provided our research team with a helpful foundation from which to prepare for our own literature review which will involve approximately 1500 papers/theses. It was decided to implement a pilot study for this large literature review in order to begin to work with the Lagrange *et al.* framework and to determine if it would be sufficient for our purposes, or may be in need of certain modifications. In the summer of 2008, we therefore began our pilot study focusing on 326 contributions dealing with CAS use in secondary/tertiary education. These papers were drawn from two well-regarded journals, namely the *International Journal for Computers in Mathematical Learning* (issues since its beginning in 1996) and the *Educational Studies in Mathematics* (since 1990). We also selected proceedings from two technology-focused conferences, namely the *Computer Algebra in Mathematics Education* (since its first meeting in 1999) and the *International Conference on Technology in Collegiate Mathematics* (since 1994 with first electronic proceedings). A sub-corpus of 204 papers dealing specifically with CAS use at the post-secondary level was also identified to further focus the analysis.

While the descriptive categories found within the Lagrange et al. template were helpful, we began to notice that several other category/theme columns would be helpful at this stage of the instrument/template development (e.g., we added fields such as "computer/calculator," "implementation scope," and "implementation issues"). An important point to note here is that in contrast to the Lagrange study where the majority of papers were those describing educational research results, our selection of papers revealed a majority that focused on practitioner innovations with very few involving educational research. Thus, we realized that in order to develop our template for reviewing the large number (1500) of papers in the research study proper, we would have to separate the practitioner report type papers from the educational research papers, and further modify the template in both of these areas. In this paper we outline preliminary results of our ongoing pilot study, with a specific focus on a series of "issues of implementation" at the tertiary level of education.

## RESULTS

The majority of the papers in the corpus are practice reports by practitioners (88%), whereas the remaining contributions are education research papers (10%) or letters to

| Table 1: Type of Contribution | |
|---|---|
| Presentation of Examples | 46% |
| Examples with practitioner reflections | 20% |
| Classroom Study | 6% |
| Classroom Survey | 3% |
| Examinations of a specific issue | 3% |
| Abstract only | 11% |
| ---------------------------------------------------- | ------- |
| Education research papers | 10% |
| Letters | 1% |

journal editors (1%) (see Table 1). Among the practice reports, different types of contributions become apparent. Some (94) are merely examples of CAS usage. Other papers (41) are mostly examples of CAS but feature reflections by the practitioner. A few (13) have the practitioners go further and include classroom data and perform some basic analysis. There are also papers (5) that focus on classroom surveys and a small set (7) that examines a specific issue in detail. The remaining contributions (23) are conference abstracts only. The analysis of the education research papers according to Lagrange et al.'s multi-dimensional framework (2003) is still in progress. In this paper, we focus our analysis mainly on practitioner reports.

In addition, nearly all papers are American (87%). The computer use is more evident (59%) than the use of graphical calculators (29%) or than the combined use of both computer and graphing calculators (10%). Furthermore, the most widely used CAS in the corpus is the graphing calculator (83 papers), followed by *Maple* (53) and *Mathematica* (43). *Derive* (21) and *Matlab* (11) are also common, as well as 27 papers dealing with other CAS. In what follows, we elaborate on one particular significant aspect of the study, namely "integration issues" that emerged from our review, and also briefly report on "integration scope."

## ISSUES OF CAS INTEGRATION

Education researchers and practitioners widely wrote about issues surrounding the use and implementation of CAS at post-secondary education (72 papers). With regard to practitioner reports, 56 papers identify some issues; of these there are 20 that go into considerable detail. These papers could be further divided into two categories: Seven of them deal with a specific problem relating to CAS (e.g., rounding error) and thirteen discuss various implementations of CAS while underlining the hurdles the authors encountered. Of the sixteen issues identified in the corpus and summarized in Figure 1, we divide them into three categories: Technical (first four columns), cost-related (fifth column), and pedagogical (last 11).

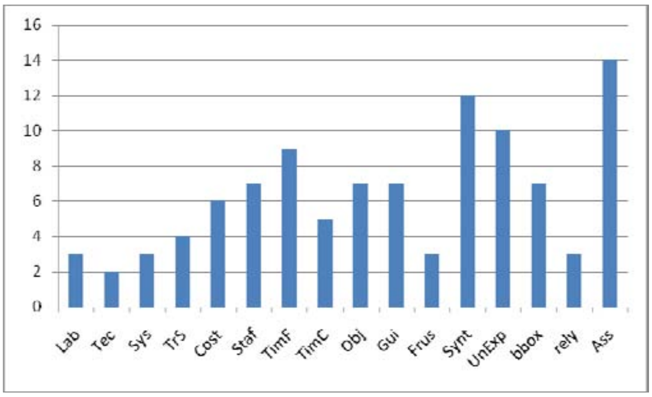


**Figure 1: Issues in integrating CAS in university education**

There are four issues discussed in the literature dealing specifically with technological aspects: Lab availability (Lab), reliability of technical support (Tec), system requirements (Sys) and troubleshooting (TrS). These issues may not be independent from each other. For example, May (1999, p. 4) urges instructors to test out their *Maple* worksheets on the lab computers rather than their own workstations due to such machines having less memory installed in them. Weida (1996, p. 3) notes that in troubleshooting, various hardware problems arise and his “experience and lots of calls to the Computer center” helps. An unexpected issue for him was the class interruption of students not enrolled in his class. While they would never think to disrupt a lecture, they would see nothing wrong with walking into his lab session to complete homework for other courses.

Many reports mention the issue of costs (Cost) incurred by integrating CAS into instructors’ courses, providing few further details beyond the existence of the financial obstacle. An exception occurs in one paper where the authors argue for a particular choice of open-source (free) technology (Hohenwarter et. al, 2007, p. 5).

Wu (1995) notes that besides the cost aspect, enacting calculus reform “requires more talent and training” (p. 1). This need for trained staff (staf) is mentioned in seven papers, often in conjunction with other issues. For example, to deal with technical difficulties during labs, Weida relies on his own experience to assist in troubleshooting (1996, p. 3). At the beginning of an attempt at CAS integration, Schurrer and Mitchell (1994, p. 1) wondered, “how they could go about motivating [sceptical mature faculty] to consider introducing the available technology and making the curricular changes this would require?”

Schurrer and Mitchel (pp. 1-2) further discuss the need for time for the faculty (TimF) to design courses and meaningful activities with technology. Their department required decisions on types of technology used and on what technology curriculum package had a “right mix.” They note that program-wide integration takes time. In their case at University of Iowa, it took seven years to implement (p. 3). Even after a curriculum change, additional time demands on faculty are reported by practitioners. Wrangler (1995, p. 8) notes that near constant improvement is needed in lab experiments and stresses that for faculty there is “no resting on laurels.” A closely related issue is the problem of time management in courses (TimC). Wrangler (p. 8) remarks that besides the time he spent outside of class, he had to take his students into the lab and walk them through basic commands. Many other practitioners, such as May (1999, p. 4), expresses similar sentiments. While this issue is discussed less frequently than time spent outside the classroom, practitioners report about both issues in conjunction (e.g., Wrangler p. 8).

CAS integration also affects classroom time management with respect to course content. Dogan-Dunlop (2003, p. 4) remarks that, “since class time was allocated for in-class demonstrations and discussions, detailed coverage of all the topics that were included in the syllabus was not possible.”

Another source of pressure on time management is the failure of students to achieve learning objectives (Obj). Krishanamani and Kimmons (1994, p. 4) note that students failed to learn material assigned in labs and they had to include it in later lectures.

One particular type of student error that clashes with learning objectives is the assumption on the part of students that their methodology is correct if their paper-and-pencil calculations match up with results obtained from the computer. As Cazes et. al. (2006 p. 342) write, "a correct answer does not mean the method is correct or is the best one. Teachers and students must be aware of such… pitfalls." Often students engaged in trial and error strategies, with students guessing the answer from feedback without making a proper mathematical argument (p. 347). Instructors sometimes failed to ensure that students found an "optimal" solution to a particular problem rather than just having a "correct" answer (pp. 342-343).

Pedagogical difficulties with learning objectives can place demands on faculty time not only inside but also outside of the lecture hall. Dogan-Dunlap (2003 p. 4) had to redesign his course and the use of CAS within it three different times because of such concerns. As previously discussed, there is an ongoing time commitment by faculty to improve their lecture and laboratory instruction and Dogan-Dunlap's experiences show that student difficulties may greatly influence the nature of those changes.

Related to the learning objectives issue, that of guidance (Gui) also emerges from the review. Often practitioners show concerns as to how much help they should give their students without compromising learning objectives. Westhoff (1997) designed a student project for Multivariate Calculus on the lighting and shading of a 3-dimensional surface. He found that the difficulty in the project, due to its complexity, lays in determining how much he could tell his students (p. 6). Another area in which guidance becomes an issue is mentioned by Weida (1996). Noting that there is a "fine line between helping students… and 'giving away' the answers," he remarks that such a problem is "particularly exacerbated at the end of a lab when the slower workers are running out of time" (pp. 3-4). Weida further presents the idea that careful scheduling could help alleviate this by ensuring that there isn't a need to leave immediately after the lab.

Student frustration (Frus) is another issue related to learning objectives. Cazes et. al. (2006, p. 344) note that students would often seek help either online or via the instructor "after having encountered the first difficulty" rather than attempting to solve the problem on their own. Krishahamani and Kimmons (1994) took steps to reduce anxiety both in course design and in providing additional help for students. Several measures, including reduced expectations, more time for tests, increased extra credit problems and a homework hotline were implemented (p. 2). Clark and Hammer (2003, p. 3) had a project for first year calculus modeling a rollercoaster. They found that "students who were not as "good" at *Maple* struggled, found the project (and *Maple* syntax) frustrating and were just happy to produce one mathematical model." This suggests possible relationship between student frustration and failure regarding activity learning objectives, and the CAS syntax issue.

Syntax (Synt) is the second most frequent concern for both practitioners and students. Cherkas (2003) found this to be a source of student dissatisfaction. He quotes a student complaining, "*Mathematica* would cause a lot of problems. If I make a mistake in the syntax, I couldn't do my work" (p. 31).

Tiffany and Farley (2004) exclusively focus on common mistakes in *Maple*, emphasizing the hurdle for practitioners caused by syntax. Practitioners employ various schemes attempting to minimize this difficulty. Some such as May (1999) design interactive workbooks that eliminate the need for teaching syntax entirely. Others like Herwaarden and Gielen (2001, p. 2) provide *Maple* handouts with expected output to their students. Some emphasize a pallet-based CAS such as *Derive* (Weida, 1996, p. 1) because it is easier to learn and has, according to them, a more straightforward notation.

Another source of student frustration is the unexpected behaviour of CAS (UnExp) even when their reasoning is syntactically and mathematically correct. Sometimes this is merely the case of paper-and-pencil calculations not easily matching up with CAS output. CAS may employ an algorithm efficient for computation and not necessarily one that matches a hand technique. For example, Holm (2003, p. 2) found that an online integral calculator would (rather than using the substitution method for $\int x(3x^2-1)^7\,dx$) simply expand the product and use the power rule. He notes that such cases provide an opportunity for learning, and that, referring to another classroom assignment, the more "savvy student would… expand $\frac{1}{48}(3x^2-1)^8$ ." Unexpected behaviour of CAS also takes the form of errors by the computers themselves. Due to the nature of floating point arithmetic and in spite of correct input by the user, roundoff error can cause the output to be wrong (Leclerc, 1994, p. 1). To encourage her students to adapt, Wu (1995, p. 2) purposely designed a lab with roundoff error. LeClerc urges students to be instructed in the nature of floating point arithmetic so that they "will be able to detect when roundoff has corrupted a result and hopefully find better ways to formulate or evaluate the computation" (1994, p. 4).

The concept of the "black box" (bbox) is examined in seven papers. Though this issue tends to be explored in more detail in education research papers, practitioners comment on it as well. O' Callaghan (1997, p. 3) writes that faculty at Southeastern Louisiana University expressed concern that "students would become button pushers rather than problem solvers." The managed used of the black box as an opportunity for students to explore complex mathematics beyond their level is discussed in great detail in education research papers (e.g., Winsløw, 2003, p. 283). Practitioners do not emphasize this potential as much. However, Cherkas (2003, p. 234) notes that CAS allows practitioners "to teach at a higher level of mathematical sophistication than is possible without such technology."

Closely related to the "black box" issue, is the fear that students become too reliant on the technology (rely). This, along with student frustration, is the least mentioned pedagogical issue. Cherkas reports on a student complaint that s/he could not do questions on tests because "Mathematica usually did them for me" (pp. 231-232). An over-reliance on technology may interfere with learning objectives. Considering this, Shelton (1995, p. 1) emphasizes her "top-down" approach and writes that "students can avoid the technology crutch and approach the goal of developing determination and mathematical maturity to perform mathematics without the technology."

The last and most commonly examined issue encountered in the literature is that of assessment (Ass). Practitioners encounter problems in evaluation. Schlatter (1999) allowed for CAS use during his exam for his multivariate calculus course. Unfortunately, in a question designed to test student understanding of the divergence theorem, several students simply used the CAS capabilities to solve the integral in a “brute force” approach (pp. 8-9). A poorly designed assessment thus leads to a failure in learning objectives. Schlatter further writes that he expected “to spend more time during this semester… more carefully designing exam questions” (p. 8), pointing again to the issue of faculty time.

Interpreting CAS output is discussed frequently. Quesada and Maxwell (1994, p.207) never accept a decimal answer (even if correct) if there is a proper algebraic expression. Many papers that discuss mathematical projects stress the use of written reports (e.g. Westhoff, 1997, p. 1). Lehmann (2006, p. 3) writes in his assignment “the important part of this assignment is the thought you put into it, the analysis you do and the presentation of your solution, not the answers themselves.” Xu (1995, p. 1) found that students were finding derivatives of easy functions by hand on assignments, but using graphing calculators to solve the more difficult questions. To show students “that the calculator could not do everything for them” he found functions in the textbook that “were easy to handle by hand but could not be done easily on the calculator.”

## CAS INTEGRATION SCOPE

Policy making regarding the curriculum in tertiary education is rather different than in school education. Hodgson and Muller (1992) mention that school mathematics curricula are in general developed by Ministries or Boards and implemented in the classroom by teachers, whereas tertiary mathematics curricula are developed and implemented by the same actors, i.e., faculty in departments of mathematics. However, change involving technology in tertiary curriculum, like in its secondary school counterpart, seems to remain very slow (Ruthven & Hennesssy, 2002). Lavicza (2006) argues that due to academic freedom, "Mathematicians have better opportunities than school teachers to experiment with technology integration in their teaching". This ad hoc basis is strongly reflected in our literature review. A large majority (67%) of the corpus restricted to practice reports discusses CAS usage with regards to one course, or in other words, CAS integration by one practitioner. While 16% has a scope that reaches across a series of courses (e.g. calculus courses), 11% discusses a CAS implementation with a grouping of courses (e.g. all first year courses). Only 6% discusses a program-wide implementation within a department.

## CONCLUSIONS

There is a need to develop a framework for the review of literature on the use of CAS at tertiary education that will integrate specificities of university-level education and technology integration. A significantly stronger majority of papers in our study stemmed from practitioner use (88%) than in Lagrange et al.’s (2003) study (60%)

which stated, "Most of the [practitioner] papers lack sufficient data and analysis and we could not integrate them into the [detailed (statistical) analysis]" (p.242). Our selection of journals and conferences for our pilot study may have influenced the above percentage. Nevertheless, this reality will clearly influence the development of our analytical framework henceforth. Lagrange et al. (2003) further state,

> [Practitioner] papers offer a wealth of ideas and propositions that are stimulating, but diffusion is problematic because they give little consideration to possible difficulties. Didactical research has to deal with more established uses of technology in order to gain insights that are better supported by experimentation and reflection. We have then to think of these two trends as complementary rather than in opposition. (p.256)

We aim at elaborating upon these complementary trends at the post-secondary level by both analyzing existing instructional practices and scrutinizing problematic issues within implementation. Lagrange et al. (2003) further state that the "integration into school institutions progresses very slowly compared with what could be expected from the literature" (pp. 237-8). This might be the case for school education, but apparently less so for tertiary education (Lavicza, 2008). The research literature about school mathematics and technology seems to pay less than adequate attention to the actual classroom implementation piece. The literature about tertiary mathematics and technology tends to inform us more about (individual) implementation than its didactical issues and benefits. This suggests that there may be a need for more education research focusing on the integration of technology in tertiary education. It also points, as suggested by Table 2, to the need of resources for departments of mathematics for systemic integration of technology in curriculum. At the recent ICME 11 conference, the results of a special survey highlighted concerns about the international trend of disinterest in university mathematics (ICME 11, n.d.). Departments of mathematics have a responsibility to question the current curriculum. We contend that part of this responsibility includes the careful consideration of the role and relevance of technology within that 21st-century curriculum and classroom.

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# THE LONG-TERM PROJECT "INTEGRATION OF SYMBOLIC CALCULATOR IN MATHEMATICS LESSONS" – THE CASE OF CALCULUS

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*A long term project (2003 – 2011) was started to test the use of symbolic calculators (SC) in grammar schools in Bavaria (Germany). The project was firstly done in grade 10. During the 2006/07 school year the project was implemented in grade 11. 732 students at 10 Bavarian grammar schools took part in an empirical investigation. The content taught was calculus: basic properties of functions, limits, continuity, derivatives, and applications of calculus. The evaluation of the project was intended give answers to the following questions: how basic mathematical skills (algebraic transformations, solving equations) changed; how the students used the SC, how they evaluated the use of the new tool. This article presents the results of this project for school year 2006/07.*

## 1. BACKGROUND

In the past, many empirical investigations concerning the use of CAS or symbolic calculators (with CAS) in mathematics teaching have been published (see Guin, Ruthven and Trouche, 2005). The central results of these projects have meanwhile been confirmed by other investigations world wide. The use of a CAS brings a greater meaning to work with diagrams, reinforces experimental work, in which the assumptions were obtained through systematic testing and CAS appears to bring an increase in computer cooperative forms of work. The effects are primarily long term. It is therefore necessary to develop a namely educational concept to evaluate the changes in knowledge and abilities over a longer time period. However, many investigations in this area restrict themselves to the applications of the computer over "just" a few weeks (Schneider, 2000, Drijvers, 2003, Pierce and Stacey, 2004 and Guin et al, 2005) and do not show the long-term effects on the knowledge and ability of the students.

In the school year 2003/04 we started a long term project to test the use of symbolic calculators (SC) – the TI-Voyage 200 and the TI-Nspire – in grammar schools (Gymnasien) in Bavaria (Germany). The project was done in grade 10 and has been repeated in the following two school years with a greater number of classes and with – concerning the use of new technologies – inexperienced teachers. An overview of the empirical investigation and especially of the theoretical background of this project gives Weigand (2008). On account of the positive results of this project, the Bavarian Ministry decided to continue the project. The follow-up project was started in September 2006.

## 2. THE TEACHING PROJECT – GRADE 11

### 2.1 The participants

During the 2006/07 school year the project was implemented in grade 11. A total of 732 students at 10 Bavarian grammar schools took part in this project. 412 students in 16 classes acted as the "pilot classes", working with Voyage 200 and/or TI-Nspire. Schools could apply for the participation in the project. The pilot schools have been chosen by the Bavarian Ministry. They are spread over the state. In addition, 320 participants from 11 classes – from the same schools as the pilot classes – formed a "control group" for the purposes of quantitative statistical investigation. The students had different previous experiences; some students had been exposed to the SC in the previous grade 10, but other students came into contact with these systems for the first time during this project.

### 2.2 The teachers

The project was mainly taught by teachers with little experience of tuition using computer algebra systems (CAS). The project teachers held two three-day meetings during which examples of possibilities and opportunities for SC use were discussed. The teachers jointly prepared a number of suggestions for a range of teaching units intended to highlight the possibilities of using SCs; during the year, the teachers were offered additional learning units[1] by the coordinator (Ewald Bichler). However, there was no uniform overall concept according to which teaching was to be organised in all classes. The personal experience, attitudes and circumstances at the individual schools were too different for this to be possible.

### 2.3 The learning contents

In grade 11, calculus is taught (in Germany). The content taught was subdivided into the following:

- basic properties of functions (symmetry, monotonicity, variations in function terms and their impact on graphs, …)
- limits, continuity
- differentiability, derivation rules, derivation function(s)
- applications of differential calculus ("classical" functions discussion, extreme value problems)

### 2.4 Teaching methods with the SC

During the meetings with the teachers at the beginning and in the middle of the school year a theoretical frame of the use of the SC in the classroom was discussed

[1] One sort of learning units developed during the project is called "Minute Made Math", more information on www.minute-made-math.com

with the teachers. Especially a short insight into the theory of instrumentation was presented and explained with examples (Artigue 2002, Trouche 2005).

Concerning the integration of the SC into the problem solving process we distinguished using the SC

- in the beginning of the problem solving process or a concept formation process (the SC as a "discoverer"),
- in the middle of the process (the SC as "solver") and
- at the end of the process (the SC as a "controller").

We also emphasized the "rule of three" while working with representations: If possible a problem or the solution of the problem should be represented on a symbolic, graphic and numeric level.

### 2.5 Research questions:

In the following we concentrate on a selection of the research questions (RQ) of the project:

RQ1. Can any differences be ascertained in terms of core mathematical abilities (substitutions, interpretation of graphs, solving equations, working with tables, and working with formulae) between the pilot and the control groups after one year?

RQ2. Can different effects of SC use be ascertained with "good", "average" and "weak" students?[2]

RQ3. To what extent have students mastered the SC at the end of the year?

RQ4. In which phases of a problem solving activity do the students use the SC?

### 2.6 Test instruments

For the purpose of answering the 1st and 2nd questions we took a (classical) pre- and post-test-design – the tests using paper and pencil but no calculator – in pilot and control classes.[3]

For the purpose of answering the 3rd and 4th questions the pilot classes took a test using a SC in February 2007 and June 2007 in which they were asked to record their working methods with the SC in a questionnaire which they completed immediately *after* the test.

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[2] The performance criteria used relate to the results of the pre-tests at the beginning of the school year.

[3] See: www.dmuw.de/weigand/2009/CERME6/

## 3. EVALUATION OF PRE- AND POST-TESTS

### 3.1 The questions

The pre- and post-test-questions (PP-questions) can be divided into the following groups:

- Questions 1 and 2: doing “classical” simplification of terms
- Question 4 and 5: solving equations
- Question 5: understanding the concept of root functions
- Questions 6 – 8: seeing the correlation between graph and term
- Question 9: interpreting graphs

### 3.2 Comparison of results of pre- and post-tests

The post-test was the same as the pre-test. In the following diagram, the *differences* between the average scores achieved for each question in the pre- and post- tests for the pilot and the control group are shown. The “average performance increase” is therefore measured for each question.

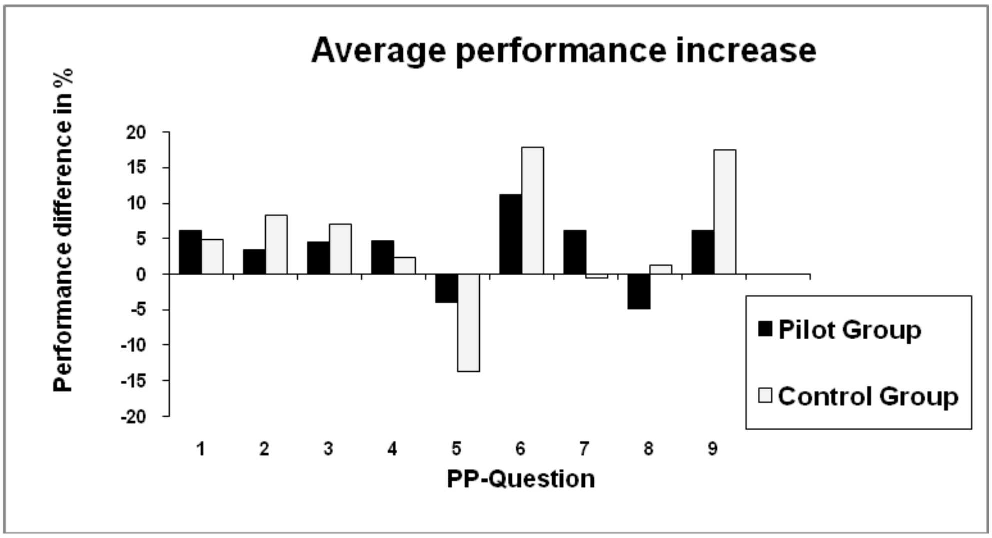



**Figure 1: Average performance increase of the pilot and the control group**

In PP-questions 5 and 7 the pilot classes' results are significantly better than than those of the control groups (t-Test: PP 5: 0.01, PP 7: 0.02). However, in PP-questions 6 and 9 they are significantly worse (t-Test: PP 6: 0.01, PP 9: 0.01).

Overall there is not a significant difference in the average performance increase between the pilot and control classes. For the comparatively worse result of the pilot classes compared with the control classes (especially for questions PP 6 and PP 9), there are two possible hypotheses. On the one hand it could be due to the fact that the students in the pilot classes were no longer adequately challenged or motivated to tackle this type of "traditional" question with enthusiasm, as they had tackled much more interesting questions during lessons – due to the SC. On the other hand the poor results of the pilot classes when determining functional equations from specified graphs (question 6) could be due to the fact that the students in the pilot classes had seen a large number of graphs – compared with the control group – during the course of the year and were therefore overtaxed by the diversity. However, the students in the control class have probably worked more often with the sine function graph which had been introduced in grade 10.

If, however, the range of performance increases is considered, an interesting picture emerges.

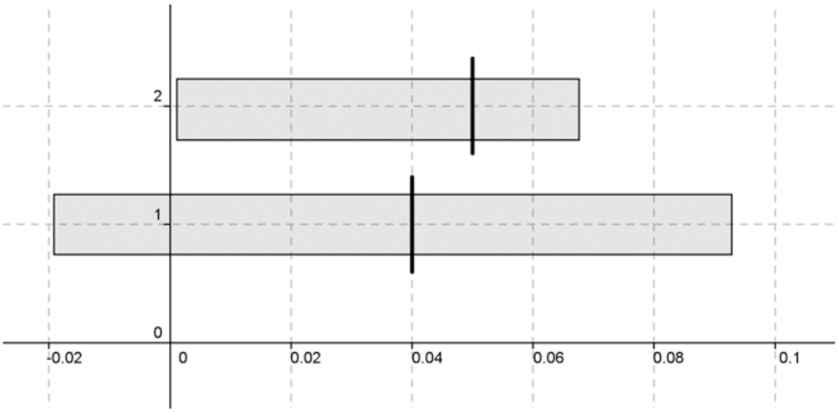



**Figure 2: Average value and range of average performance increases in pilot (1) and control groups (2)**

With an almost identical average value, it becomes apparent that the differences in performance are more varied with the students in the pilot classes than with the students in the control groups. Therefore, there are students in the pilot classes who benefit more from SC use than students in the control classes. However, there are also students whose results deteriorate compared with the initial test.

The test results can also be interpreted in a positive way for the pilot classes, as there are no differences in terms of classical technical and manual abilities and skills. However, this investigation has deflated hopes that the ability to interpret graphs and transfer between different forms of representation are automatically improved by the use of the SC.

### 3.3 Scores for "good", "average" and "weak" test participants

In accordance with the results of the pre-test, we divided the test participants into "weak", "average" and "good".[4] The following result is produced when the performances of these groups are compared in terms of pre- and post tests.

[4] The "good" students form the upper performance quartile, the "weak" students the lower performance quartile, and the "average" students are represented by the two central performance quartiles.

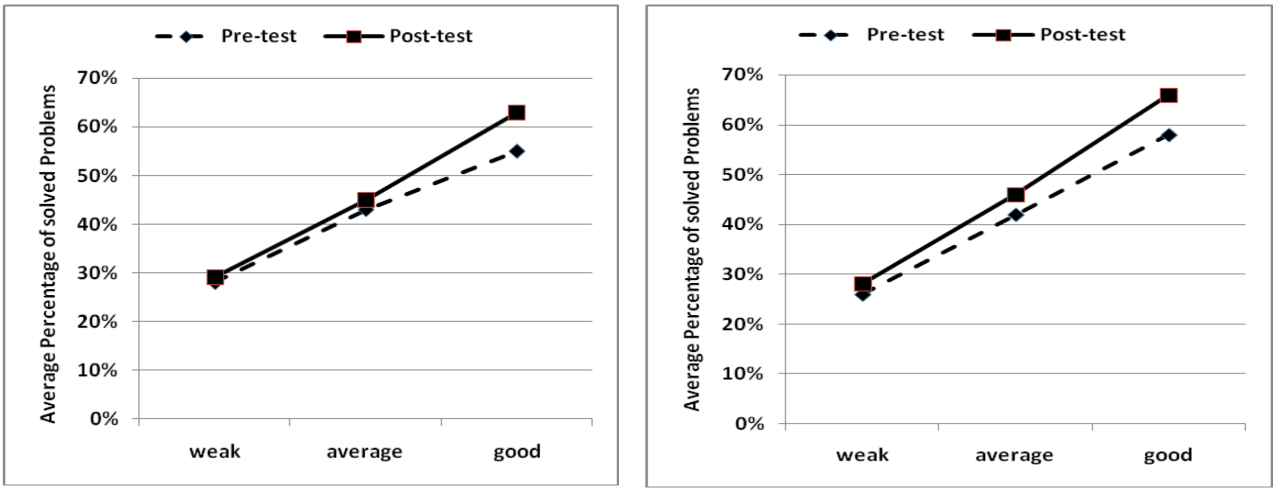



**Figure 3: Performances of the pilot group (left) and the control group (right)**

Compared with tests carried out in recent years in grade 10 (see Weigand 2008), different behaviour was demonstrated here. Whilst the "weak" students achieved a greater performance increase than the "average" and "good" students in grade 10, the "good" students – both in the control and pilot groups – improved more markedly (by 8 percentage points) than the "average" and "weak" students (by 3 percentage points and 1-2 percentage points respectively) in the grade 11 test.

The differences between the "weak" and "good" groups can be found in the understanding of concepts (question 5) and the transfer between different forms of representation (between graph and equation - questions 8 and 9)). The lack of performance increase in the case of weak students is attributable to the greater cognitive challenges posed by calculus, which may have taken some students to the limits of their capacities so that they were no longer able to follow lessons ("dropout effect").

## 4. THE SYMBOLIC-CALCULATOR-TESTS (SC-TESTS)

### 4.1 Research questions

In February and in June the pilot classes took a test where they were allowed to use the SC. Use of the SC was optional for the students, i.e. they decided themselves whether or not they would use the calculator. The two tests consisted of four questions each.[5] In order to establish how calculators were used, we applied a new investigation method: the students completed a questionnaire on SC-use immediately *after the test*, giving details of whether and how they used the calculator. This test was intended to answer the following questions:

1 How do students use the calculator?

2 In which phases of a problem solving process do the students use the calculator?

[5] See: www.dmuw.de/weigand/2009/CERME6/

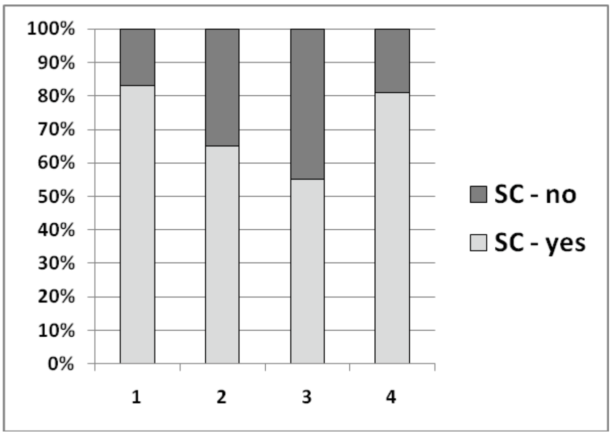
3 Which functionalities (symbolic – graphic – numerical) do the students use?

In addition, the teachers were presented with a questionnaire regarding the questions immediately *before the test*, in which they were intended to provide details of the difficulties expected in terms of the questions.

In the following, only a few spotlights of the results will be given.

### 4.2 Actual use of the SC

The following diagrams show how many students used the SC during the tests in February and in June – according to their own statements:



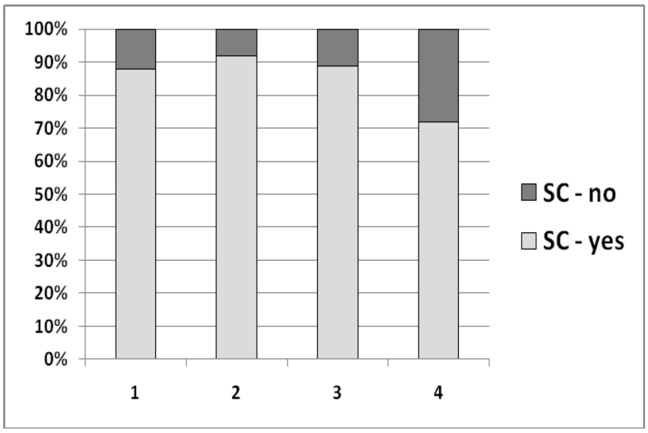
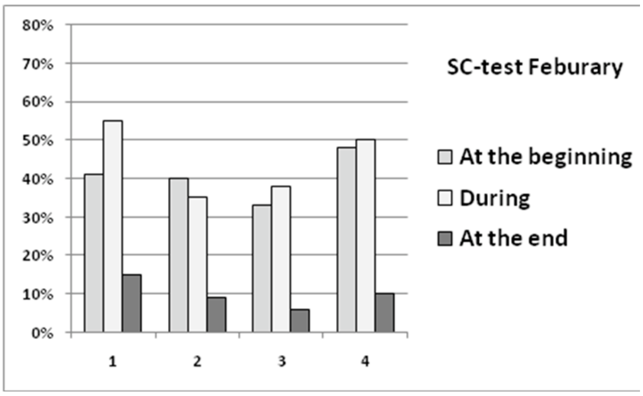



**Figure 4: Results of the SC-test in February (left) and June (right) 2007**

The difference between SC use in February and in June shows an increase in use of the calculator. More over, those students who used the SC in June when solving the questions scored significantly better than those who did not use it. We attribute this to the fact that it takes a full school year for students to acquire adequate confidence in the SC, as well as knowledge of the benefits of its use as a tool when solving problems, to be able to use these for the purpose of solving problems.

### 4.3 The SC-use during the problem solving process

The students also provided information in the questionnaire as to whether they used the SC in the beginning, during or at the end of the problem solving process.



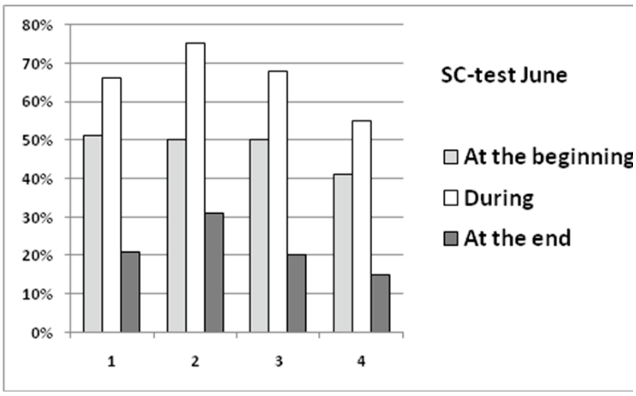



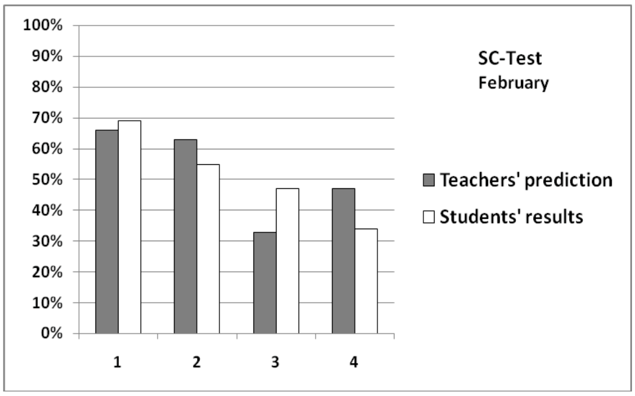

**Figure 5: Use of the SC during the course of the solving process (according to statements made by the students themselves)**

When students integrate the SC into their solving process, it is predominantly used at the beginning and during the solving process. If we compare the middle of the school year with the end, we can observe a clear increase in the frequency of positive responses to “during”. This allows us to conclude that the SC is more strongly integrated into the solving process by the students at the end of the school year. A slight increase can also be observed “at the end”, which makes us aware that the use of checking the solution is gaining in importance.

We also asked the students which representations they used while solving a problem with the SC. It appears that the students mainly use the symbolic and graphic possibilities of the SC. Numeric use is very limited. More over they are not familiar with the special advantages or diadvantages of the representations nor do they use the relationship between the different representations. The type of the used representation depends on the one hand very strongly on the way problems are given to the students. If it is asked for a “solution of an equation”, they mainly work on a symbolic level, if it is asked for an “intersection point of two graphs” they work on a graphic level. This shows that the SC is used in a very mechanical way, guided not by the type of problem but by the expressions used in the problem. On the other hand the type of use depends also very strongly on the classes and indicates the significance of the teacher and his or her didactic approach.

## 4.4 Teachers' predictions

Before each test was carried out, the teachers provided an assessment of the extent to which students would solve the problems. The question has been: “For each problem, a student gets 100 % of the marks for a completely right answer. What do you suggest will be the average score of marks your class gets for problem 1 (2, 3, 4)?” The results are as follows:



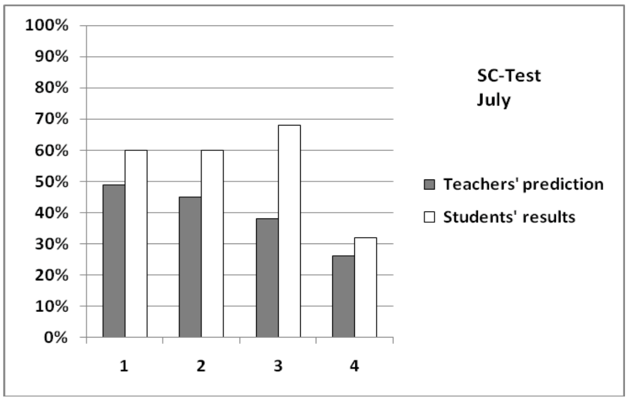




**Figure 6: Comparison of teachers' predicted and student results in the SC tests**

It is noticeable that the teachers underestimated the students in the June test.

## 5. Questions for the future

If we summarise the core results of this one-year school project there are some questions for up-coming investigations.

- **Methodology of pre- and post-tests**.

Hopes have not been fulfilled that students in the pilot classes would improve to a greater degree in terms of dealing with and interpreting graphs than students in the control classes. The hypothesis is that students in the pilot classes are not have been adequately challenged or motivated as the result of the largely traditional nature of the test problems. This raises the question whether the used pre- and post-test methodology is an adequate method to answer this question.

- **Polarisation**.

When working with new technologies, polarisation occurs in that some students benefit greatly from SC use, whereas for other students, SC use inhibits performance or even decreases performance. Two thirds of students are of the opinion that the SC was helpful and made them more secure and they classify lessons as “interesting”. Approximately one third of students do not share this view. Are there ways to get all students convinced of the benefits of the SC?

- **Calculator use**.

The reasons for non-use of the calculator are on the one hand the uncertainty of students regarding technical handling of the unit and on the other hand a lack of knowledge regarding use of the unit in a way which is appropriate for the particular problem. Is there a correlation between these two aspects?

- **Period of adjustment**.

The responses of the students confirm that familiarity with the new tool requires a very long process of getting used to it. It is surprising that it took almost a year to establish familiarity with this tool for students to use it in an adequate way. After one year of SC use, confidence in and familiarity with the SC grow. However there is still a large group of students who experience technical difficulties when operating the SC. Will there be ways to shorten this period of adjustment?

- **Solution documentation.**

Students have problems how to record solutions when using the SC. Difficulties with the type and manner in which to document the solution decreased over the year, but still remain at a high level. This latter point will continue to be a permanent challenge when working with the SC, as there is no algorithmic solution for the procedure. Are there documentation rules for all or a special type of problems?

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# ENHANCING FUNCTIONAL THINKING USING THE COMPUTER FOR REPRESENTATIONAL TRANSFER

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*The area of functional thinking is complex and has many facets. There are several studies that describe the specific difficulties of functional thinking. They show that the main difficulties are the transfer between the various representations of functions, e.g. graph, words, table, real situation or formula, and the dynamic view of functional dependencies (process concept of a function). Interactive Geometry Software allows the visualization of the dynamic aspect of functional dependencies simultaneously in different representations and offers the opportunity to experiment with them. The author presents and discusses the potential of two interactive learning activities that focus on the dynamic aspect of functional thinking in a special way. Some preliminary results from a first adoption of the activities in class are presented. Resulting research questions and plans for further research are stated.*

**Keywords:** Functional thinking, representational transfer, Interactive Geometry Software, Interactive learning activity, empirical study.

## THEORETICAL BACKGROUND

### Functional Thinking – Concept and Relevance

In Germany the term 'functional thinking' was first used in the 'Meraner Reform' of 1905. The 'education to functional thinking' was a special task of the reform. Functional thinking was meant in a broad sense: As a common way to think which affects the whole mathematics education (Krüger 2000). In the 60s and 70s the impact of functional thinking in the above sense on the mathematics curriculum in Germany was very low. Since the 80s it regains importance although not in the broad sense of the Meraner Reform. A common definition of functional thinking derives from Vollrath (1989): 'Functional thinking is the typical way to think when working with functions'. Functional thinking in this sense is strongly connected to the concept of function. In the german mathematics curriculum the 'idea of functional dependency' is one of five central competencies, which form the mathematics education (Kultusministerkonferenz 2003).

The concept of function and functional thinking includes many aspects and competencies: On one hand functional dependencies can be described and detected in several representational systems like graphs, words, real situations, tables or formulas. On the other hand the nature of functional dependencies has different characteristics (Vollrath 1989 or Dubinsky, Harel 1992): Functional dependency as a pointwise relation (horizontal, static aspect), functional dependency as a dynamic process (aspect of covariation and change, vertical aspect), Functions viewed as objects or as a whole.

There are many studies (e.g. Janvier 1978, Müller-Philipp 1994, Swan 1985, Kerslake 1981) describing the following main difficulties and misconceptions concerning functional thinking:

The interpretation of functional dependencies in different representations and the representational transfer is a main difficulty. Especially the interpretation of functional dependencies in situations and the transfer to e.g. the graphical representation and vice versa causes problems. For example: graphs are often interpreted as photographical images of real situations ('graph-as-image misconception'), which is mainly caused by the inability to interpret the functional dependency dynamically. Especially distance-time graphs are often interpreted as movement in the plane.

The above difficulties were affirmed by written tests the author gave to either 10$^{th}$ class students and to university students who just started their study on mathematics. Based on the problems in the test the interactive learning activities, which we describe below, were built. Figure 1 shows one of the problems (Schlöglhofer 2000) from the tests.

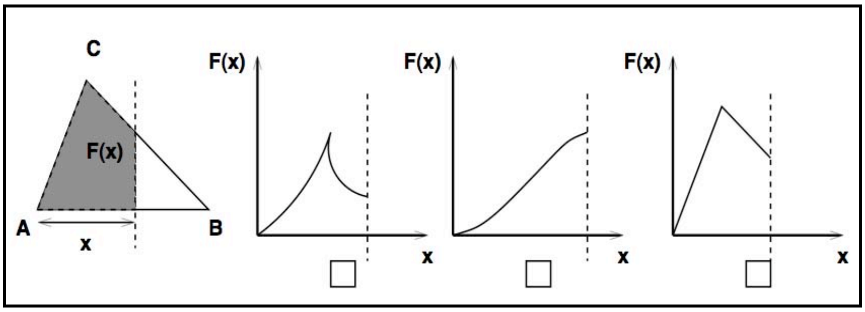


**Fig. 1: The dashed line moves rightwards. F(x) is the area of the grey part of the triangle dependent on the distance x. Which graph fits and why?**

Only 66% of about 100 university students made their cross at the graph in the middle. Giving the problem to sixteen 9$^{th}$ and 10$^{th}$ grade high school students, resulted in only 37% correct answers. The main mistake was to put a cross at the graph on the right side. The reason for this choice was usually given by a statement like: The area [of the graph on the right side] is just like the area F(x).

## The chances of Interactive Geometry Software

When using the computer in classrooms on the topic functions one might think immediately of using Computer Algebra Systems (CAS). Most studies about the use of the computer when working with functions are about using CAS, e.g. Müller-Philipp (1994), Weigand (1999), Mayes (1994). While CAS is input/output based and gives back information and changes asynchronously, the use of Interactive Geometry Software (IGS) allows interactivity and gives immediate response. This difference will be used to emphasize the dynamic view of functional dependencies.

Especially the software Cinderella includes a functional programming language called *CindyScript*. This enables the teacher to create learning activities and own teaching material like the ones described below by using standard tools (Kortenkamp 2007).

## DESIGN OF THE ACTIVITIES AND CONCEPTUAL BASIS

### Main research question

The learning activities are designed with regard to the following research question:

Is it possible to enhance the dynamic aspect of functional thinking by dynamically visualizing functional dependencies simultaneously in different representations and by giving the opportunity to experiment with them?

### General design ideas and concept

We developed two interactive learning activities (joint work with Andreas Fest). The activities consist of single Java applets embedded into a webpage and can be used without prior installation with a standard Internet browser. The applets are built with the IGS Cinderella and are accessible by using the links on the webpage http://www.math.tu-berlin.de/~hoffkamp.

Figure 2 shows the typical design of a learning activity. Next to the applet there is a short instruction on how to use the applet and some work orders. The students are asked to investigate and describe the functional dependency between the distance A-D and the dark (if coloured: blue) area within the triangle.

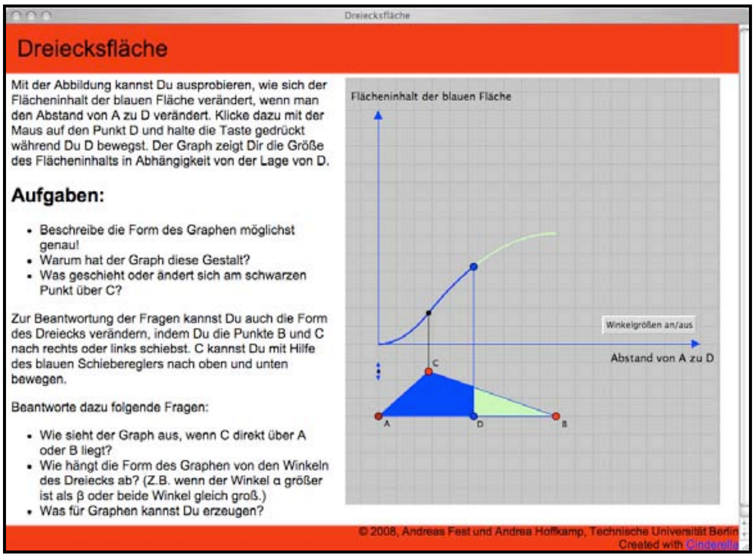


**Fig. 2: Interactive activity 'Dreiecksfläche' ('Area of a triangle'). Moving D makes the dynamic aspect visual. Moving B and C changes the triangle and the function itself.**

The learning activities have the following conceptual and theoretical ideas in common:

*Connection situation-graph:* The starting point is a figurative description of a functional dependency, which is simultaneously connected to a graphical representation. The graphical representation was chosen, because it relates to the covariation aspect in a very eminent way. As analysed by von Hofe (1995) students are able to establish 'Grundvorstellungen' (GV) more easily when an imaginable situation is given. GV's are mental models connecting mathematical concepts, reality and mental concepts of students. Rich GV's of the functional dependencies are necessary to succeed in problem solving processes.

*Language as mediator:* The students are asked to verbalise their observations in their own words. Janvier (1978) emphasizes the role of the language as a mediator between the representations of the functional dependency and the mental conceptions of the students.

*Active processing assumption:* According to the cognitive theory of multimedia learning of Mayer (2005) humans are actively engaged in cognitive processing in order to construct a coherent mental representation. The activities are conceptualized as attempt to assist students in their model-building efforts. Therefore the activities allow to experiment with different representations of the functional dependencies. At the same time the actions of the user are limited to focus on the dynamic view of the functional dependencies.

*Two levels of variation:* The activities allow two levels of variation. First, one can vary within the given situation. This visualizes the dynamic aspect. To understand a dynamic situation one needs to construct an 'executable' mental model to achieve mental simulation. The idea is to support the mental simulation processes visually (Supplantation, Salomon 1994). Secondly, one can change the situation itself and watch the effects on the graph. We will call this **meta-variation**. Meta-variation allows the user to investigate covariation in several scenarios. It is variation within the function that maps the situation to the graph of the underlying functional dependency and changes the functional dependency itself. This leads to a more global view of the dependency. Therefore meta-variation refers to the object view of the function. To understand the covariation aspect one needs to find correlations between different points of the graph in order to describe changes. This requires a global view of the graph. For example the property 'strict monotony' of a graph is a global property and therefore refers to the object view of a functional dependency. But to describe it in terms of 'if $x>y$ then $f(x)>f(y)$' one has to understand the covariation of different points of the graph.

*Low-overhead technology and practicability:* To work with the interactive activities there is no special knowledge of the technology necessary. The activities make use of the students' experience with Internet browsing (actions like dragging, using links, using buttons etc.). The students (and the teachers) can work directly on the problems without special knowledge of the software and the software's mathematical background. This is important especially with regard to time economy.

## Learning activity 'Die Reise' ('The journey')

Based on the conceptual ideas above the learning activity 'Die Reise' ('The journey') was developed. Like the activity 'Dreiecksfläche' it is adapted from a problem (Swan

1985) the author gave to university students and 10th grade students within a written test. After using 'Die Reise' in classroom within a first study the activity was worked over. Some results of the study are presented below. The activity in its current version consists of three parts. Part one is about the transfer situation-graph (Fig. 3): A car advances from Neubrandenburg (top of the map) to Cottbus (bottom of the map). The graph shows the corresponding distance-time graph for the journey.

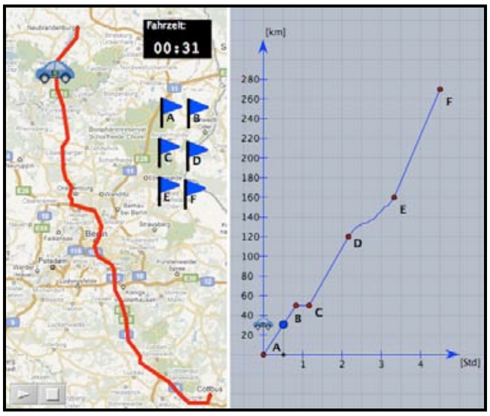


**Fig. 3: Applet within the first part of 'Die Reise'. The point on the distance-time-graph is movable. The students are asked to mark the positions A-F with the flags on the map.**

Part two of 'Die Reise' (without figure) refers to the first level of variation (visualization of the dynamic aspect in the given situation). It shows the distance-time graph of part one again together with the corresponding velocity-time graph. The work orders aim at interpreting the slopes in the distance-time graph in connection with the velocity-time graph.

Part three refers to the level of meta-variation (figure 4).

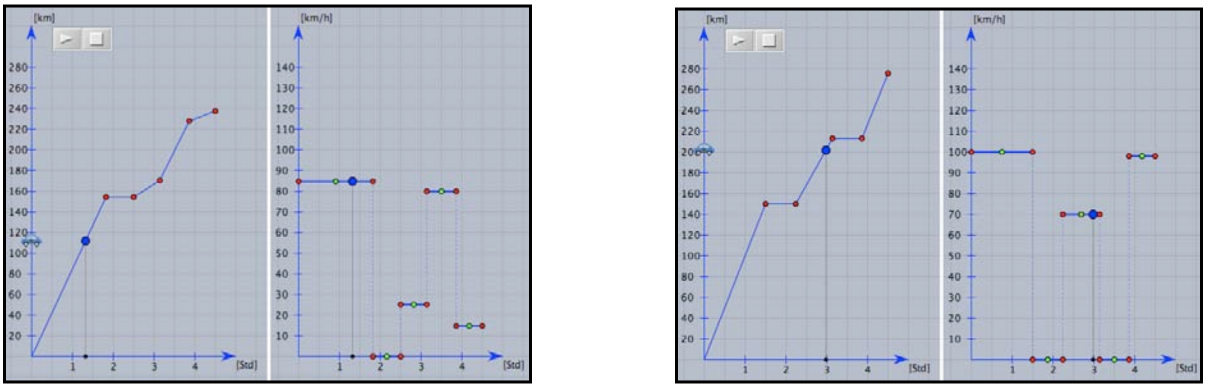


**Fig. 4: Meta-variation in 'Die Reise'. Besides moving the points on both graphs, the bars in the velocity-time graph can be moved vertically and the width of the bars can be changed.**

## FIRST STUDIES

### Setting and methods

The activities 'Dreiecksfläche' and 'Die Reise' were tested with 19 respectively 32 secondary school students of age 15-16 (10th class) in a block period of 2x45 minutes in each case. The students were not prepared to either the topic or the special use of technology. A worksheet was prepared which contained the Internet address of the interactive activity and some questions to work on. The students had to start on their own using the instructions of the worksheet. To provoke discussion and first reflection about the problems two or three students worked together. Afterwards the solutions were discussed in class. The results of the studies are based on student observations during their work with the computer, general impression of the discussion in the class, a short written test and a questionnaire. In addition the computer actions and student interactions of one student pair was recorded while working with the activity 'Die Reise'. All teaching material, tests and questionnaires can be found on www.math.tu-berlin.de/~hoffkamp.

The studies were conceived as preliminary studies with the following aims: Test the interactive activities and work them over for further studies, specify further research questions, create a study design for a larger study based on the experiences made.

### Results and discussion

*Computer-aided work and work with the activities in general:*

The concept of low-overhead technology and practicability was successful. The students were able to work with the interactive activities without further instruction. This is also important concerning time economy, especially from the teachers' point of view.

The use of the computer had a very positive effect on the students' motivation. This is caused by many factors. For example the students appreciated to work autonomously in their own tempo following their own train of thoughts. They also highly appreciated that the computer takes over annoying actions like drawing or calculating. This is a crucial point especially for slow-writers and was observed when watching a recorded sequence of the students' working phase. The sequence shows that the order 'Draw a suitable distance-time graph' really blockaded and frustrated the student. The following student statements from the questionnaire confirm the above comments:

> Question: Is there something special you like when working with the computer?
>
> Answer 1: It is less monotonous and the lesson is organized differently. You learn by means of a different learning aid, which allows a better imagination. The studious atmosphere is more comfortable. You do not have to follow the group's train of thoughts.
>
> Answer 2: The computer makes the calculations and I do not have to write so much, which means that it cannot be smeared and illegible.

Answer 3: That I can work independently (without teacher). One can use his own mistakes to come to the right result.

Statements like answer 3 were made several times. The students had the impression that they were able to use their mistakes in a productive way. Moreover the computer-aided work allowed for a better internal differentiation of the learner group. Slowly learning students asked the teacher for help more often than more advanced students, but they still worked independently for longer periods.

*Effects on functional thinking:*

By visualizing the representational transfer dynamically the students were forced to focus on the dynamic aspect of functional thinking and they seem to have established (more or less) adequate mental models integrating the dynamic view. Many student answers on the questionnaire aim at the aspect of 'dynamic visualization':

Question: What is different for you when you use the computer to work on mathematical problems?

Answer: Because of the visualization I am able to watch the problem from different perspectives and this makes it easier to solve it.

Question: Can you say what exactly you understood better by using the computer?

Answer 1: That I could **see** the problem.

Answer 2: How the graph changes when changing the triangle.

Answer 3: I liked this form of figurative illustration that was given directly when changes were made because it is easier to understand something by watching it.

Answer 4: The motion. When one graph moves although you use the other graph.

The second question was used to find out what aspects of the activities where considered by the students as showing them something new. In this sense many student answers aim at the level of meta-variation. As explicated above the level of meta-variation is connected with the object view of a function, a view, which is not (fully) attained in the age group the author is looking at (Sfard 1991). The student answers lead to the assumption that meta-variation makes the object view accessible for cognitive processes (in a may be implicit way) and could be a step towards the perception of a function as an object. This assumption is strengthened by the results of the tests. Figure 5 and 6 show some results from the written test.

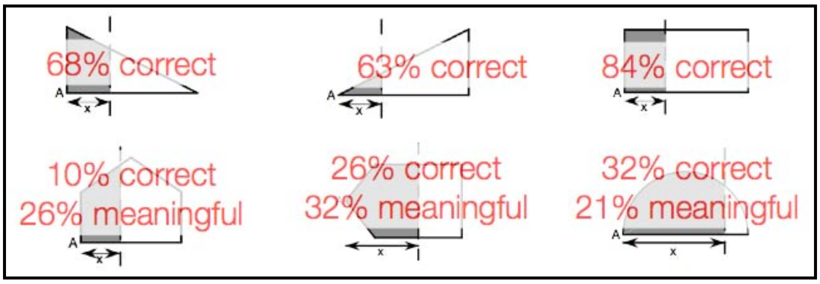

**Fig. 5: The students had to sketch graphs describing the dependency between x and the grey area F(x). The figure shows percentages of correct and meaningful graph sketches. An answer was 'meaningful' when the graph was strictly increasing, but e.g. left and right turn were mixed up.**

As seen from figure 5 the students by majority seemed to have created an 'idea' of the dynamics of the functional dependency as far as the solution of problems like the one above is concerned, although it was still difficult to adapt the concept to other situations (here: other forms in line two of figure 5). However the students got aware that changes, variations, certain points (e.g. inflection points) and properties (e.g. symmetry, monotony) have a graphical correspondent, which gives qualitative information about the functional dependency.

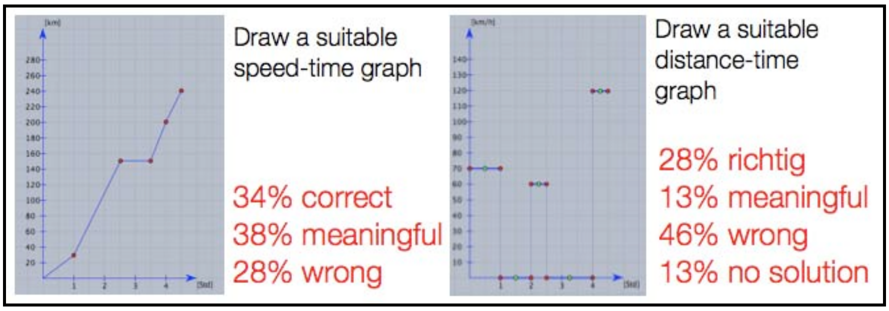


**Fig. 6: The students had to draw suitable graphs to the given graphs above. Graphs were 'meaningful' when the graphs had 'correct shapes', but some slopes where done wrong.**

Figure 6 shows some results of the post-test within 'Die Reise'. Most of the students were able to draw suitable speed-time graphs to given distance-time graphs. The other way round – from speed-time graphs to distance-time graphs – was more difficult. The results confirm the assumption that the potential of meta-variation in order to enhance the understanding of the dynamic aspect of the functional dependencies seems to be high. Furthermore the activities seem to allow an easy qualitative approach to concepts of calculus. In case of 'Die Reise' the applets visualize the physical intuition of the fundamental theorem of calculus. When discussing the question 'Can you see from the speed-time graph how far the journey is?' in class, the students finally ended up with an intuitive concept of integration.

The results from the preliminary studies lead to the following hypothesis: *Although the object view is more advanced, it facilitates the understanding of the covariation aspect and the establishment of mental models with regard to the dynamic view of functional dependencies.*

The class discussion of the results – which mainly consist of verbalisations of the properties of the functional dependency – ran pretty smooth. The students were highly engaged in making contributions to the discussion. But it was obvious that there was a high need for reflection of the students' train of thoughts since the student answers were mostly superficial. Concerning 'Die Reise' some test results showed, that the 'graph-as-image misconception' is very persistent in the sense of interpreting distance-time graphs as movement in the plane. Based on these experiences the first two parts of the learning activity 'Die Reise' were modified to their current version.

## OUTLOOK

The preliminary results of the first studies give valuable hints for the direction of further research. Basing on the conceptual ideas described above a third interactive activity will be developed and pretested. It is planned to conduct a larger qualitative study using the three activities. The leading question is how the work with functions within the activities affects functional thinking itself. The level of meta-variation is a central idea. It leads to the concepts of calculus and may be used as a qualitative approach to school-analysis in the context of propaedeutics.

The following research questions are of interest and will guide our future research:

*Main question:* Is it possible to enhance the dynamic aspect of functional thinking by dynamically visualizing functional dependencies simultaneously in different representations and by giving the opportunity to experiment with them?

*Further questions:*

- Do the students establish GVs concerning the dynamic view of functional dependencies? What sort of GVs do the students establish?
- Which elements of the applets have a positive effect on the dynamic view of functional dependencies?
- Is it possible to distinguish types of students who get along better or worse with the learning units?
- How do slow learners deal with the units compared to more advanced students?
- How can we use computer-based activities like these as diagnostic tools?

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# THE ROBOT RACE:

# UNDERSTANDING PROPORTIONALITY AS A FUNCTION WITH ROBOTS IN MATHEMATICS CLASS

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*This paper presents and discusses the use of robots to help 8th grade students learn mathematics. An interpretative methodology was used and data analyses were supported by Situated Learning Theories and Activity Theory. These tools allowed the accurate description and analysis of student's practices in mathematics classes. The results indicate that the use of robots to study proportionality as a function aided and supported student learning.*

## INTRODUCTION

Educational systems the world over are investigating new and engaging mechanisms in order to better present complex concepts and challenging domains such as mathematics. The implementation and exploration of technologies in classrooms is a promising general approach. However, we should not neglect the real world where the actual students live – a world more and more dependent on technologies. Consequently, it is essential to combine computation aids and new educational aims with a redefinition of teaching processes and teachers role's in the classroom. It is in this context that the project DROIDE was initiated in 2005.

DROIDE[2]: "Robots as mediators of Mathematics and Informatics learning" is a project with three main objectives:

**(1)** to create problems in Mathematics Education/Informatics areas which are suitable to be solved using robots; **(2)** to implement problem solving using robotics at three points in the educational system: mathematics classes at K-9 and K-12 levels; Informatics in K-12 levels; Artificial Intelligence, Didactics of Mathematics and Didactics of Computer Science/Informatics at the university level; **(3)** to analyze and understand students' activity during problem solving using robots.

This paper discusses research on the second issue (the implementation of problem solving using robots in mathematics class) at the K-9 level. It addresses the following research problem: to describe, analyse and understand how students learn mathematics using robots as mediators of learning. It particularly focuses on the mathematical concept of proportionality as a function.

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## THEORETICAL BACKGROUND

The research approach is derived from Situated Learning Theories (Lave & Wenger 1991, Wenger, 1998, Wenger et al, 2002) and Activity Theory (especially the 3rd generation introduced by Engeström, 2001). A key element of Situated Learning theories is the notion of a community of practice and the suggestion that learning is a situated phenomenon. In this paper, this viewpoint is used to reflect upon emergent learning within students' mathematical practices.

### The Concept of Practice

According to Wenger, McDermott and Snyder (2002) practice[3] is constituted of a set of "work plans, ideas, information, styles, stories and documents that are shared by community members" (p.29). Practice is the specific knowledge that the community develops, shares and maintains. Practice evolves as a collective product integrated in participants' work and the organisation of knowledge in ways that are useful and reflect the community's perspectives (Matos, 2005).

Wenger (2002) proposes three dimensions in which practice is the source of coherence in a community: mutual engagement, joint enterprise and shared repertoire. Mutual engagement is a sense of "doing things together"; the sharing of ideas and artefacts, with a common commitment to interaction between community members. Joint enterprise is having (and being mutually accountable for) a communal common goal, a procedure which rapidly becomes an integral part of practice (Matos, Mor, Noss and Santos, 2005). Shared repertoire refers to a set of agreed resources for discussions and negotiations. This includes artefacts, styles, tools, stories, actions, discourses, events and concepts.

### The Concept of Mediation

Engeström (1999) conceptualizes an activity model formed by three elements – the subject, the object and the community – with mediation relations between them. In the context of this research, the mathematics classroom forms such an activity system. The subject is part of a collective; reflecting the fact that we do not act individually in the world. The subject is part of a system of social relations.

The concept of mediation has a central role in Activity Theory[4]. It is based on the presupposition that the subject does not act directly on the environment; that it has no direct access to the objects. The relation between subject and object is mediated by artefacts (Werstch, 1991); things constructed by individuals and maintaining a dialectic relation between people and activity (Werstch, 1991). To say that a tool or

[3] The term practice is sometimes used as an antonym for theory, ideas, ideals, or talk. In Situated Learning theories that is not the idea. In Wenger's sense of practice, the term does not reflect a dichotomy between the practical and the theoretical, ideals and reality, or talking and doing. The paper extension does not allow the development of the idea of practice. For discussion of practice related with mathematics education see Fernandes (2004).

[4] For a more general vision of Activity Theory see http://pparticipar-t-act.wikispaces.com/

artefact is mediator of learning means that it gives power to the process of transformation of objects; that it is a tool with which people think (Piteira, 2000).

This paper claims that robots can be artefacts, mediators of the learning of functions. The veracity of this claim is demonstrated in the following sections.

## METHODOLOGY

The work reported in this paper was organised into three stages:

**First stage** – analysis of School Mathematics and Informatics curriculum; selection of didactical units where robotics can be used; creation of problems/tasks to be solved in Mathematics and Informatics classes.

**Second stage** –implementation of problems/tasks in Mathematics and Informatics classes; data collection through video recordings of students.

**Third stage –** analyses of student activity during learning with robots using interpretative methods introduced in Situated Learning Theories and Activity Theory. The unit of analysis was "(...) the activity of persons-acting in setting" (Lave, 1988, p.177).

## LEARNING AS PARTICIPATION: ANALYSING STUDENTS MATHEMATICAL ACTIVITY WHEN USING ROBOTS

### A brief description of mathematics class

In mathematics classes students worked in small groups. In the initial phase, the work involved construction of the robots and basic programming to solve simple tasks. This activity took place on a Windows® desktop environment and the students used a visual programming tool that ships with the robot kits. Subsequently, students used the robots to recognise and apply concepts in coordinate systems, to understand the meaning of function, to represent one function (proportionality) using an analytic expression and to intuitively relate a straight line slope with the proportionality constant, in functions such as $x \mapsto kx$.

### General plan of work for functions unit

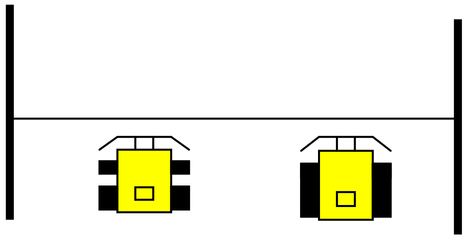
The first mathematical unit students worked on involved functions. Four sets of problems were prepared. **Problem set 1** presented examples and counterexamples of functions explaining things that take place in everyday situations. **Problem set 2** showed more complex graphs (beyond straight lines) and taught students to also recognize then as functions. In **problem set 3** it was intended that students learn proportionality as a function. The definition of proportionality emerged from the mathematical activity of students as they used robots. Finally, **problem set 4** was concerned with affin functions, such as y-intersect and slope. It also dealt with the relation between the graphical appearance of these kinds of function those of proportionality shown earlier. This paper[5] analyses students solving problem 3.

## In the classroom

We will describe and analyse mathematical activity of two groups of students. One group consisted of four girls with similar mathematical levels and abilities (**C**, **La**, **Li** and **S**). When they started to work together, they had experienced considerable difficulty, even going so far as to repeatedly suggest that the problem could not be solved, at least individually. Eventually, they understood the problem could be solved if they teamed up and learned to work cooperatively. The other group featured three boys (**M**, **P** and **Ma**), in which one of them had a higher level of mathematical ability than the other two.

The class started with the teacher distributing materials to each group: one robot (either *Roverbot* or *Tank*), one laptop, one tape-measure and a worksheet including the following tasks[6]:

I. Let's compare the two robots speed: Roverbot and Tank. Probably the first idea that occurs to you is to hold a robot race, to find out which is the quickest. However, that is not the best way to determine speed values and compare them accurately.




a) Through experimentation of Roverbot (programming, test and registration of data) complete the following table:

| Time(seconds) | 1 | 3 | 6 |
|---|---|---|---|
| Distance covered (cm) | | | |

(i) Calculate the quotient between distance covered and time. (ii) Do the values 'distance covered' and 'time' vary in direct proportion? Justify your answer. (iii) Which is the proportionality constant? In this situation what does the proportionality constant mean? (iv)Comment upon the following affirmation: "The correspondence between the distance covered by Roverbot and the time spent to cover that distance is a function."

[5] For a more general discussion about mathematical activity of students using robots to learn functions see Fernandes, Fermé and Oliveira (2006, 2007) and Oliveira, Fernandes and Fermé (2008).

[6] After the realization of several tests we verified that the time that the robot needs to reach the standard velocity as well as the braking time are negligible. So we can assume that, to the end of this question, time and distance covered vary in proportion.

## Practice as meaning

According to Engeström (1999), in the structure of an activity we can identify subjects that act over objects, in a process of reciprocal transformations that culminates with the achievement of certain results.

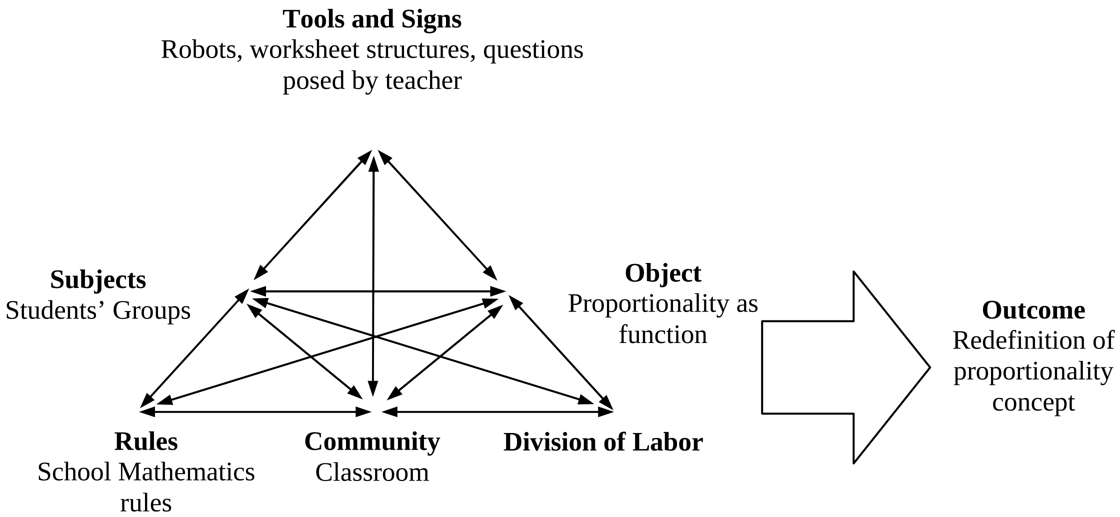



**Figure 1 – School mathematics activity structure**

Figure 1 shows activity during school mathematics class when robots were used to study proportionality as a function. In this case the term *subject* (figure 1) is collective and is represented by the different groups of students. The *community* is the class and its work methodology. The *object* is the 'raw material' at which the activity is directed and which is transformed (with the help of mediating instruments) as its outcome. In the situation considered here, the object is proportionality as a function and the instruments were the robots, the worksheet structure and the way the teacher posed questions to students. The episode described below shows how one of the groups solved the task described above.

Each student read the task. **C** programmed the Roverbot to move forward one second, then measured the distance covered. 33cm was recorded in the table. **S** followed the some process for 3 seconds and they registered 99cm. Then **C** programmed the robot to move forward 6 seconds. However, the desk on which they were working was too short for this last course. **Li** suggested they try out on the floor. This was done and 178 cm was recorded in the table. The students then began to discuss the results for the first time. They started to calculate the quotient between space covered and time, more or less the first times they speak. There dialogue is shown below:

1. C: 33/1 = 33 [data recorded on the worksheet].
2. C: 99/3 = 33
3. Li: 178:6 = 29.6666
4. S: It can't be. It has to be 33.
5. C: Let's programme and measure all again. Something is wrong. [They repeat all the process and the values were again 33, 99 and 178].
6. S: But it can't be. It has to be 33 (referring to the value of the quotient between the two variables)
7. La: 33 x 6 is 198. Let's put 198 on the table.

They erased 178 on the table and wrote 198. Teacher came near to the group and saw 198 (but he had previously seen 178).

8. Teacher: Wasn't the result of measuring 178?
9. C: Yes, but 33/1 is 33, 99/3 is 33
10. La: So we changed 178 by 198 because 33 times 6 is 198.
11. S: Let's programme and measure all again.

Meanwhile another group calls teacher. They programmed again the robot to forward one second and then they measured the distance covered over the desk.

12. La: Oh! I know… We measured in two different places. We have to measure always on the floor.

The results obtained of measuring the distance covered were 30, 89 and 178 for 1, 3 and 6 seconds respectively.

13. The results of the quotient were 30, 29,(6) and 29,(6) respectively. Students accepted them as good and answered that time and distant covered are in direct proportion.

Wenger (1998) states that "meaning is a way of talking about our (changing) ability - individually or collectively – to experience our life and the world as meaningful" (p. 5). He describes meaning as a learning experience.

The concept of proportionality is studied in mathematics class from 5$^{th}$ grade onwards. It refers to a constant relationship between two variables and is usually discussed abstractly, such as in the example below:

Verify that there is no proportionality between the following variables.

| | | | | |
|---|---|---|---|---|
| a | 13 | 26 | 39 | 52.08 |
| b | 1 | 2 | 3 | 4 |

Many times, in school mathematics, proportionality is discussed without context; only numbers matter and the emphasis is on the mathematical concept instead of in the meaning of mathematical concept. This process makes difficult the learning experience in Wenger (1998) sense.

In the episode presented above, the students believe that the variables time and distance should be in proportion. Analysing the episode we can not determine the origin of that belief. But we can conjecture that it comes from the presence of the robot (a car) or from the way the question is written in the worksheet (question iii). Although we are guessing at its source, it is clear that the idea of proportionality is meaningful for the students, as they choose to recapture their data several times in the face of results that violate this principle. Only when an inconsistency appears, do the students begin to discuss where they made a mistake and what to do in order to solve it. But the idea that time and distance should be in proportion is really meaningful for them. This can be seen when they changed the result (from 178 to 198) to ensure that the calculations adhere to the rule and neglecting the fact of the last quotients are not equal. In spite of the evidence of the measurements, students believed that values should be in proportion. This shows that the 'dogmatic' knowledge of direct proportionality is more *entrenched*[7] than their confidence in their ability to successfully run experiments and, consequently, they neglected the evidence of the experiment.

The use of unusual artefacts in mathematics class (tape-measure, robots, laptops) associated to a methodology of work where students can stand up, measure, program the computer and experiment with data helped students to construct and rebuild meaning about the concept of proportionality.

From the perspective of activity theory, students groups acted on robots, which were mediators' elements, between them and the object. The robots were a facilitator of activity that they empowered students during the process of object transformation.

In the second student group, students had a different experience. After programming the robot for 6 seconds they had the following discussion:

> M: It's 172 cm [referring to the space covered by the robot in 6 seconds].
>
> P: 172?
>
> M: 172 or 173.

[7] The term entrenchment refers to Goodman (1954). He claims that the criterion to decide between two predicates (in our case, the rule and the evidence) is the degree of entrenchment of the predicates. The entrenchment of a predicate depends of the history of the past projections and their success or failure. In our case, the students have more history records where they must leave their proper ideas when confronted with the formal concepts (teacher knowledge, textbook).

P: But it can't be. It's not correct. It should be 180. And the other value should be 90 [referring to the space covered by the robot in 3 seconds].

Ma: Why?

P: I have done it in the calculator. If in one second the robot covers 30cm, I multiplied it by 3 and it's 90. And for 6 seconds it is 180.

M: But it's not correct. Aren't you seeing the tape measure? It's 173cm.

In this dialogue we can notice that one of the students of the group knows the scholarly notion of proportionality well and applies it to compare with the results of the experiment. He seems to trust more in the mathematical rules that he knows than in the evidence of the measurement experiment.

The two students groups reacted differently to the inconsistency between mathematical rules and the empirical evidence: one believed the values they obtained through measurement and considered that the values they obtained by approximation from the quotient were enough to guaranty the proportionality (as shown the episode above); the others calculated values after they knew the space covered by the robot in one second. Where does this difference in attitude (in the face of the same evidence) come from?

*The division of labour* (figure 1) refers both to the horizontal division, of the tasks between different members of the community, and the vertical, of power and status. The vertical division of labour is connected with the fact that, in the groups, there are students with more power than others (due to their superior performance in mathematics class, assessed through evaluation by their co-students) and these lead the search to solve the problem. Therefore, by analysing the horizontal division of labour we can say that it has emerged naturally between different students of the groups and represents the way how they organized their work in order to solve the problem proposed by teacher.

Finally the *rules* (figure 1) refer to the explicit or implicit regulation, to norms and conventions that constrain actions and interactions in the activity system. What students believe to be mathematics class, the way they see mathematical rules, the way they interpret the question put by the teacher and the worksheet structure (that is connected with the way they see mathematics class and mathematics) impose a certain form to the students' actions. As we have said before we have two different reactions to the inconsistency between correctness of mathematical rules and the inexactness of the empirical evidence – for one group the rules won and for other the empirical evidence.

## FINAL CONSIDERATIONS

Robots helped students to renegotiate the meaning of proportionality that they had previously encountered (during seven years of school mathematics) as depending uniquely and exclusively of the quotient between two variables. The negotiation of the meaning evolves through the interaction of two process – participation and reification (Wenger, 1998). When concepts are presented to students as reified objects participation (in Wenger's sense) becomes difficult. Learning through experience, essentially negotiating meaning through participation, helps students' better grasp mathematical concepts. Most of the students in the study described here redefined the concept of proportionality as a function directly because of the work done in this mathematics class and the robots had an important role in this process (Fernandes *et al.*, 2006, 2007, Oliveira *et al.* 2008). Furthermore, as this result was not explicitly intended. Instead, it was an emergent aspect of the students' mathematical practice and study of functions. In the course of their experience with robots, students transitioned from the abstract perfection of mathematics (the definition of proportionality in school mathematics) to the practical reality (proportionality in action) of everyday experience.

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# INTERNET AND MATHEMATICAL ACTIVITY WITHIN THE FRAME OF "SUB14"

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*In this paper we analyze and discuss the use of ICT, particularly the Internet, in the context of a mathematical problem-solving competition named "Sub14", promoted by the University of Algarve, Portugal. Our purpose is to understand the participants' views regarding the mathematical activity and the role of the technology they've used along the competition. Main results revealed that the participants see the usage of Internet quite naturally and trivially. Regarding the mathematical and technological competences elicited by this competition, evidences were found that develops mathematical reasoning and communication, as well as it increases technological fluency based on the exploration of everyday ICT tools.*

## A GLIMPSE OVER THE MATHEMATICAL COMPETITION "SUB14"

Sub14 (www.fct.ualg.pt/matematica/5estrelas/sub14) is a mathematical problem-solving web based competition addressed to students attending 7$^{th}$ and 8$^{th}$ grades.

It comprises two stages. The *Qualifying* consists of twelve problems, one every two weeks, and takes place through the Internet. The Sub14 website is used to publish every new problem; it provides updated information and allows students to send their answers using a simplified text editor in which they can attach a file containing any work to present their solution. The participants may solve the problems working alone or in small teams and using their preferred methods and ways of reasoning. They have to send their solution and complete explanation through the website mailing device or using their personal e-mail account. Every answer is assessed by the organizing committee, who always replies to each participant with some constructive feedback about the given answer.

The word problems are selected according to criteria of diversity and involve several aspects of mathematical thinking not necessarily tied to school mathematics. Their aim is to foster mathematical reasoning, either on geometrical notions, numbers and patterns, and logical processes, among others. There is a concern on presenting problems that allow different strategies and also some that have multiple solutions.

In Iona's class the students had to elect a delegate and a co-delegate. Each student wrote two names in a voting sheet by order: the first for the delegate and the second for the co-delegate. There are 13 students in the class. How many ways have a student to vote if his or her own name is allowed?

**Fig. 1: A problem aiming to elicit the abilities of organizing and counting**

The *Final* consists of a one-day tournament where the finalists solve five problems, individually, with paper and pencil, and explain their reasoning and methods. This *Final* also provides some recreational activities addressed both to contestants and accompanying persons, namely parents and teachers.

Joanna, Josephine and Julia are all very fond of sweets. As the summer aproaches they decide to go on a diet. Their father has a large scales and they used it to weigh themsleves in pairs.
Joanna and Josephine together wheighed 132 kg
Josephine and Julia together wheighed 151 kg
Julia and Joanna together weighed 137 kg.
What is the weight of each one?

**Fig. 2: A problem from the Final on identifying and relating variables and numbers**

Demanding a clear description of the reasoning, methods and procedures was a strong concern of the committee. Moreover, the feedback sent to each participant had an essentially formative role (Diego & Dias, 1996), aimed at stimulating self-correction and valuing students' own ideas. Every two weeks the Sub14 committee publishes a proposal of the solution of the previous problem, stressing the diversity of strategies that students could have applied. Hence, the committee selects noteworthy excerpts from student's solutions, whether due to the originality of their reasoning, their creativity or the interesting usage of technological tools.

## A THEORETICAL FRAMEWORK

In this paper we are addressing a part of a larger study and consequently we refer to a few theoretical aspects of the overall framework. There are four main focuses in the theoretical approach: (a) looking at mathematics as a human activity, (b) taking problem solving as an environment to develop mathematical thinking and reasoning, (c) exploring the concept of being mathematically and technologically competent and finally (d) considering the role of home ICT in out of school mathematics learning.

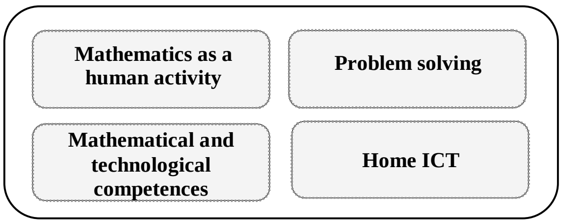


**Fig. 3: Main conceptual elements of the theoretical framework**

### Mathematics as a human activity

*Doing mathematics* may be recognized as a human activity based upon a person's empirical knowledge, in search of a formalized understanding of the everyday problematic situations. From this point of view, Freudenthal (1973, 1983) states that human activity, which comprises empirical knowledge, guides oneself from the simple observation and interpretation of phenomena – horizontal mathematizing – to its abstract structuring and formalization – vertical mathematizing.

One of the criteria observed in launching a problem in Sub14 refers to the expectation that participating students will be able to activate their empirical knowledge and their experience to tackle mathematical problems. This perspective on mathematical activity is shared by many authors who emphasize the importance of exploring mathematical situations starting from common sense knowledge (Hersh, 1993, 1997; Ernest, 1993; Ness, 1993; Matos, 2005). As Schoenfeld (1994) claims, easiness in the use of mathematical tools, like abstraction, representation or symbolization, does not guarantee that a person is able to think mathematically. Rather mathematical thinking requires the development of a mathematical point of view and the competence to use tools for understanding.

This is the perspective that is present in Sub14 and which expresses the prevailing concept of mathematical activity arising from the perspective of Realistic Mathematics Education: bringing student's reality to the learning situation so that he/she is the one who does the mathematics, drawing on his/her knowledge and resources.

## Mathematical knowledge and problem solving

Several authors from the field of mathematics education have proposed problem solving as a privileged activity "for students to strengthen, enlarge and deepen their mathematical knowledge" (Ponte et. al, 2007, p. 6).

This view on mathematical problem solving entails a conception of mathematical knowledge that is not reducible to proficiency on facts, rules, techniques, and computational skills, theorems or structures. It moves towards broader constructs that entails the notion of mathematical competence (Perrenoud, 1999; Abrantes, 2001) and problem solving as a source of mathematical knowledge. In solving a problem there are several cognitive processes that have to be triggered, either separately or jointly, in pursuing a particular goal: to understand, to analyze, to represent, to solve, to reflect and to communicate (PISA, 2003).

According to Schoenfeld (1992), the concept of mathematical problem can move between two edges: (i) something that needs to be done or requires an action and (ii) a question that causes perplexity or presents a challenge. The educational value of a problem increases towards the second pole where the solver has the possibility of coming across significant mathematical experiences. One of the purposes of mathematical problems should be to introduce and foster mathematical thinking or adopting a mathematical point of view, which impels the solver to mathematize: to model, to symbolize, to abstract, to represent and to use mathematical language and tools (Schoenfeld, 1992, 1994).

The formative aims of the problems proposed in Sub14 are essentially in line with the perspective of giving students the experience of mathematical thinking and also the opportunity to bring forth mathematical models and particular kinds of reasoning.

## Communication, home technologies and learning

Considering that mathematics is a language that allows communicating your own ideas in an accurate and understandable way (Hoyles, 1985), Sub14 intends to develop that relevant communicational aspect, as stated in the current National Curriculum: “students must be able to communicate their own ideas and interpret someone else’s, to organize and clearly present their mathematical thinking” and “should be able to describe their mathematical understanding as well as the procedures they use” (Ponte et. al, 2007, p. 5). Conversely, the importance of developing the competence of mathematical communication draws on a strong connection between language and the processes that structures human thought, as it is referred by Hoyles (1985). Accordingly, language takes up two different roles in mathematical education: communicative, where students show the capacity to describe a situation or reasoning act; and cognitive, which may help to organize and structure thoughts and concepts. Hence, there is a multiplicity of capacities and competences, both mathematical and technological, which are triggered through the combination of facts and resources in order to solve each problem of the competition.

Technologies and particularly the Internet, which gave life to Sub14, had a somewhat “neutral” or “trivial” role since the main focus of students’ concerns was on the actual mathematical activity involved. Noss and Hoyles (1999) used the “window image” to emphasize this phenomenon: a window allows us to look beyond, and not only at the object itself. Although every new technology tends to draw attention to the tool itself, we soon need to “forget” the tool and concentrate on the potentialities it has to offer, namely on the learning and cognition field.

Using Lévy’s (1990) ideas, Borba and Villarreal (2005) claim that technology mediates the processes that are responsible for the rearrangement of human thinking. In fact, knowledge is not only produced by humans alone, but it’s an outcome of a symbiotic relationship between humans and technologies – which the authors entitled *humans-with-media*: “human beings are impregnated with technologies which transform their thinking processes and, simultaneously, these human beings are constantly changing technologies” (p. 22).

Indeed, human thought used to be defined as logical, linear and descriptive. Nowadays it is hastily changing into a *hypertextual thinking*, comprising many forms of expression that go beyond verbal or written forms, such as image, video or instant messaging. These social changes allow youngsters to develop a large number of competences, which grants them the skills and sophistication required to learn outside the school barriers.

Towards the conclusions of the ImpaCT2 project, that took place in Great Britain, Harrison (2006) asserted that the model used to measure the influence of new technologies on youngster’s school achievement was too simplistic and induced to settle on the absence of such influence. This author then proposed a new model that emphasized the importance of social contexts in which learning takes place. Harrison (2006) was thus able to conclude that learning at home must not be neglected, but be faced as a partner of the school curriculum.

Although knowledge gathered outside the school is frequently seen as worthless, it is clear that children are capable of watching a YouTube's video, talk to their friends through MSN, and also solve the Sub14 problems and express their thinking using an ordinary technological tool. These "digital natives" (Prensky, 2001, 2006) access information very fast, are able to process several tasks simultaneously, prefer working when connected to the Web and their achievement increases by frequent and immediate rewards.

## METHODOLOGY

The purpose of this study was to identify and understand the participants' perceptions regarding the (i) mathematical activity, (ii) the competences involved and (iii) the role of the technological tools they've used along the competition.

A case study methodology reveals itself appropriated in cases where relevant behaviours can't be manipulated, but it is possible and appropriate to proceed to focused interviews, attempting to understand the surrounding reality (Yin, 1989). Since we intended to get diversity and interpret results, eleven participants were chosen intentionally, from the 120 finalists, hoping they would provide interesting data according to the research questions.

The field work began collecting data that would allow a complete understanding of the competition, in order to adjust the approach to the participants. Later on, we used other data collecting techniques: a *questionnaire* to the finalists, *video records* from the Final, *documental data* from participants (such as their solutions to the Sub14 problems, or their interactions with the Sub14 committee, using e-mail). That data allowed the planning of interviews to the eleven participants, as well as to their parents, aiming at collecting descriptive data, in their own language, hoping for an understanding on how they viewed certain aspects of Sub14 and of their involvement.

For the data analysis we used an interpretative perspective (Patton, 1990) and an inductive process (Merriam, 1988), based on content analysis. Thus, the objective was to understand the significance of the events from the interviewees' perspective, within the scope of the theoretical assumptions defined prior to the interviews.

## THE INTERNET – THE SUB14 LIFE SUPPORT

The first evidence produced about students' perceptions on the problem solving environment was the fact that the Internet and the technologies used within Sub14 assumed, from the point of view of students, a neutral role in the development of their mathematical activity. However several aspects of their products and statements showed evidence of the importance and usefulness of different tools, behind their apparent indifference to technology if put in abstract terms. Therefore, we may state that the Internet undoubtedly is the technology that brings Sub14 to life; all the learning processes and the competences involved derive from the interaction provided and nourished by this tool.

## Trivializing Technology

Resorting to the Internet and other technologies was seen as absolutely natural by some participants.

> "As I see it, reasoning comes from the mind; therefore I think no technology will help us to really solve a problem." [Bernardo]

Trivializing the role of the Internet and the technology involved in the competition can be found in the model proposed by Harrison (2006), which highlights the importance of the social context surrounding the learning process. These participants show all the characteristics of a digital native (Prensky, 2001), i.e., they start using computers at an early age, with a great variety of purposes, which can be related or not to school learning. Furthermore, these participants can also be considered as "humans-with-media", or particularly, "humans-with-Internet", according to the definitions proposed by Borba and Villarreal (2005), since their personality is being built, simultaneously, through the daily interaction with the Internet and other technologies.

## The Role of Communication and Feedback

Essentially, the participants like the feedback sometimes provided immediately by the Sub14 committee, resulting from the analysis of their answers to each problem. The possibility of correcting little mistakes or even change the resolution completely, using the hints from the feedback, increase their self-esteem and motivation to remain in the competition. For the interviewed students, this is the characteristic that distinguish Sub14 from other similar competitions.

> "This year I also participated in another competition. We send an answer to a problem, but they don't reply to us, and the Sub14 committee keeps sending hints". [Isabel]

As students pointed out there is someone who receives their answer to the problem, their questions or even their complaints.

> "It's not something that we send and no one will care about, they are always there." [Lucia]

As mentioned above, the feedback is almost immediate and this is only possible due to the communicability that the Internet enables. The constant request for auto-correction forces the participants to reflect on their own reasoning and the mistakes given, stimulating them to submit a correct answer as quickly as possible. Some of them sent messages to Sub14 several times a day, until they get the confirmation that their answer was correct.

Another positive aspect of this bilateral communication is the request of a complete, coherent and clearly written explanation of the participant's reasoning. This way, the feedback provided by the organizing team respects and nourishes the reasoning of each participant, as well as the processes used. We have even noticed a development on the correctness of the answers that the participants submitted throughout the competition.

> "In the beginning it was somehow strange. I wasn't used to it. I'd put the calculations and that was it. But we had to present all our thinking. It was as if I had to write what I was thinking. Thus, I would think out loud and split it into parts. But from the 3rd or 4th problem I was already used to it." [Isabel]

This feedback originated a change of attitudes in some participants within their mathematics classroom when facing assessment situations. The students themselves observed they took more care while answering to questions posed by the teachers, presenting all the necessary justifications and showing a greater predisposition to interpret a problematic situation, find a reasoning path or procedure in order to explain the solution in a convincing way.

> "[…] I now pay more attention to little details that sometimes others don't, and it reflects on the tests and on the problems that the teacher gives us, some of them really tricky… but now I am tuned!" [Lucia]

**"Home Technologies"**

The dynamic nature of the bidirectional communication can be felt in other aspects revealed by the participants. First off all, we note the usage of the Sub14 website: the participants use it frequently and think that the available information is important and useful, they like the design, the way it is organized and the fact that it is permanently updated:

> "I like having an organized website (…) the 'Press Conference' page was always updated." [Ana]

The purpose of posting submitted solutions was to show the methods used by some of the participants, hoping to improve their performance by the positive reinforcement of seeing their works and their names posted online.

> "Yes! Sometimes I would go there to see if any of the posted solutions was mine! Once or twice I found my answer and I was very happy and shouted… 'Daddy, daddy, come here!'" [Bernardo]

Bernardo's enthusiasm, as well as many other participants', supports the pedagogical and motivational aspects of the methodology adopted in Sub14. Not only it promotes the diversification of reasoning strategies and points out the several problem solving phases, but it also increases self-esteem and improves innovation and creativity as "special" answers are selected to be published online.

Moreover, the fact that Sub14 is a digital competition allowed the participants the opportunity of communicating their reasoning in an inventive way, since they could resort to any type of attachments, particular the ones they felt more comfortable with or the ones they found adequate to the problem itself. Therefore, the participants used mainly the text editor, MSWord, but they also used drawing and spreadsheet programs, like MSPaint and MSExcel, all examples of home technology.

MSWord was used to compose text, organize information in tables, and insert images, automatic shapes, WordArt objects or Equation expressions. It was elected the favorite between the participants, since it is the one they better understand and constantly are asked to use for several school assignments.

> "[Word] is the simplest to use, it's the one that I have more confidence on to do school tasks, and I'm used to it. It's the one I'm good at." [Lucia]

Using images was a strategy that seven of the interviewee used. Nevertheless, some of them only inserted images that had something to do with the problem context, more like an illustration. In this case, we may consider that resorting to images had mainly an aesthetic function, as it didn't help presenting or clarifying the reasoning and strategy used to solve the problem. However, other interviewees sketched their own images using MSPaint in order to improve the intelligibility of their thoughts:

> "Anything that I thought that could help to improve the reasoning, I would draw it [in paper] and then I'd put it in the computer." [Bernardo]

> "We were playing with some straws and we reached the solution by trial and error. Then [we took some pictures] with the digital camera [and] put them in the computer so that we could send them." [Alexandra]

In this way the image usage assumed, essentially, two roles in the answers of these participants. Firstly, it was merely a visual detail, which may be influenced by the type of work done in students' school assignments. Secondly, the creation of images within the context of their interpretation of the problems is an evidence of their efforts on expressing their reasoning in the best possible way. Moreover, we can notice their awareness of the different representations that could materialize their reasoning and even some decision ability when facing the options they had at hand.

Two interviewees used Excel to present their answers. One of them used this tool to solve every Sub14 problem, showing however a narrow usage of the program as a means to organize the information and his answer. Seldom using the function "SUM", he essentially resorted to tables and images, considering that the spreadsheet was better than a text editor. The referred simplicity seems to come from the fact that he has been exposed to this tool from an early age:

> "Sometimes, when I was a kid – I got my first computer when I was six – I liked to get there [MS Excel] and do squares with the cells, paint them and that sort of things…" [Bernardo]

Another participant used the spreadsheet to solve five of the twelve problems, showing that he knew some of the advantages of this tool. Therefore, these participants were confident enough in using MSExcel, nonetheless not as a result of work within the school context, but rather of their domestic "findings".

## ANOTHER LOOK AT SUB14 AS A LEARNING ENVIRONMENT

Solving the Sub14 mathematical problems requires looking at a problem situation from a mathematical perspective. This can be seen as a mathematizing process, since the participant is stimulated to express the way in which thinking was organized and progressed. In this competition, the participants found a place where they could freely communicate their ideas, had someone who listened and advised them, helping to make their mathematical thinking and expression become clearer. Moreover, when solving a problem, they faced the transition from convincing themselves to convincing the others (Mason, 2001). This led participants to develop their own understanding of the problem, promoting the usage of domestic technologies to communicate, thus adding competences that sometimes school neglects or forgets.

As a learning environment, although being external to the school context, Sub14 is aligned with school mathematics, and promotes a set of competences that fit within current mathematical education purposes and curricular targets. The fact that the competition occurs in a loose institutional context allows a greater family commitment and complicity with the participant's learning process, fostering the discussion on mathematical questions and problems outside the school environment, especially at home, maybe around dinner table.

Further work on this field shall include a future experience to investigate the possibility of allowing participants to communicate amongst them, within the website, bearing in mind the idea of a connected learning environment.

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# A RESOURCE TO SPREAD MATH RESEARCH PROBLEMS IN THE CLASSROOM

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*In this communication we intend to present a digital resource the aim of which is to give aid to teachers to use research problems in their classes; in a first part we are going to present the theoretical framework which was used by the team in the conception of the resource and the consequences on its model; we will present the results of a study dealing with the role and the impact of the resource used by teachers preparing lessons.*

## INTRODUCTION

Different works have shown the benefits of the use of research's problems [Polya, 1945, Schoenfeld, 1999, Brown and Walter, 2005, Harskamp and Suhre, 2007, Arsac et al., 1991, Arsac and Mante, 2007], in the construction of knowledge and both the interest of teachers and the difficulties to deal with in the classrooms; moreover, the institutional injunctions of using research problem are important in France and are going to take part in the final evaluation of the secondary school [Fort, 2007].

As far as we are concerned, and in the framework of the Piagetian psychological theory, we assume that the construction of knowledge has to go through an adjustment to the milieu as we will define it in the next section, and in this context, research problems are elements of the "material milieu" that teachers offer to learners.

We also assume that amongst all hindrances of generalization of research problems in the classroom, the following points are decisive:

1. the important part of the experimental dimension in problem solving clashes with the main representation of mathematics amongst maths teachers but also in the society;
2. the focus on heuristics and reasoning skills in maths research problem is in contradiction with the institutional constraints of teaching maths notions, particularly regarding French maths curricula;
3. difficulties for teachers to pick out in the students' activity the mathematics part of their work, and, as a result the notions which can be institutionalized;
4. the difficulties teachers have to assess such a work, the usual assessment modalities being not appropriate.

In this context, a team of teachers and researchers[1] from different institutions (IREM de Lyon, IUFM de Lyon, INRP and LEPS[2]), has worked on the construction of a numerical resource the aim of which is to give aid to maths teachers in order to use research problems in their teaching. In this paper, we will present the main theoretical frameworks used in the construction of this resource and will show, through the results of a particular study, the role this resource can play in the activity of teachers from the preparation of a lesson to the implementation in the classroom.

## THE THEORETICAL CHOICES

This resource was written to be a part of the milieu of the teachers in the meaning Brousseau [Brousseau, 1986,Brousseau, 1997, Brousseau, 2004] and after him [Margolinas, 1995, Bloch, 1999, Bloch, 2005, Houdement, 2004] give to this concept. More precisely, learners learn through regulations of their links with their milieu. Going a bit deeper in this concept, Margolinas [Margolinas, 1995] described the structure of the milieu as a set of interlocked levels which can be described as follow:

| **Level** | **Teacher** | **Pupil** | **Situations** | **Milieux** |
|---|---|---|---|---|
| 3 | Noosphere-T | | Noospherian situation | Construction milieu |
| 2 | Builder-T | | Construction situation | Milieu of project |
| 1 | Project-T | Reflexive pupil | Project situation | Didactical milieu |
| 0 | Teacher | Pupil | Didactical situation | Learning milieu |
| -1 | Teacher in action | Learning pupil | Learning situation | Reference milieu |
| -2 | Teacher observing | Pupil in action | Reference situation | Objective milieu |
| -3 | Teacher organising | Objective pupil | Objective situation | Material milieu |

[1] Gilles Aldon, Pierre-Yves Cahuet, Viviane Durand-Guerrier, Mathias Front, Michel Mizony, Didier Krieger, Claire Tardy

[2] IREM : Institut de Recherche sur l'Enseignement des mathématiques ; IUFM : Institut Universitaire de Formation des Maîtres ; INRP : Institut National de Recherche Pédagogique ; LEPS : Laboratoire d'Etude du Phénomène Scientifique, Université de Lyon.

**Table 1 Structuring of the milieu**

In this table, the milieu of level *n* is the situation of level *n-1* and is made up of the existing relationships between *M*, *P* and *T*. Using the symmetry of the table and, in our case, proposing to the teachers a situation, (in the acceptation of the didactical theory of situations) in which the a-didactical situations of action had as aim to allow teachers to construct, by themselves, the knowledge necessary to conduct a situation of problem research in the classroom [Peix and Tisseron, 1998] we speak of the material and objective milieu of the teachers. In this study, the resource appears to be a part of the material milieu of the teacher and the question is: is it possible, for a teacher, to use the resource to facilitate his tasks:

- organizing the material milieu of the pupils,
- understanding the objective milieu of the pupils and the links between their knowledge and conceptions
- choosing the pertinent notions to be institutionalized in the reference milieu of the pupils, and anticipating the conflicts between misconceptions and tools to solve…

Moreover, the theoretical framework of cognitive ergonomics through its concepts and methods allows us to study the competencies of the teacher in his interaction with the work system, and more particularly in the relationship between the prescribed tasks and his activity. Lastly, and in the field of using a numerical resource in professional tasks, the concepts of utility, usability and acceptability [Tricot et al., 2003] have been sounded out in two different ways:

- by an evaluation by inspection in order to construct and organize the resource,
- by an empirical evaluation in a professional situation.

Utility is "the question of whether the functionality of the system in principle can do what is needed" [Nielsen, 1993]

Usability can be defined as: "the capability to be used by humans easily and effectively" [Schackel, 1991], but also "the question of how well the users can use that functionality" [Nielsen, 1993]

Acceptability refers to the decision to use the artefact, and answers the questions: is this artefact compatible with the culture, the social values, global organisation in which the artefact has to be included.

## PRESENTATION OF THE RESOURCE

### Structure

It is possible to use this resource in different ways; theoretical texts about the experimental dimension in mathematics [Dias and Durand-Guerrier, 2005, Kuntz, 2007] can be read as well as different presentations made in conferences [Aldon, 2007, Aldon and Durand-Guerrier, 2007]. It is also possible to understand the sense of the resource by reading a curriculum-vitae [Trouche and Guin, 2008] of the different steps and reflections of its building. The different situations are outlined using a common structure:

- Maths situation out of the classical literature on open problems developed in particular in IREM de Lyon (nowadays, there are seven maths situation):
    - Egyptian fractions: break down 1 into the sum of fractions of numerator 1.
    - Trapezoidal numbers: study of the sum of consecutive whole numbers.
    - The river: study of the shortest distance between two points with constraints.
    - The number of zeros of n!: study of the digits of n! in different numeration systems.
    - The greatest product: study of the product of integers of fixed sums.
    - Polya's urns: study of the dynamic of the composition of an urn in a repeated experience.
    - Inaccessible intersection: find a line going through an inaccessible point.
- Maths objects that may be used to solve the given problem: for each of the situations, the a-priori analysis allows to extract the mathematics objects that are part of the mathematics situation and can be used in the process of resolution.
- Learning situation: how the maths situation has been transformed into a didactical situation? In this part of the resource, reports of real experiments can be read.
- References
- Synthesis: a ten pages synthesis of the situation allows teacher to familiarize themselves with the content of the section.
- Connected situations: how is it possible to protract the situation and what are the extensions in the maths researches nowadays?

**Introduction of the resource**

In order to confirm the hypothesis and to evaluate the utility, usability and acceptability of the resource, we built an experimentation with teachers from the first handover of the resource to the real experiment of a research problem in the classroom. In this section we are going to focus on the first handover in order to evaluate the usability of the resource.

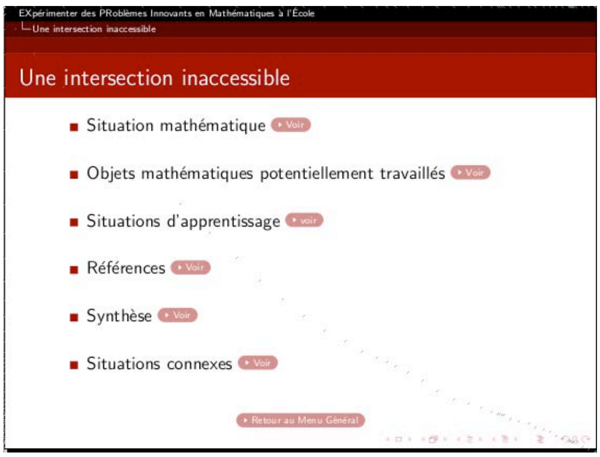
The methodology of this part of the experimentation was built using an observation of teachers faced to a professional problem (preparing a lesson using research problem); the context was a training course with sixteen teachers involved. They discovered the resource during this course as an artefact in the sense that the functioning of the resource has not been explained; the observer (the first author of this paper) recorded dialogues of two teachers and in the same time recorded the computer screen.

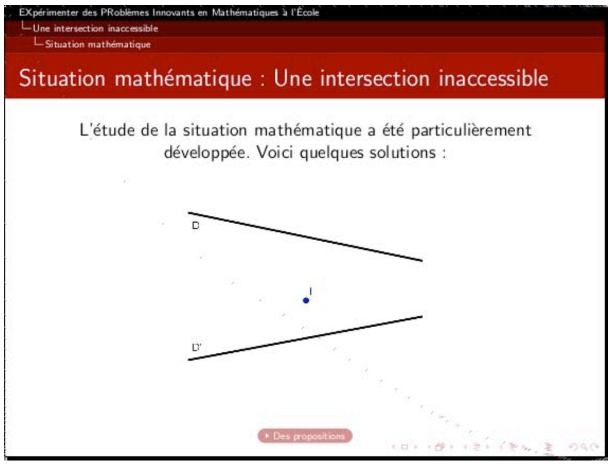
There is a confrontation, for the same person, between the position of expert (a teacher preparing a lesson, hence choosing objectives, a problem linked to these objectives, organising time of the lesson …) and the position of beginner in two different ways: using research problem in his (her) preparation and using a new tool. The theoretical framework of the didactic situation theory gave us the possibility to observe the position of the resource in the milieu of the teacher and to observe why this resource gives a possibility to the teacher to have a look into the pupils' objective milieu as described above. The cognitive ergonomics framework gives us keys to analyse the activity of teachers in this professional situation. Moreover, the concept which is tested was the usability of the resource, using the following criterions [Tricot et al., 2003]:

- Possibility of learning the system
- Control of the errors
- Memorization of the functioning
- Efficiency
- Satisfaction

But also, and we will see why later, its acceptability, that is to say the degree of confidence the teachers have.

The first result that we can highlight is the very quick adaptability of the observed teachers in front of the resource. After à three minutes wandering, the teachers used the different path in the resource to find exactly what they want as it can be possible




to see when teachers changed from one situation to another. In the first time, the mouse hesitated on the screen, going from one button to the others before the click, and progressively, the structure became clearer and the adequacy between the given objective and the browsing into the resource became safer:

After nine minutes:

Are you interested?

Yes

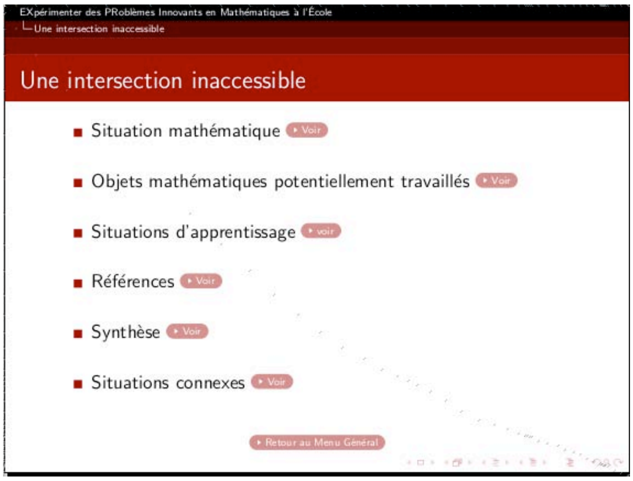
(*click on "situation mathématique*[3]")

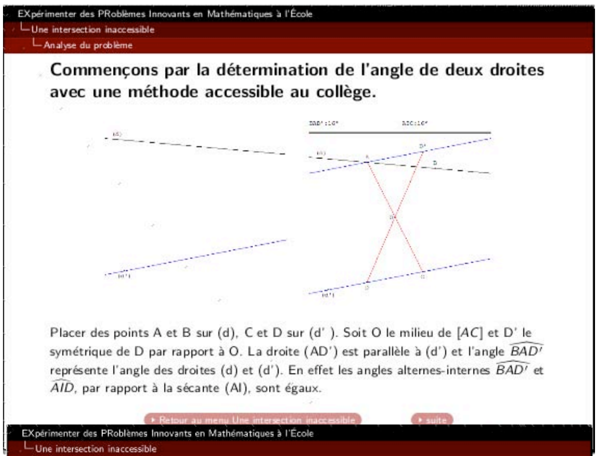
(*two clicks and two screens in one second*)

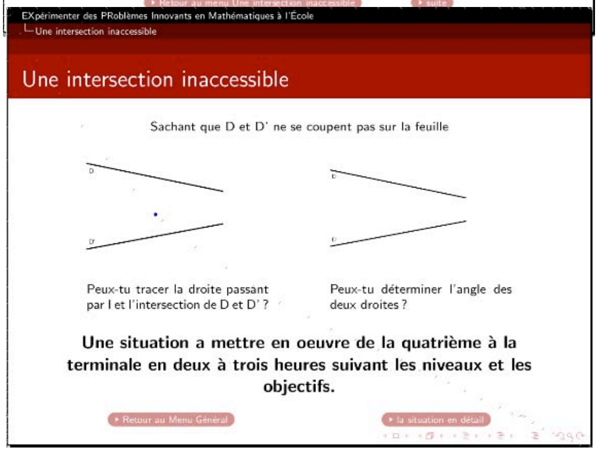
The mathematical situation… (*they read*)

Possible for our pupils (*click, click*)

I would like to see that (*the mouse turn over the menu "possible maths objects…"*)




[3] Mathematical situation

The second important observation, linked to the concept of acceptability can be seen by the feeling of trust in the authors of the resource; at the beginning of the exploration, the two teachers click on the menu: theoretical framework, and after some seconds says:

> "We are not going to read the whole text…"

And, later, in front of a situation, one of the teachers said:

> "We are going to read what they say…"

These two brief sentences show us the growing of the confidence during the use of the resource and can be considered as a clue of the acceptability of the resource. The other observations and particularly the use of the resource to construct a real lesson confirm us in the feeling of the acceptability of the resource.

**Realisation**

In order to go on in the evaluation of the resource, a second experimentation has been built with the goal of testing the utility and the acceptability of the resource; we observed a research problem lesson focusing more particularly on the interactions between pupils and teacher during the situation of action. The teacher who was observed and interviewed, participated to the training course described above.

The chosen mathematical situation was the trapezoidal numbers and the question given to the pupils of a scientific eleventh class[4] was:

What are the whole numbers which are sum of at least two consecutive integers?

Interrogating the two theoretical frameworks, the interview with the teacher allows us to bring to light utility and acceptability of the resource, but also to understand the position of this resource in the teacher's milieu.

Utility of the resource is in this case obvious, the teacher having prepared the lesson with the resource:

"Yes, yes I use it… I read all you wrote about this problem. Oh, yes, without the resource, I think I should not give this problem to my pupils, because I would have spent too much time to do this work… I would not do that!"

Regarding acceptability, a lot of clues allow us to consider, for this teacher and in this experimentation, the resource as acceptable, for example the feeling that the lesson created using the resource brought a new dimension to her course:

"I think I'll do that earlier next year, to create something in the class, precisely, this dynamic which makes the pupils actors, as I said to you, a pupil was speaking from the board, and I was at the back of the class, and the other pupils ask questions… I think it's a good way to involve pupils in the maths lessons, to put a lot of them in maths… For me, it's

[4] Pupils are sisteen-seventeen years old

very confident, visibly they enjoy this time, and I think it's something important to insert pleasure in maths lessons, it's something which questions me, because it's so easy to do maths without pleasure!"

Moreover, the interview confirms the position of the resource in the objective milieu of the teacher in a posture of preparation of a lesson including a research problem.

In an other hand the observation of a group of pupils gives us interesting feedback about the mathematical objects students deal with and shows that the *a-priori* analysis of the resource corresponds to the reality of the class; for example, one of the mathematical object which was highlighted by the authors of the resource related to this problem was the powers of two. In other words, the hypothesis was that powers of two belong to the objective milieu of the pupils and, consequently are a field of experiencing; the confrontation of pupils with these objects allows them to change their position in the milieu and to bring with the help of the institutionalisation these objects in the reference milieu of the pupils:

F2: (using her calculator) two to the power five gives thirty two… Yes ; two to the power seven gives one hundred twenty eight

G: two hundred and fifty six, five hundred and twelve, thousand and twenty four, two thousands and twenty eight …

F2: how do you calculate to obtain the results so quickly?

G: you multiply by two

F2: Ah yeah right!

In this small excerpt, the two definitions of the powers of two as an iterative or recursive process are called up and the link between these definitions is made by F2; it is possible to think that the recursive definition belongs now to her objective milieu and a necessary work must be done to institutionalize it in her reference milieu. The fact that this object was present in the resource allows the teacher to pay attention to this dialogue and to use it in her lesson:

## CONCLUSION

The described engineering and the results of observations and interviews show the place of the resource in the milieu of the particular teachers involved in this experiment, and clearly show the utility, usability and acceptability of this resource. Regarding the didactical theory of situations, this experimentation shows that the resource emplaced in the material milieu of the teachers can be mobilised in their objective milieu and used in the setting up of research problem lessons in the classroom. The resource also allows teachers to launch themselves in the different milieu of the students and to understand the position of mathematical objects in these milieus, and consequently it facilitates the institutionalization.

However, new questions appear, in particular linked to the genesis of this resource and its transformation from an external resource possibly used by a teacher to a document available in his/her environment.

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# THE SYNERGY OF STUDENTS' USE OF PAPER-AND-PENCIL TECHNIQUES AND DYNAMIC GEOMETRY SOFTWARE: A CASE STUDY

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*This study is part of an ongoing research[1] on the interpretation of students' behaviors when solving plane geometry problems in Dynamic Geometry Software and paper-and-pencil media. Our theoretical framework is based on Rabardel's (2001) instrumental approach to tool use. We seek for synergy relationships between students' thinking and their use of techniques by exploring the influence of techniques on the resolution strategies. Our findings point to the existence of different acquisition degrees of geometrical abilities concerning the students' process of instrumentation when they work together in a computational and paper-and-pencil media. In this report we focus on the case of a student.*

## INTRODUCTION

We report research on the integration of computational technologies in mathematics teaching, in particular on the use of Dynamic Geometry Software (DGS) in the context of students' understanding of plane geometry through problem solving. We focus on the interpretation of students' behaviors when solving plane geometry problems by analyzing connections and synergy among techniques used in environments, DGS and paper-and-pencil, and geometrical thinking (Kieran & Drijvers, 2006). Many pedagogical environments have been created such as Cinderella, Geometer's Sketchpad, and Cabri Géomètre II. We focus on the use of GeoGebra because it is a free DGS that also provides basic features of Computer Algebra Software. As said by Hohenwarter and Preiner (2007), the software links synthetic geometric constructions (geometric window) to analytic equations, coordinate representations and graphs (algebraic window). Our aim is to analyze the relationships between secondary students' problem solving strategies in two environments: paper-and-pencil (P&P) and GeoGebra (GGB). Laborde (1992) claimed that a task solved using DGS may require different strategies to those required by the same task solved with P&P; this fact has an influence on the feedback provided to the student.

Our broadest research question aims at how the use of GGB in the resolution of plane geometry problems interacts with the students' paper-and-pencil skills and their conceptual understanding. We analyze and compare resolution processes in both environments, taking into account the interactions (student-content, student-teacher and student-GGB). In this report we focus on two research goals as being interpreted in the case of one student, Santi. We analyze this student's instrumentation process,

[1] The research has been funded by Ministerio de Educación y Ciencia MEC-SEJ2005-02535, 'Development of an e-learning tutorial system to enhance student's solving competence'.

and we compare his resolution strategies when using P&P and GGB within each problem. In the whole research we work with a total of fourteen individual cases from the same class group and establish some commonalities and differences among them.

## THEORETICAL FRAMEWORK

We first draw on the instrumental approach (Rabardel, 2001). According to Kieran and Drijvers (2006), a theoretical framework that is fruitful for understanding the difficulties of effective use of technology, GGB in our case, is the perspective of instrumentation. The instrumental approach to tool use has been applied to the study of Computer Algebra Software into learning of mathematics and also to Dynamic Geometry Software. The instrumental approach distinguishes between and artifact and an instrument. Rabardel and Vérillon (1995) claim the importance of stressing the difference between the artifact and the instrument. A machine or a technical system does not immediately constitute a tool for the subject; it becomes an instrument when the subject has been able to appropriate it for her/himself. This process of transformation of a tool into a meaningful instrument is called instrumental genesis. This process is complex and depends on the characteristics of the artifact, its constraints and affordances, and also on the knowledge of the user. The process of instrumental genesis has two dimensions, instrumentation and instrumentalization:

- Instrumentation is a process through which “the affordances and the constraints of the tool influence the students’ problem solving strategies and the corresponding emergent conceptions” (Kieran & Drijvers, 2006, p. 207). “This process goes on through the emergence and evolution of schemes while performing tasks” (Trouche, 2005, p. 148).
- Instrumentalization is a process through which “the student’s knowledge guides the way the tool is used and in a sense shapes the tool” (Kieran & Drijvers, op. cit., p. 207).

In our research, we select different problems for being solved first with P&P and then with the help of GGB. In order to analyze the connectivity and synergy between the students’ resolution strategies in both environments, the problems are to be somehow similar. The basic space of a problem is formed by the different paths for solving the problem. We transfer the similarity of the problems to the similarity of their basic spaces. For example, the problems considered in this article, share common strategies for reaching the solution such as equivalence of areas due to complementary dissection rules, application of formulas (area of a triangle), particularization, etc.

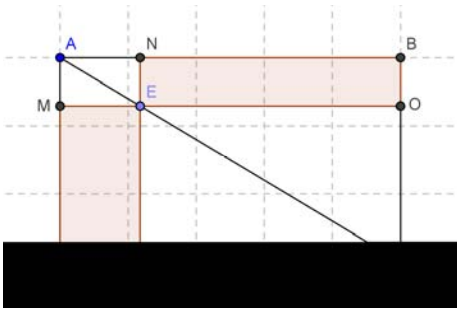
We plan to design an instructional sequence, focusing on a systematization of the interactions produced between artifacts (P&P, GGB), the mathematical actions and the didactical interactions. The theoretical framework is based on instrumental approach and activity theory (Kieran & Drijvers, 2006). We connect the activity theory as part of the “orchestration” (Trouche, 2004). The actions consist in different problem sequences to be proposed by the teacher to the students, to be solved in both media. The teacher proposes different indications or new problems. For each problem, we prepare a document with pedagogical messages that provide differing levels of information, and we group them according to the phases of the solving processes which are being carried out: familiarization, planning, execution, etc. We classify the pedagogical messages, for each phase, in three levels. Level 0 contains suggestions that do not imply mathematical contents or procedures in the solving process. The messages of level 1 only convey the name of the implied mathematical contents or procedures. Level 2 provides more specific information on these contents or procedures. For the problems to be solved in a technological environment we also prepare contextual messages. These messages are related to the use of GGB. The teacher can help the students in case they have technical difficulties with GGB.

We also specify some terms that will be used in this study of students'GGB resolutions such as figure and drawing. We use these terms with their usual meaning in the context of the Dynamic Geometry Software (Laborde & Capponi 1994). We use this distinction between Figure and drawing in order to describe the way in which students interpret the representations generated on the computer.

## CONTEXT AND METHOD

The study is conducted with a group of fourteen 16-year-old students from a regular class in a public high school in Spain. These students are used to working on Euclidean geometry in problem solving contexts. They have been previously taught GGB. The main source of data for this paper comes from the experimentation with two problems:

1. Rectangle problem: *Let E be any point on the diagonal of a rectangle ABCD such as AB =8 units and AD=6 units. What relation is there between the areas of the shaded rectangles in the figure below?*




2. Triangle problem: *Let P be any point on the median [AM] of a triangle ABC. What relation is there between the areas of the triangles APB and APC?*

These problems have to do with comparing areas and distances in situations of plane geometry. They admit different solving strategies; they can be solved by mixing graphical and deductive issues, they are easily adaptable to the specific needs of each student, and they can be considered suitable for the use of GGB. For all the problems, we start by exploring the basic space of the problems in the P&P and GGB environments. After having identified the different resolution strategies and conceptual contents of the problems, the focus is on analyzing the necessary knowledge to solve them. Finally, we prepare a document with the pedagogical and technical messages that provide differing levels of information.

All the activities with students are planned to take four sessions of one hour each with an average of two problems per session. The two problems above were developed in the first two lessons in which the students worked on their own. The inquiry-based approach to the lessons leads the students to assume the responsibility for the development of the task. The teacher fosters the students' autonomy by only intervening in certain moments and giving some messages, established a priori, concerning the resolution.

For the experimentation with each problem, the whole set of data is: a) the solving strategies in the written protocols (P&P and GGB); b) the audio and video-taped interactions within the classroom (student-teacher, student-content and student-GGB); and c) the GGB files. All these data were examined in order to inform about our research goals. The integration of data concerning these goals led us to the description of the students' process of instrumentation. For the description, different variables were considered, among them: the students' heuristic strategies (related to geometric properties, to the use of algebraic and measure tools or to the use of both…); the use of GGB (visualization, geometrical concepts, overcoming difficulties…); the obstacles encountered in each environment (conceptual, algebraic, visualization, technical obstacles…); etc.

For each case, we first analyze the P&P resolution with data coming from the tapes and the protocols. We consider the student's solving strategies and the use of mathematical contents. Then we analyze the GGB resolutions with data coming from the tapes and especially from those tapes that show the screen. We consider again the student's solving strategies, the use of mathematical contents and now we also pay attention to instrumented techniques and technical difficulties. After having developed these two types of analysis, we compare GGB and P&P resolutions by looking at the use of the two environments within each problem. To analyze the problem solving process, we also consider the phases of the problem solving process (Schoenfeld, 1985) as a whole in each group of problems (GGB and P&P).

## THE CASE OF SANTI: An episode of exploration/analysis

The mathematical content of the problem was dealt with in courses prior to the one Santi is currently taking. Santi has procedural knowledge relating to the application of formulas for calculating the area of the Figure, and sufficient knowledge of the concepts associated with geometric constructions. He is a high-achieving student. Santi is asked to solve the first problem with P&P and the second problem with the help of GGB. In this section we summarize his problem solving process for both problems.

- Resolution of the rectangle problem (P&P):

In the resolution of the first problem, after reading the statement of the problem, Santi observes the figure and then he states that he does not have enough numerical data. The teacher suggests the student to consider a particular case (heuristic cognitive message of level 1 in the planning /execution phase). Santi reacts to this message, considering the particular case in which E is the midpoint of the diagonal and he conjectures that both areas should be equal. Then he tries to prove the conjecture for the particular case in which the length AE is 2 units. The student reaches a solution to the particular case by using trigonometry. He obtains the angles in the triangle EAN (Figure 1) and he calculates the measures of the sides, AN and AM. Finally he obtains the numerical value of both areas and he observes that he gets different values. Santi requests a message about the solution because he expected to obtain equal values. The teacher remarks that there is an algebraic mistake in his resolution and suggest Santi to review the process he has followed because there are algebraic mistakes (metacognitive message of level 1 in the verification phase). The student finds the mistake and obtains the equal values of both areas (Figure 1). He then tries to use the same strategy for the general case using the relation: $\frac{8}{6} = \frac{AN}{AM}$.

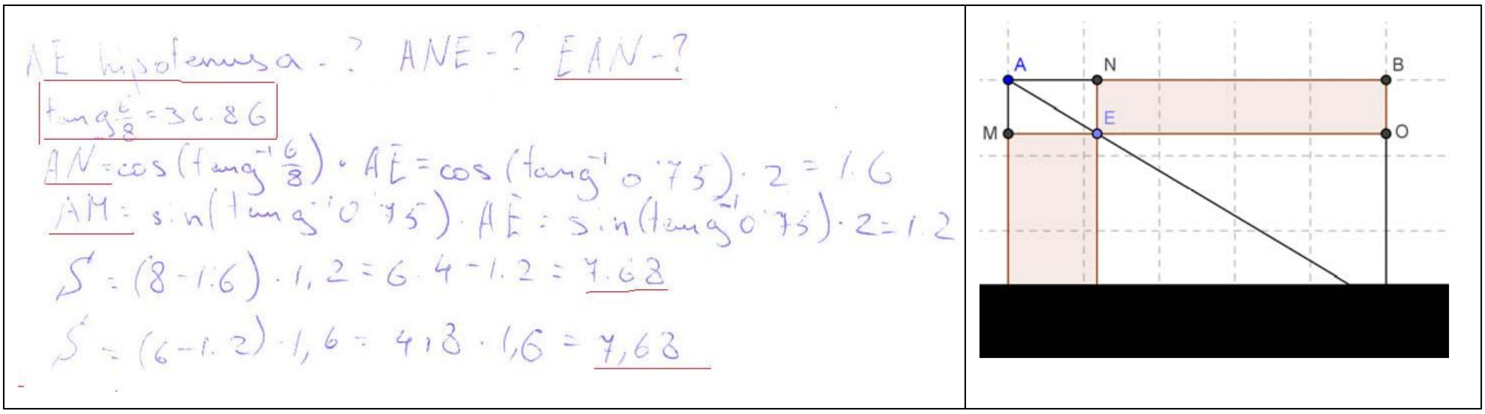



Figure 1: Resolution with paper and pencil of the first problem (Santi)

Santi bases his resolution strategy on applying trigonometry and he does not try to use the strategy based on comparing areas of congruent triangles (strategy based on equivalence of areas due to complementary dissection rules). The teacher proposes other problems to be solved with P&P and with GGB. In the following paragraph we consider one of these problems.

- Resolution of the triangle problem (GGB):

After reading the statement of the problem, Santi draws a graphic representation without coordinate axes before constructing the figure with GGB. The teacher observes that Santi has considered the point P in the side AC of the triangle instead of the median. The teacher gives Santi the following message: "*Try to understand the conditions of the problem*" (metacognitive message of level 0 in the familiarization phase). Santi constructs a new figure with GGB (Figure 2) and he observes the figure trying to find a solving path. Then he proposes a conjecture and asks the teacher for verification: "*the triangles APC and APB have a common side and the same area (he* *verifies this with the tool area of a polygon). How could I prove that these two triangles are equal* [congruent]*”? I have tried to prove that they have the same angles but I don’t see it...”*

We observe that Santi does not validate his conjecture with the help of GGB (using measure tools for instance). The teacher gives him a validation message of level 1*“Are you sure that these triangles are congruentl?* Santi reacts to this message changing the triangle ABC. He drags the vertex A (Figure 3) and he observes without measure tools that the triangles are different.

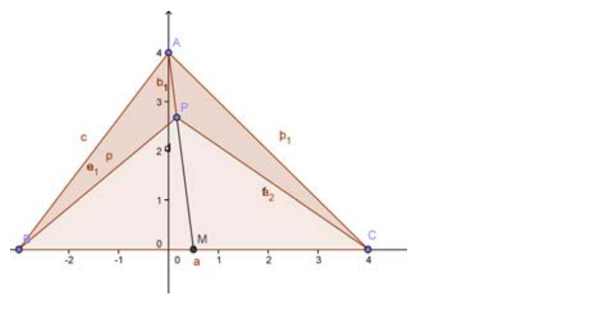


**Figure 2**: Construction with GGB of the triangle ABC and its median. Santi uses the tool polygon to construct the triangles.

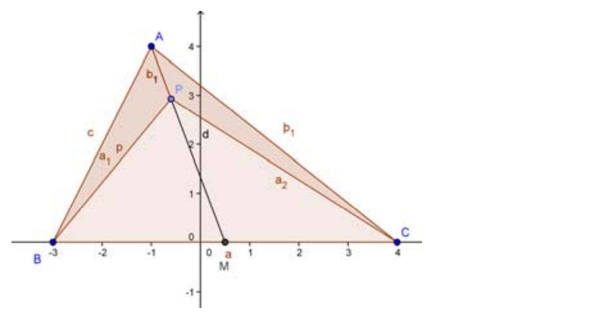


**Figure 3:** He moves the vertex A to obtain a general triangle. We observe that he tries to define vertices with coordinates that are integer numbers.

The last graphic deduction marks the beginning of the search for a new strategy. He observes the figure, without dragging its elements. More than five minutes have gone without doing anything in the screen. Santi requests again the help of the teacher (Table 1, line 1) for the familiarization phase of the problem.

| | | Interactions |
|---|---|---|
| 1 | Santi | Is P any point in the segment AM? Isn’t it the midpoint? [Santi tries to consider particular cases] |
| 2 | Teacher | P is any point in the median [AM]. The triangle ABC is also a general triangle [cognitive message of level 1 for the familiarization phase] |
| .... | Santi | [Santi reacts to this message modifying the initial triangle. He drags again the vertices to obtain the triangle in Figure 3]. |
| 3 | Santi | I think that I see it!...The triangles have a common side and the same height [the segments [BM] and [MC] (wrong deduction)] |
| 4 | Teacher | Are you sure about that? |
| 5 | Santi | [Santi reacts to this message observing the triangle without doing any action on the screen. Then he states: ]<br>No. These lines are not perpendicular! [(AM) and (BC)]. But, this was a good trial... |

| | | |
|---|---|---|
| | | Have they the same base? [he refers to the common side of both triangles ] |
| 6 | Teacher | Yes |

Table 1: How Santi tries a new solving path

For the first time, Santi tries to drag the vertices of the triangle trying to find invariants. While he drags the vertexes he looks in the algebraic window for invariants. We observe here the simultaneous use of the algebraic window and the geometric window. He observes again that the triangles have the same area in all the cases and a common side. He tries to prove that the heights are equal but he wrongly considers that the side [BM] is the height of the triangle BAP (Figure 3). The teacher gives him a message of level 0 for the validation phase (Table 1, lines 3 to 6). Santi reacts to this message constructing with GGB the perpendicular line from the vertex B to the base of the triangle (Figure 4). He tries to follow the same strategy (proving that the heights have the same length) and he drags continuously the vertexes A, B and C, changing the orientation of the triangle, and observing the constructed lines on the geometric window.

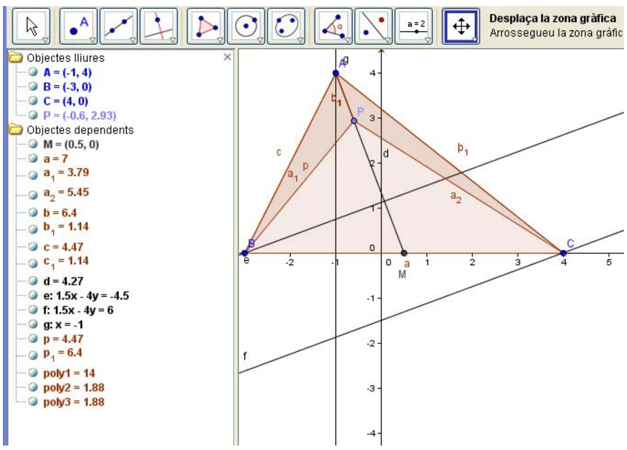



Figure 4: Construction of the height of the triangle BPA and perpendicular line through C to the median.

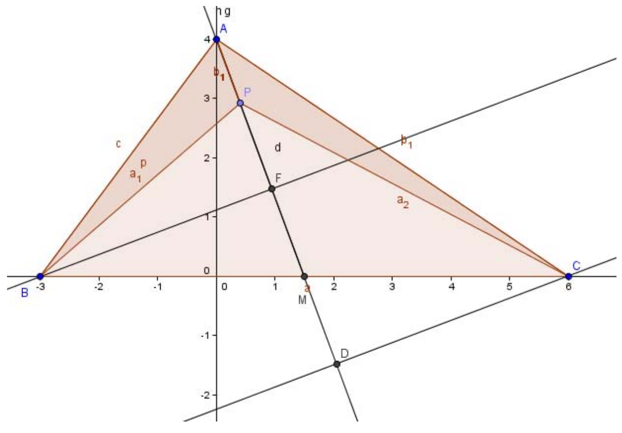


Figure 5: the heights have the same length (congruent triangles BFM and MCD)

In this time, he observes again the figure (Figure 4) without dragging. He is lost. This is the beginning of a new phase. We wonder if Santi had found a proof for his conjecture if he had constructed the heights of both triangles. Nevertheless, he does not construct the points F and D (Figure 5) and he abandons the solving strategy. Santi requests again the help of the teacher for the planning/execution phase and he states: “*Is it possible to solve the problem with trigonometry?*”.The teacher gives him a new message: *“Could you think of some way of breaking the triangle ABC into triangles and look for invariants with the help of GGB”* (cognitive message of level 2 for the planning phase). Santi reacts to the previous message of the teacher and starts a new exploration phase. He erases the perpendicular lines and drags continuously the vertexes of the triangle ABC. He observes in the algebraic window the changing values looking for invariants. He extracts the inner triangles BPM and CPM which have the same area (Table 2, line 1) from the initial configuration. This observation will suggest him a new solving path based on comparing areas. He makes a new conjecture and requests the help of the teacher for validating his deductions (Table 2).

| | | Interactions |
|---|---|---|
| 1 | Santi | Are the triangles BPM and PMC equal? (Figure 2) |
| 2 | Teacher | What do you mean by equal? |
| 3 | Santi | The triangles have the same area |
| 4 | Teacher | Yes. You should justify this fact. |
| 5 | Santi | If I subtract two equal areas from two equal areas, do I get the same area? |
| 6 | Teacher | Yes |
| 6 | Santi | Ok! I justify this with paper and pencil. |

Table 2: Strategy based on comparing areas

Finally Santi justifies his deductions with P&P, he proves that the median of a triangle divides the triangle into two triangles of same area. We wonder if the use of GGB helps Santi to find a strategy based on comparing areas.

## FINAL REMARKS

We observe in this study that Santi appropriates the software in few sessions of class and he bases his constructions on geometric properties of the figures. He also combines the simultaneous use of the algebraic window and the geometric window and he tends to reason on the figure. We consider that the affordances of the software and teacher's orchestration have influenced Santi's resolution strategies. We have identified the following instrumented schemes: 'dragging combined with perceptual approach to find a counter-example' and 'dragging combined with perceptual approach to distinguish geometric properties of the figure (perpendicularity, congruence of triangles, equality of areas). In the ongoing research (longer teaching experiment) we have also observed some common heuristic strategies in both environments such as the strategy of supposing the problem solved and the strategy of particularization. We have also observed that Santi tends to use more algebraic strategies when he works only with P&P than when he works in a technological environment. Moreover he tends to produce more generic resolutions, independent of numerical values, fostered by a proposal of problems that accept these kinds of solving strategies. Nevertheless, given that students have different relationships with the use of GGB and the detailed study of Santi gives us some insight of a future classification of typologies in the instrumental genesis. In our broader research we try to follow the instrumental genesis for a group of fourteen students to observe different students' profiles. Future research should help to better understand the process of appropriation of the software and to analyze the co-emergence, connectivity and synergy of computational and P&P techniques in order to promote argumentation abilities in secondary school geometry.

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# STUDENTS' UTILIZATION SCHEMES OF PANTOGRAPHS FOR GEOMETRICAL TRANSFORMATIONS: A FIRST CLASSIFICATION[♦]

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*The activities with the Mathematical Machines are very rich from educational and cognitive points of view. In particular, the use of pantographs has revealed educational potentialities for the acquisition of some important mathematical concepts and for the emergence of argumentation and proving processes, at any school level. In this paper, we propose a cognitive analysis of the processes involved in the manipulation of the mathematical machines, providing a first classification of utilization schemes of pantographs for geometrical transformations. This classification can be efficiently used to observe, describe and analyse cognitive processes involved in the exploration of mathematical properties incorporated in the machines.*

Keywords: Mathematical Machines, utilization schemes, pantographs, geometrical transformations and cognitive processes.

## INTRODUCTION

The Mathematical Machines Laboratory (MMLab: www.mmlab.unimore.it), at the Department of Mathematics in Modena (Italy), is a research centre for the teaching and learning of mathematics by means of instruments (Ayres, 2005; Maschietto, 2005). The name comes from the Mathematical Machines (working reconstruction of many mathematical instruments taken from the history of mathematics), the most important collection of the Laboratory. These machines concern geometry or arithmetic:

> "a geometrical machine is a tool that forces a point to follow a trajectory or to be transformed according to a given law"…"an arithmetical machine is a tool that allows the user to perform at least one of the following actions: counting; making calculations; representing numbers" (Bartolini Bussi & Maschietto, 2008).

The MMLab research group carried out various activities with the Mathematical Machines, namely: laboratory sessions in the MMLab, long-term teaching experiments in classrooms, workshops at national and international conferences and also exhibitions (see chapters 2 and 5 of the forthcoming volume by Barbeau and

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Taylor, from ICMI Study n. 16) in collaboration with the members of the association "Macchine Matematiche" (http://associazioni.monet.modena.it/macmatem).

The laboratory sessions in the MMLab are designed in order to offer hands-on activities with mathematical machines for classes of students in secondary schools (an average of 1300-1500 Italian secondary students a year come with their mathematics teacher to experience hands-on mathematics laboratory), groups of university students, prospective and practicing school teachers (Bartolini Bussi & Maschietto, 2008). As the Mathematical Machines activities in school classrooms concerns, the MMLab research group organized different long-term teaching experiments in primary and secondary schools (Bartolini Bussi & Pergola, 1996; Bartolini Bussi, 2005; Bartolini Bussi, M. G., Mariotti M. A., Ferri F., 2005, Maschietto & Martignone, 2007).

All the activities quoted above are based on two fundamental components: the idea of the "mathematics laboratory"[1] and the didactical research on the use of tools in the teaching and learning of mathematics (Bartolini Bussi & Mariotti, 2007).

The MMLab researches aim at the development of different activities that should foster, through the use of the mathematical machines, the acquisition of some important mathematical concepts and the emergence of argumentation processes.

In order to implement the studies on MMLab laboratory activities, and to set up new teaching experiments, we consider important to carry out a cognitive analysis of the processes involved in the manipulation of the Mathematical Machines. The aim of our research is identifying Mathematical Machines utilization schemes and the connected exploration processes, providing a first classification. In the paper we shall present the first steps of this new research.

## THEORETICAL FRAMEWORK

According to the educational goals that the activities with Mathematical Machines intend to realize, we investigate students cognitive processes involved in exploration of open-ended problems (in particular the problem of identifying the geometrical laws that explain how a machine works), in generation of conjectures and argumentations and in concept formation (for example: the concepts of geometrical transformations, of conic, of central perspective…). First of all, to analyse deeply these processes we propose a classification of Mathematical Machine utilization schemes [2]. This classification is suitable not only for describing the interactions between machines and subjects but also for analysing both their exploration and argumentative processes.

The processes through which a subject interacts with a machine have been studied by Rabardel in cognitive ergonomics: he grounded his research in constructivist epistemologies, primarily in *activity theories*, but also in the Piagetian and post-Piagetian developmental approach to the cognition-action dialectic (Rabardel, 1995; Béguin & Rabardel, 2000).

Rabardel proposed an original approach blending anthropocentric and technocentric approaches: as a matter of fact, in line with activity theory, he conceived the instruments as psychological and social realities and studied the instrument-mediated activity. According to Rabardel (1995) an *instrument* (to be distinguished from the material -or symbolic- object, the *artefact*) is defined as a hybrid entity made up of both artefact-type components and schematic components that are called *utilization schemes*.

> "What we propose to call " utilization scheme" (Rabardel, 1995) is an active structure into which past experiences are incorporated and organized, in such a way that it becomes a reference for interpreting new data" (Béguin & Rabardel, 2000)

An artefact only becomes an instrument through the subject's activity. This long and complex process (named *instrumental genesis*) can be articulated into two coordinated processes: *instrumentalisation*, concerning the individuation and the evolution of the different components of the artefact, drawing on the progressive recognition of its potentialities and constraints; *instrumentation*, concerning the elaboration and development of the utilization schemes (Béguin & Rabardel, 2000).

For the importance of these schemes, for their specificity in interacting with Mathematical Machine and for the limits that this paper has to respect, we focus here on utilization schemes in the case of pantographs.

## METHODOLOGY

The method used for investigation was the clinical interview: subjects were asked to explore a machine and to express their thinking process aloud at the same time. In particular, after having explained to the student that the machines to be explored are pantographs for geometric transformations, we asked:
1. To define the mathematical law made locally by the articulated system.
2. In particular, to justify how the machine "forces a point to follow a trajectory or to be transformed according to a given law" and then to prove the existing relationship between the machine properties (structure, working…) and the mathematical law implemented.

The interviews were videotaped and the analysis is mainly based on the transcripts of the interviews. The interviews were analysed with special attention to verbal tracks and hands-on activities in order to detect mental processes developing during the exploration of the machines. Every protocol is analysed in a double perspective: as bearer of new information about possible exploration processes and as evidence for the existence of recurrent schemes.

The subjects were three pre-service teachers, two university students and one young researcher in mathematics. The choice to interview subjects which are familiar with (Euclidean) geometry and with problem-solving has allowed us to collect observations of complete machine exploration: namely, the generation of conjecture about the mathematical law implemented by the machine and, subsequently, argumentation and proof of mathematical statements that can explain the functioning of the machine. Moreover, the subjects were new in working with this environment: in this way we could assume that they did not have an a priori specific knowledge about these machines.

The artefacts selected for this first research are machines concerning geometry, in particular pantographs: for the axial symmetry, for the central symmetry, for the translation, for the homothety and for the rotation. These machines establish a local correspondence between points of limited plan regions connecting them physically by an articulated system; they were built to incorporate some mathematical properties in such a way as to allow the implementation of a geometrical transformation (i.e. axial symmetry, central, translation, homothety, rotation).

## CLASSIFICATION OF THE UTILIZATION SCHEMES

In this paper we present the first part of our research that aimed to introduce a classification of utilization schemes observed during the explorations of pantographs for geometrical transformations. The identified utilization schemes were divided into two large families: utilization schemes linked to the components of the articulated system (as the constraints, the measure of rods, the geometrical figures representing a configuration of rods, etc.) and utilization schemes linked to the machine movements. As regards the first family, we have identified the following utilization schemes: the research of fixed points, movable points (with different degrees of freedom), plotter points and straight path; the measure of rods length; the research of geometric figures representing the articulated system or some part of it; the construction of geometric figures that extend the articulated system components; the individuation of relationships between the recognized geometric figures; the analysis of the machine drawings.

As regards the utilization schemes linked to the machine movements [3], we distinguish between the movements aimed at finding particular configurations obtained stopping the action in specific moments and the continuous movements aimed to analyse invariants or changes. We summarize this classification in a table:

| **Linkage Movement that stops in** | **Movements description:** |
|---|---|
| Generic Configurations | Movement that stops in a configuration which is considered representative of all configurations observed (that does not have "too special" features) |

| | |
|---|---|
| Particular Configurations | Movement that stops in a configuration that presents special features (i.e. right angles, rods positions…) |
| Limit Configurations | Movement that stops in configurations in which the geometric figures that represent the articulated system become degenerate |
| Limit zones | Movement that stops in the machine limit zones: i.e. the reachable plane points |

| **Linkage Continuous movements** | **Movements description:** |
|---|---|
| Wandering movement | Moving the articulated system randomly, without following a particular trajectory |
| Bounded movement<br>(For example: Movements by fixing one point or one rod…) | Moving the articulated system, blocking particular points or rods |
| Guided movement | Moving the articulated system, forcing a point to follow a line or a specific figure |
| Movement of a particular configuration | Moving the articulated system, maintaining a particular configuration |
| Movements between limit configurations | Moving the articulated system so that it can successively assume the different "limit Configurations" |
| Movement of dependence | Moving (in a free, guided or bound way) a particular point and see what another particular point does |
| Movement in the action zones | Moving the articulated system in a such a way that all the possible parts of the plane are reached |

## A PROTOCOL

In this paragraph we present the first part of one clinical interview transcripts dealing with the exploration phase (i.e. the beginning of the machine exploration, before the identification of the geometrical transformation made by the machine), where we can identified some of the utilization schemes described in the previous paragraph [4]. The subject of the protocol, Anna, is a pre-service teacher graduated in mathematics and she explored the pantograph of Scheiner (see Fig. 1-2).

**Anna**: (*she touches a rod which seems to remain blocked*) all motionless!...(*she moves the articulated system*) Ah, no, only a single fixed point … I saw that leads are useful, and then… … (*opening and closing the linkage, she draws lines that converge in the fixed point*) … then (*she turns the machine and she draws again "concentric lines"*)…

She starts controlling which part of the linkage is pivoted to the wood plane (*research of fixed points)* and then, in order to explore the linkage movements, she puts the leads in both plotter holes (*individuation of plotter points*) and draws curves produced by the linkage closing movement (*guided movements that end in a limit configuration*: see Fig. 3)

**Anna**: I do not see anything then………(*she is looking the motionless machine and the curves drawn*)…(*she moves the linkage and she stops in a generic configuration*) well, this is a parallelogram, I would say… That is… then, parallelogram, and in a vertex there is a lead… (*with the ruler she measures two rods: in the fig. 2 CQ and CP*)… are congruent (*she points them out)*

The analysis of the drawn curves does not seems to help her to discover what transformation the machine makes, therefore she starts an analysis of the linkage structure (*research and individuation of a generic configuration and recognition of particular geometric figures in the linkage structure*): at first she identifies a parallelogram (see Fig.4), and then she focuses on other linkage rods (those parts that do not form the parallelogram). She recognizes the parallelogram without using the ruler (probably the visual perception of congruence has been supported by the previous exploration of movements during which the rods remained parallel). Differently, to discover the other characteristics of the linkage geometric structure, Anna feels the need to *measure the rods length*, so she discovers that there are two congruent rods (CQ and CP).

**Anna**: … so this (*she looks at the linkage and she uses two fingers to show the "virtual segment" PQ that completes the triangle PQC:* see Fig. 5) is an isosceles triangle

The identification of these congruent rods arouses the construction of a new geometric figure (an isosceles triangle) created completing, with an imaginary segment, the sequence of the congruent rods (*extending and individuation of geometric figures in the linkage structure*).

**Anna**: but I will not see anything… but it doesn't say anything to me at this moment…… (*she moves the machine*, *drawing always concentric lines*) well they are always circumferences…(*she is looking at the drawings*) I do not understand if they are or not circumferences …

Also the exploration of linkage characteristics does not seem to help her, for this reason she comes back to the previous strategy: she starts again to draw lines that follow the machines closing movement (*guided movements that end in a limit* *configuration and analysis of these drawings*), but, as before, she is not aware of the drawn lines characteristics; therefore, not knowing which properties designed curves have, she can not understand how they are transformed by the machines.

**Anna**: (*she makes a zigzag movement*) well, but it seems to me that they trace the same thing (*she makes the zigzag movement in another area of the paper*)… (*she points the zigzag drawing and she moves away the linkage*)… the leads then trace the same, the same image, it seems to me, but I dare say that (*she makes a gesture*: see Fig.6)…that it is reduced in scale.

Anna changes the *guided movements* (zigzag movements) and, this time, the *analysis of the drawings* leads to the recognition of the transformation (the homothety). Therefore it seems that what lets Anna to do the discovery of the transformation incorporated in the machine, is the drawings analysis more than the machine structure; but not all the drawings seem to be successfully: in fact each of them gives only partial information about the transformation. In particular, for Anna is determinant the choice to change the movement (and consequently, the drawing): as a matter of fact in the zigzag lines it can be seen that the correspondent segments are modified, while the angles are not (in the previous drawings these proprieties are "hidden", while it came out the presence of a fixed point).

In conclusion, it is interesting to underline that also in a brief excerpt, it is possible to see the variety, the complexity of their relationships and, in particular, the plot of the different utilization schemes. After the individuation of the schemes, we can make a cognitive analysis of the exploration processes linked to these schemes. For example, we intend to examine closely how (and then why) Anna swings between two different strategies that remain separated (the drawing/analysis of lines and the study of linkage structure). This analysis brings important information for the understanding of subsequent processes: in fact, in the continuation of this protocol, the lack of interweaving of the information acquired through the different utilization schemes used, seems to be an obstacle in the Anna's proof construction (about how the machine incorporates the transformation properties). This part of the research is still in progress, but the first results raise the hypothesis that successful strategies are those that maintain a tension and integration between the analysis of the articulated system proprieties, the drawings and the invariants of the movement.

## CONCLUDING REMARKS

The studies on the interaction between a subject and a machine have to take into account an intriguing complexity because several components are involved. From a cognitive point of view and with educational goals, in this paper, we have presented a study to better understand the exploration of some geometrical machines: in particular, we have proposed a first classification of utilization schemes of pantograph for geometrical transformations and we have shown an analysis carried out through this classification. In this analysis we have underlined the importance of the identification of the different schemes in describing the aspects of mathematical machines exploration.

Further researches are needed in two directions. On the one hand, we will study how these schemes are intertwined with the processes involved in conceptualisation, in argumentation and in proving; on the other hand, we will explore the evolution of the utilization schemes and its relationship with argumentation processes and subject's cultural resources.

Moreover, this study will be developed to offer teachers tools that could be efficient to set up activities with educational goals and to intervene in students' interactions with the machines, promoting those processes that are considered relevant for the activities with the mathematical machines.

## NOTES

1. "A mathematics laboratory is a methodology, based on various and structured activities, aimed at the construction of meanings of mathematical objects. (…) The mathematics laboratory shows similarities with the concept of Renaissance workshops where apprentices learned by doing and watching what was being done, communicating with one another and with the experts" http://umi.dm.unibo.it/italiano/Didattica/ICME10.pdf.

2.In literature there are not previous cognitive studies of this type on mathematical machines. A classification of utilization schemes of instruments of different nature is proposed in Arzarello et al. (2002) where different modalities of dragging are discussed.

3.In addition to the linkage movements, there are also the movements of the machine wood base (on which the linkage is set): i.e. the rotations of the base that permit to look the machine from other points of view.

4.In these extracts there are not all the utilization schemes identified during our research. For the limit of this article we should not make an example for each of the utilization schemes previously listed.

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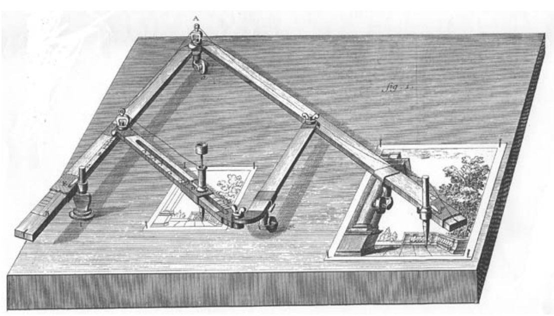
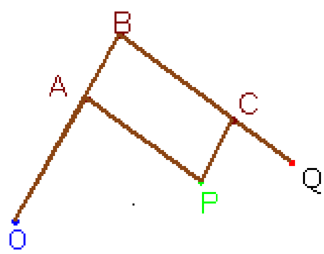
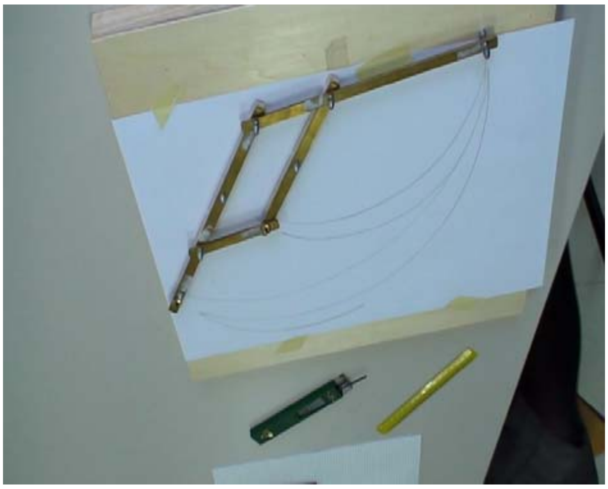
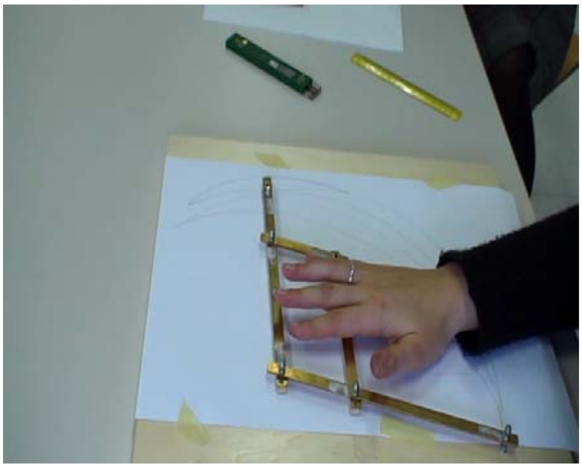





| | |
|---|---|
| **Fig 1:** Encyclopédie ou dictionnaire raisonné des sciences, des arts et des metiers (1751- | **Fig 2**: An image from Scheiner pantograph graphic animation: Four bars are pivoted so |

| | |
|---|---|
| 1780) | that they form a parallelogram APCB. The point O is pivoted on the plane. It is possible to prove that the points P, Q and O are in the same line and that P and Q are corresponding in the homothetic transformation of centre O and ratio BO/AO. |





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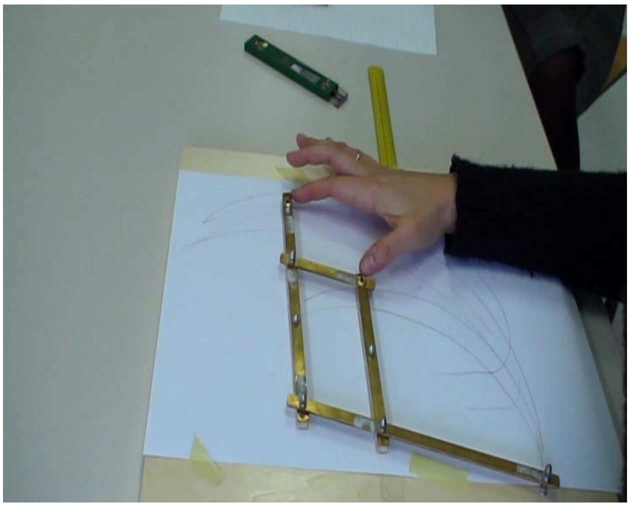
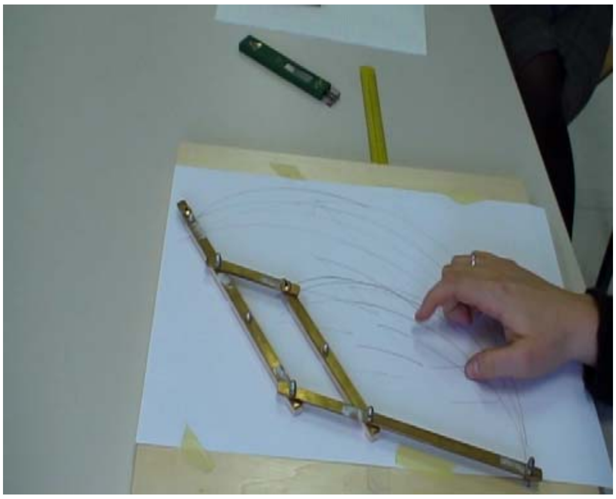
|---|---|
| **Fig. 3:** Anna's drawings | **Fig. 4**: Anna identifies the parallelogram |





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|---|---|
| **Fig. 5:** Anna shows the isosceles triangle | **Fig. 6**: Anna's gesture for indicating the "reduction in scale" of the zigzag lines |

# THE UTILIZATION OF MATHEMATICS TEXTBOOKS AS INSTRUMENTS FOR LEARNING

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*The mathematics textbook is one of the most important resources for teaching and learning mathematics. Whereas a number of studies have examined the use of mathematics textbooks by teachers there is a dearth of research into the use of mathematics textbooks by students. In this paper results of an empirical investigation of the use of mathematics textbooks by students as an instrument for learning mathematics are presented. Firstly, a method to collect data on student's use of mathematics textbooks is introduced. It is explicated, that this method is capable to explore the actual use of the mathematics textbook by students, and a way of recording the use of the mathematics textbook whenever and wherever students use it. Secondly, results from the study are presented. The results outlined in this paper focus on typical self-directed uses of the mathematics textbook by students.*

## INTRODUCTION

Research in mathematics education has been concerned with the role of new technologies in the teaching and learning of mathematics from the very beginning computers and information technologies entered the mathematics classroom. In the first ICMI study the computer is even considered to be a new dimension in the mathematics classroom: "We now have a triangle, student-teacher-computer, where previously only a dual relationship existed" (Churchhouse et al., 1984). But, this perspective disregards the fact that tools have always been incorporated in teaching and learning mathematics and thus the relationship in the mathematics classroom has never actually been dual. The mathematics textbook was and still is considered to be one of the most important tools in this context. According to Howson, new technologies have not affected its outstanding role: "despite the obvious powers of the new technology it must be accepted that its role in the vast majority of the world's classrooms pales into insignificance when compared with that of textbooks and other written materials." (Howson, 1995)

Valverde et al. (2002) believe that the structure of mathematics textbooks is likely to have an impact on actual classroom instruction. They argue, that the form and structure of textbooks advance a distinct pedagogical model and thus embody a plan for the particular succession of educational opportunities (cf. Valverde et al., 2002). The pedagogical model only becomes effective when the textbook is actually used. Therefore, mathematics textbooks should not be a subject to analysis detached from its use. It is an interactive part within the activities of teaching and learning mathematics In order to develop a better understanding of the role of the mathematics textbooks within the activities of teaching and learning mathematics an activity theoretical model was developed (Rezat, 2006a):

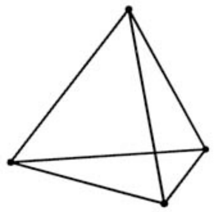



**Fig. 1: Tetrahedron model of textbook use**

This model is based on the fundamental model of didactical system: the ternary relationship between student, teacher, and mathematics (Chevallard, 1985). The mathematics textbook is implemented as an instrument at all three sides of the triangle: teachers use textbooks in the lesson and to prepare their lessons, by using the textbook in the lesson teachers also mediate textbook use to students, and finally students learn from textbooks. Thus, each triangle of the tetrahedron-model represents an activity system on its own. From an ergonomic perspective it is argued that artefacts have an impact on these activities, because on the one hand they offer particular ways of utilization and on the other hand the modalities of the artefacts impose constraints on their users (cf. Rabardel, 1995, 2002). Thus, the mathematics textbook has an impact on the activity of learning mathematics as a whole that is represented by the didactical triangle on the bottom of the tetrahedron.

Whereas a number of studies have examined the role of new technologies in terms of tool use (cf. Lerman, 2006) the role of the mathematics textbook as an instrument for teaching and learning has not gained much attention. So far, a number of studies have examined the use of mathematics textbooks by teachers (e.g. Bromme & Hömberg, 1981; Haggarty & Pepin, 2002; Hopf, 1980; Johansson, 2006; Pepin & Haggarty, 2001; Remillard, 2005; Woodward & Elliott, 1990) whereas there is a dearth of research into the use of mathematics textbooks by students (Love & Pimm, 1996). This is striking, because as pointed out by Kang and Kilpatrick (1992), textbook authors regard the student as the main reader of the textbook.

In order to develop a better understanding of the impact that textbooks have on learning mathematics a qualitative investigation was carried out in two German secondary schools that focused on how students use their mathematics textbooks.

## METHOD AND RESEARCH DESIGN

The difficulty of obtaining data on students working from textbooks is one reason that Love and Pimm (1996) put forward in order to explain the dearth of research into student's use of texts. Therefore, developing an appropriate methodology to collect data on student's use of mathematics textbooks can be regarded as a major issue in this field.

First of all, the method of data-collection has to be in line with the situation of textbook use. In Germany, schools either provide mathematics textbooks to students for one year or students buy the books. Accordingly, students have access to their mathematics textbook at school and at home. From previous research there is evidence, that German teachers rely heavily on the textbook in the preparation of lessons and also during lessons. (Bromme & Hömberg, 1981; Hopf, 1980; Pepin & Haggarty, 2001).

The method to collect data on student's use of mathematics textbooks was developed within the framework of the activity theoretical model of textbook use. According to this model the use of mathematics textbooks is situated within an activity system constituted by the student, the teacher, the mathematics textbook, and mathematics itself. First of all, this implies that a method to investigate the use of mathematics textbooks by students has to take all four vertices of the tetrahedron-model into consideration.

In addition, three criteria were established for an appropriate methodology to collect data on student's use of mathematics textbooks:

1. The actual use of the mathematics textbook should be recorded in detail.
2. Biases caused by the researcher, by the situation or by social desirability should be minimized.
3. The use of the textbook should be recorded at any time and any place it is used.

Criterion 1 leads to the rejection of quantitative methods and of methods that are likely to reveal only verbalized uses of the textbook, e.g. interviews. Experimental settings and artificial situations are refused due to criterion 2. Approaches that are solely based on observation are discarded because of criterion 3.

The methodological framework that was developed according to the three criteria combines observation and a special type of questioning. First of all, the students were asked to highlight every part they used in the textbook. Additionally, they were asked to explain the reason why they used the part they highlighted in a small booklet by completing the sentence "I used the part I highlighted in the book, because …". By assigning more than one comment to a highlighted book section the reuse of book sections becomes apparent. This method of data-collection was developed in order to get the most precise information about what the students actually use and why they use it by keeping the situation of textbook use as natural as possible. Nevertheless, highlighting sections in a textbook is not the natural way to use the textbooks and therefore a bias on the data cannot be totally excluded.

Provided that the students take their task seriously, this method enables to collect data on the use of the textbooks whenever and wherever students use it and therefore meets criterion 3.

In addition, the lessons were observed and field notes were taken. On the one hand the overall structure of the lesson was recorded in the field notes using a table comprising three columns: time, activity/content and remarks. On the other hand all utterances concerning the textbook were transcribed literally. Furthermore, a focus was put on all utilizations of the textbook. Both, the use of the textbook by the students and by the teacher was taken into account. This is important for several reasons:

First of all, there is evidence from previous research that the teacher plays an integral part in mediating textbook use. Because of that, the teacher was included as a variable in the model of textbook use.

Secondly, the observation provides an insight into the way the teacher mediates textbook use in the classroom. It makes a difference if the students only use the textbook when they are told to by the teacher or if they use it of their own accord. This difference will become apparent through classroom observation.

Thirdly, the methodological triangulation provides a measure for the validity of the data. Collecting data on how the textbook has been used in the classroom makes it possible to compare the markings and comments of the students with the field notes. The degree of correspondence between these two sources relating to the use of the textbook in the classroom indicates how serious the students took their task.

While the method of highlighting and taking notes especially satisfies criterion 3 and at the same time aims at both, providing a precise record of the actual use of the textbook by students (criterion 1) as well as keeping biases low (criterion 2), the intention of the observation is threefold. On the one hand the idea is to lower biases that might be caused by the method of highlighting (criterion 2) and on the other the triangulation of two different data-sources provides a measure for the validity of the student's data.

In addition to the previously described methods interviews were conducted with selected students.

Data was collected for a period of three weeks in two $6^{th}$ grade and two $12^{th}$ grade classes in two German secondary schools. Within the German three partite school system, these schools are considered to be for high achieving students. All four classes were taught by different teachers.

The coding process followed the ideas of Grounded Theory by Strauss and Corbin (Strauss & Corbin, 1990). Categories were established in the process of analysing the data. Each highlighted section in the textbook was categorized according to the kind of block it belongs to (introductory tasks, exposition, worked example, kernels, exercises) (cf. Rezat, 2006b), the activity it was involved in, and finally whether the use of the section was mediated by the teacher or not.

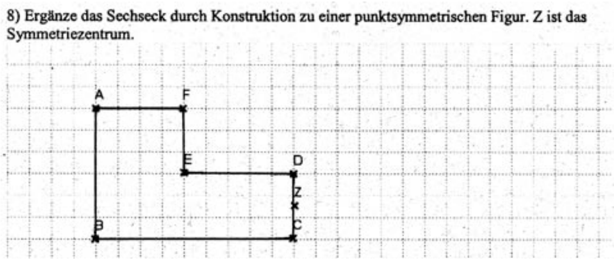
In order to understand the role of the mathematics textbook as an instrument within the activity system represented by the tetrahedron model Rabardel's (1995, 2002) theory of the instrument was used. As Monaghan (2007) points out, this theory has proven fruitful to provide insights into the use of new technologies as instruments for learning mathematics. According to Rabardel an instrument is a psychological entity that consists of an artefact component and a scheme component. In using the artefact with particular intentions the subject develops utilization schemes which are shaped by both, the artefact and the subject. Vergnaud (1998) suggests that schemes are characterized by two operational invariants: theorems-in-action and concepts-in-action. Since these two operational invariants are put forward in order to describe the representation of mathematical knowledge, it is not self-evident to apply them to knowledge related to the use of an artefact like the mathematics textbook. Therefore, it is suggested to generalize Vergnaud's notion of theorems-in-action and concepts-in-action to the notion of beliefs-in-action. As well as concepts-in-action beliefs are supposed to guide human behaviour by shaping what people perceive in any set of circumstances (Schoenfeld, 1998). Like theorems-in-action beliefs are propositions about the world that are thought to be true (Philipp, 2007). The appendix 'in-action' is supposed to underline that beliefs-in-action might be inferred from actions. They do not necessarily have to be expressed verbally. Because of its universality, the notion of beliefs-in-action offers an appropriate means to characterize operational invariants of utilization schemes linked to any artefact.

## RESULTS

A first and a major result of the study is, that students do not only use the mathematics textbook when they are told to by the teacher. But, they also use the textbook self-directed. The following analysis focuses on utilizations of the mathematics textbook that students perform in addition to teacher mediated textbook use.

Students incorporate their mathematics textbook as an instrument into four activities: solving tasks and problems, consolidation, acquiring mathematical knowledge, and activities associated with interest in mathematics. From the data it was possible to reconstruct several individual utilization schemes of the mathematics textbook related to these activities. Comparing the individual schemes of different students related to the same activity revealed that some of the schemes were analogous in terms of the underlying beliefs-in-action. These schemes were generalized to utilization scheme types (UST). USTs are general in the way, that they allow to classify individual utilization schemes of the textbook into USTs and thus make individual utilizations comparable. Nevertheless, different students might show different USTs. The USTs are not general in the way that they are common to all students.

Solving tasks and problems is associated with activities where students utilize their mathematics textbook in order to get assistance with solving tasks and problems. Three different USTs were found related to this activity. It was observed that students repeatedly utilize specific blocks from the textbook as an assistance to solve tasks and problems. Worked examples and boxes with kernels were instrumentalized in most of the cases. This scheme could be traced back to the belief-in-action that a specific block from the textbook is useful in order to solve tasks and problems. It was also observed that students choose sections from the textbook that show similarities to the task. For example, Oliver is working on the following task that is not from the textbook:




He looks for assistance in the textbook and reads a task in the textbook that is located next to an image, which is identical to the image in the task. From this behaviour it can be inferred that Oliver expects information concerning the image next to it. In his case, the information is not useful for solving the task, because it is a task itself.

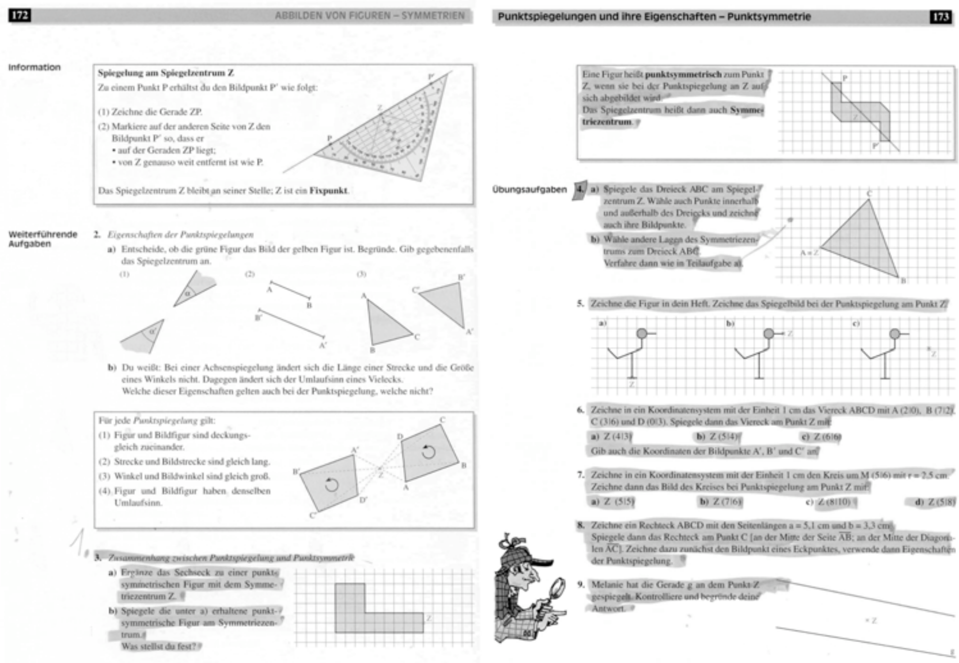



**Fig. 2: Passage Oliver used from the textbook "because he was looking for something" (Griesel *et al.*, 2003)**

In order to get assistance with solving tasks and problems it was also observed that students search an adequate heading in the book and start reading from there until they find useful information. From this behaviour it was inferred that these students expect useful information related to a subject at the beginning of a lesson in the textbook.

All three USTs reveal that students are looking for information in the book that can be directly applied to the task. The only difference is the way they are approaching the information. Hardly ever does it seem like students want to understand the mathematics first and then apply it to the task.

Consolidation is associated with all activities that students perform in order to improve their mathematical abilities related to subject matters that were already dealt with in the mathematics class. One UST of students using their mathematics textbook for consolidation is strongly related to teacher mediated exercises from the textbook. They either recapitulate tasks and exercises from the book that the teacher mediated or they pick tasks and exercises that are adjacent to teacher-mediated exercises. This was traced back to the belief-in-action that effective practising means to do tasks and exercises that are similar to teacher-mediated exercises. If students pick tasks that are adjacent to teacher mediated tasks this is also supported by the belief-in-action that adjacent tasks in the textbook are similar. The use of specific blocks for consolidation was also observed. One UST is that students only read the boxes with the kernels of several lessons in the textbook.

So far, consolidation seems to comprise learning rules, recapitulating teacher mediated tasks and solving tasks that are similar to teacher mediated tasks respectively. But, it was also observed that students either utilize special parts at the end of a unit that are designed especially for recalling and practising the main issues of the unit or they scan the section in the book relating to the actual topic in the mathematics class and read different parts of it in order to consolidate their understanding of the topic. Both UST are less dependent on teacher mediation and show more proficiency in the utilization of the textbook.

Whereas consolidation related to previously treated topics, acquisition of knowledge is associated with activities where students use parts of the book that have not been a matter in the mathematics class so far. The UST identified in this context is that students use parts from the proximate lesson in the textbook. This is supported by the belief-in-action that the chronological succession of topics in the mathematics class will follow the order of the textbook.

Students also used parts of their textbook because they thought they were interesting. These utilizations are associated with activities related to interest in mathematics. In this case the UST is connected to the use of images and other salient elements from the book. Students either only look at the images or they read passages that are next to images or other salient elements. Looking just at the pictures does not seem to be associated with learning mathematics though. This UST usually is observed in the context of other utilizations of the textbooks. It seems like this UST is not based on a belief-in-action, but that salient elements in the textbook catch the attention of the students while there utilizing it for another purpose.

## CONCLUSIONS

The activities the mathematics textbook is involved in do not only give an insight into student’s utilizations of mathematics textbooks, but they also give an idea of what learning mathematics is about for students. The USTs show that learning mathematics with the mathematics textbook comprises activities as solving tasks and problems, consolidating mathematical knowledge and skills, acquiring new contents.

The USTs show how the textbook is used as an instrument within these activities. Furthermore, these USTs reveal interesting insights into student's dispositions towards mathematics. Learning mathematics comprises mainly learning rules, applying rules and worked examples to tasks, and developing proficiency in tasks that are similar to teacher mediated tasks.

Consciousness about student's USTs could affect teacher's ways of implementing the mathematics textbook in the teaching process. Some USTs show that the use of mathematics textbooks by teachers in the classroom is an important reference for student's utilizations of the textbook. For example, the UST that is characterized by the utilization of tasks that are adjacent to teacher mediated tasks for consolidation is dependent on the mediation of tasks from the textbook by the teacher. Therefore, it is important that the teacher uses tasks from the textbook in order to support student's individual learning of mathematics. Another example is the anticipation of the next topic in the mathematics class by reading parts of the proximate lesson in the textbook. This UST shows that students belief that the course of the mathematics lessons will follow the order in the book. Accordingly, the textbook provides orientation for students, and it can therefore be considered important that teachers follow the succession of the topics in the book.

It was pointed out, that Valverde et al. (2002) argue that the structure of mathematics textbooks advances a distinct pedagogical model and is likely to have an impact on actual classroom instruction. From an ergonomical perspective it can be argued that the structure of the book also has an impact on the USTs of the students. This raises the question of how a textbook must be structured in order to promote desirable USTs.

Furthermore, this study provides evidence that Rabardels theory of the instrument is not only capable of conceptualizing human-computer-interaction, but is also applicable to non technological resources. The conceptualization of student-textbook-interaction on the basis of this theoretical framework provides interesting insights into different aspects of learning mathematics. The UST do not only provide a better understanding of student's utilizations of mathematics textbooks, but also reflect student's ways of learning mathematics. Furthermore, it can be inferred from student's USTs how the textbook is effectively used in the classroom by the teacher. Accordingly, a better understanding of student's utilizations of mathematics textbooks is a prerequisite for effective implementation of mathematics textbooks into teaching.

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# TEACHERS' BELIEFS ABOUT THE ADOPTION OF NEW TECHNOLOGIES IN THE MATHEMATICS CURRICULUM

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*The purpose of the present study was to examine elementary mathematics teachers' concerns in relation to the expected implementation of the new technology based mathematics curriculum in Cyprus. A questionnaire examining teachers' concerns towards this innovation was administered to seventy four elementary school teachers. Results provide evidence that the majority of teachers were positive towards the innovation. Results revealed the existence of four factors related to teachers' concerns and beliefs towards the innovation, namely the concerns about the nature of the curriculum, teachers' self-efficacy beliefs, concerns about the consequences on the organization of teaching, and concerns about the effectiveness of the curriculum.*

## INTRODUCTION AND THEORETICAL FRAMEWORK

Based on the premise that Information and Communication Technologies (ICT) can have a positive impact on mathematics teaching and students' learning outcomes, technology based activities have been implemented in mathematics curricula in a number of countries (Hennessy, Ruthven, & Brindley, 2005). This implementation is, however, not an easy yet straightforward task; a number of factors such as mathematics teachers' beliefs and concerns about the adoption of this innovation, facilities, in-service teachers' training, and available resources might influence the successful implementation of the innovation (Hennessy, et al., 2005).

Gibson (2001) argues that technology by itself will not and can not change schools. It is only when reflective and flexible educators integrate technology into effective learning environments, that the restructuring of the classroom practices will benefit all learners. The introduction and implementation of ICT in the teaching and learning of mathematics has not been successful in a number of cases in different countries (Hennessy, et al., 2005). As reported by the British Educational Communications and Technology Agency (2004), only few teachers succeed in integrating ICT into subject teaching in a fruitful and constructive way that can promote students' conceptual understandings and can stimulate higher-level thinking and reasoning. In most of the cases, teachers just use technology to do what they have always done, although in fact they often claim to have changed their teaching practice. Further, a number of teachers report that they do not feel comfortable with the integration of ICT in subject teaching, since their role was predetermined and designed by educational authorities and teachers feel that they face a lack of professional autonomy (Olson, 2000). Olson (2000) proposes that integrating new technologies challenges teachers and, thus, requires innovators to understand and be engaged “in conversations with teachers about their work culture, the technologies that sustain it and the implications of new approaches for those technologies” (p.6).

Among the factors that have been identified as crucial for the successful integration of ICT in the mathematics curricula are teachers’ concerns and beliefs about this change (Van den Berg et al., 2000). To this end, a number of studies focused their research efforts on examining teachers’ concerns towards the adoption of ICT in general (Gibson, 2001) or towards an innovation in education (Hall & Hord, 2001). According to Hord and colleagues (1998), concerns can be described as the feelings, thoughts, and reactions individuals develop in regard to an innovation that is relevant to their job (Hord, Rutherford, Huling-Austin & Hall, 1998). In this framework, innovation concerns refer to a state of mental arousal resulting from the need to cope with new conditions in one’s work environment (Hord et al., 1998). Furthermore it is argued that teachers are also important as representatives of their students’ needs. In this respect, the opinions and views of teachers can be considered to be reflective of opinions and views from two major stakeholder groups instead of one, and this further underlines the importance of studying teachers’ concerns before and during implementing a new innovation in education (Hossain, 2000).

A model that has been widely used for the evaluation of the innovations in education is the Concerns-Based Adoption Model (CBAM) (Hord, et. al., 1998). This model can be used to identify how, for example, teachers (who feel that they will be affected by the new technology based curriculum in mathematics) will react to the implementation of the innovation (Christou et al., 2004). The CBAM includes three tools that are used for collecting data related to teachers’ concerns and beliefs. These tools include: (a) the levels of use questionnaire, (b) the innovation configurations, and (c) the stages of concerns questionnaire. The stages of concerns questionnaire was adopted, modified and used in the present study to measure elementary school teachers concerns and beliefs about the innovation of introducing a technology based mathematics curriculum (Hall & Hord, 2001). The stages of concerns questionnaire includes items for measuring teachers’ concerns towards seven stages of concern, namely the Awareness, Informational, Personal, Management, Consequences, Collaboration, and Refocusing stages.

Briefly, in the awareness stage teachers have little knowledge of the innovation and have no interest in taking any action. In the informational stage teachers express concerns regarding the nature of the innovation and the requirements for its implementation. In the personal stage teachers focus on the impact the innovation will have on them, while in the management stage their concerns begin to concentrate on methods for managing the innovation. In the consequences and collaboration stages their concerns focus on student learning and on their collaboration with their colleagues. Finally on the refocusing stage teachers evaluate the innovation and make suggestions for improvements related to the innovation and its implementation (Hord et al., 1998).

## PURPOSE AND RESEARCH QUESTIONS

The purpose of the present study was to examine teachers' beliefs about an innovation that will soon take place in Cyprus, namely the adoption of a new mathematics curriculum. The new curriculum is expected to incorporate an inquire based approach and to integrate technological tools into the teaching and learning of mathematics. The study aimed at investigating how well prepared teachers feel about implementing the new curriculum and whether teachers are positive towards this innovation.

The research questions of the study were the following:

(a) What beliefs do teachers have regarding the adoption of a mathematics curriculum that integrates technology?

(b) Do teachers' beliefs differentiate in accordance to their teaching experience and their studies?

(c) Do teachers feel capable to implement the new curriculum and if not, what do they reported that they need to be appropriately prepared?

## METHODOLOGY

### Participants

The participants in this study were 74 teachers from nine elementary schools in Cyprus. Schools were randomly selected from the district of Nicosia. One hundred questionnaires were mailed to schools and 74 were returned to researchers. Teachers were grouped according to their teaching experience and their studies, in three categories and in two categories, respectively. The numbers of teachers in each group are presented in Table 1.

Table1. *Teachers involved in the study by years of teaching experience and level of studies*

| Studies | Teaching experience | | |
|---|---|---|---|
| | 1-5 | 6-15 | >15 |
| Postgraduate studies | 16 | 13 | 9 |
| Undergraduate studies | 6 | 16 | 14 |
| Total | 22 | 29 | 23 |

## Batteries

The questionnaire included 23 likert-scale items. Part of the items was adopted from previous stages of concerns questionnaires (e.g., Hall & Hord, 2001; Christou et al., 2004). Since these studies focused on teachers' adoption of innovations in general, the items were modified to serve the purposes of investigating teachers' concerns of the adoption of the innovation of using ICT in the teaching of mathematics. The 23 items were on a 7-point likert scale, from 1 (strongly disagree) to 7 (strongly agree); all responses were recorded so that higher numbers indicated greater agreement with the statement. The questionnaire also included two open-ended questions in which teachers were asked to report on: (a) what they need in order to feel confident and well prepared to implement the new technology-based mathematics curriculum, and (b) their beliefs and concerns in general about their new role in teaching mathematics after the implementation of the innovation.

The data were analyzed using the statistical package SPSS. An exploratory factor analysis and an multiple analysis of variance were conducted. Descriptive statistics were also used.

## RESULTS

The exploratory factor analysis resulted in four factors, including the 21 items of the teachers' questionnaire. The following four factors arose: (a) Concerns/Beliefs about the nature of the new mathematics curriculum, (b) Teachers' self-efficacy beliefs, (c) Concerns about the consequences on the organization of teaching, and (d) Concerns/Beliefs about the effectiveness of the new curriculum. The loadings of each statement in the four factors are presented in Table 2.

Furthermore, teachers that participated in the study appeared to have positive beliefs about the nature of the proposed new curriculum ($\bar{x}$=5,1). Particularly, the majority of teachers reported that the new curriculum will put emphasis on pupils' way of thinking and their reasoning skills, on problem solving and on the enhancement of students' conceptual understanding. The mean score of the 'Self-efficacy beliefs' factor ($\bar{x}$=4,1) might claim that teachers feel quite confident and well prepared to use the new curriculum. Although the mean score can be considered quite large, it is important to underline that the majority of teachers reported that there is a strong need for in-service teachers' training before the implementation of the innovation.

Furthermore, it seems that teachers' beliefs concerning the consequences on the organization of teaching are also rather positive. The mean score ($\bar{x}$=4,0) reveals that many teachers who participated in this study believe that after the implementation of the curriculum the stress of the teacher regarding the organization of teaching will be reduced and that this innovation will relieve the teacher from a great deal of

Table 2: *Factor analysis results*

| | Factors | | | |
|---|---|---|---|---|
| Statements | F1 | F2 | F3 | F4 |
| The adoption of the new curriculum will place sufficient emphasis on the development of pupils' thinking. | ,831 | | | |
| The use of the computer in mathematics develops pupils' mathematical thinking and reasoning skills. | ,744 | | | |
| The new curriculum that takes advantage of the computer in the teaching of mathematics promotes problem solving. | ,730 | | | |
| The use of computers promotes conceptual understanding in mathematics. | ,704 | | | |
| The new curriculum places emphasis on investigation. | ,618 | | | |
| The knowledge that students acquire through the use of computers is not superficial. | ,572 | | | |
| I do not feel confident about teaching mathematics with computers. | | ,808 | | |
| I do not face difficulties in teaching mathematics with computers. | | ,759 | | |
| The implementation of the new curriculum requires the use of methods that I am not familiar with. (recoded) | | ,723 | | |
| I do not need guidance to teach mathematics with the use of computers. (recoded) | | ,715 | | |
| I know how to use computers effectively in mathematics in the classes that I teach. | | ,541 | | |
| The computer based activities that will be included in the new curriculum will reduce teacher's preparation. | | | ,856 | |
| With the implementation of the new curriculum, teachers' stress about the organization of teaching will be reduced. | | | ,846 | |
| Pupils' homework will be reduced. | | | ,578 | |
| Teaching of mathematics with the use of computers will allow me to follow the progress of each pupil. | | | | ,775 |
| The adoption of the new curriculum is a useful innovation. | | | | ,613 |
| I believe that the adoption of the new curriculum will improve students' achievement. | | | | ,557 |
| The integration of computers in mathematics teaching will result in major changes in the teaching of mathematics. | | | | ,418 |

preparation. They also reported that they expect that pupils' homework will be reduced and that the integration of technology will improve the organization/management of the classroom.

Similarly, the mean score for the forth factor was also quite large ($\bar{x}$=5,3). Teachers appeared to be positive that the new curriculum will introduce major changes in the teaching of mathematics and that it will improve results. They also consider the mathematics curriculum that integrates technology as a useful innovation in primary education mathematics and as a means that will allow them to follow the progress of each pupil.

Table 3: *The four factor model mean scores*

| Factors | Mean | SD |
|---|---|---|
| F1: Beliefs about the nature of the new mathematics curriculum | 5,1 | 0,9 |
| F2: Teachers' self-efficacy beliefs | 4,1 | 1,2 |
| F3: Concerns/Beliefs about the consequences on the organization of teaching | 4,0 | 1,2 |
| F4: Concerns/Beliefs about the effectiveness of the new curriculum | 5,3 | 0,9 |

In order to investigate whether teachers' beliefs in four factors differentiate in accordance to the years of teaching experience and level of studies, a multivariate analysis of variance (MANOVA) was conducted, with the statements of teachers in four factors as dependent variables and years of teaching experience and studies as independent ones. The results of the multivariate analysis showed that there were significant differences between teachers beliefs across the years of teaching experience (Pillai's $\underline{F}_{(2,64)} = 2{,}211$, $p<0{,}05$). More concretely, the results indicated that there were statistically significant differences between the three groups only in the first factor, 'Beliefs about the nature of the new mathematics curriculum' (F=5,667, p<0.05). It was found that the significant differences related to this factor appeared only between inexperienced teachers (years of teaching experience: 1-5) and experienced teachers (6-15) (p<0.05) and between inexperienced teachers and teachers with more than 16 years of experience who probably possess administrative places (16+) (p<0.05). As the years of experience increase the beliefs about the nature of the curriculum get higher. In the other three factors there were no significant differences between the three groups of teachers. The results of the multivariate analysis indicate that there were no significant differences between teachers' beliefs in the four factors in relation to their level of studies (Pillai's $\underline{F}_{(1,68)} = 0{,}661$, $p > 0{,}05$).

Of importance are also teachers' responses to a number of individual items of the questionnaire. The item with the highest mean score ($\bar{x}$=6,1) was the one that referred to the need for training courses. Specifically, the majority of teachers (60 teachers), agreed strongly (chose 7) or very much (chose 6), and only two teachers disagreed that training courses are necessary for the successful implementation of the technology based curriculum in mathematics. The items with the lowest mean score were the 'The knowledge that students acquire through the use of computers is superficial' ($\bar{x}$=2,7) and 'The adoption of the new curriculum for the integration of computers in the teaching of mathematics is a useless innovation' ($\bar{x}$=2,1). Teachers' responses to these items also showed that teachers consider the integration of technology in the teaching of mathematics as a useful innovation that will enforce learning, something that is in line with the high mean score ($\bar{x}$=5,2) which refers to the improvement of students' achievement after the implementation of the new curriculum. Their positive beliefs and willingness to integrate technology into teaching appears also from the high mean score ($\bar{x}$=5,2) of the item 'I would like to teach mathematics lessons using computers'.

Table 4: *Mean scores for questionnaire items*

| Items | Mean | SD |
|---|---|---|
| The knowledge that students acquire through the use of computers is superficial. | 2,7 | 1,2 |
| Training courses for the integration of computers in the teaching of mathematics are necessary for teachers. | 6,1 | 1,3 |
| I would like to observe and participate in technology based mathematics lessons taught by more experienced teachers. | 5,2 | 1,4 |
| I believe that the adoption of the new mathematics curriculum that integrates technology into teaching will improve students' achievement. | 5,2 | 1,1 |
| The adoption of the new curriculum for the integration of computers in the teaching of mathematics is a useless innovation. | 2,1 | 1,7 |

Teachers' need for training courses came also up from their answers in the first open-ended question. Fifty-five teachers answered this question and some of the answers consisted of a combination of different ideas. For this reason some of the teachers are included in the percentage of more than one category of answers. Forty-six teachers (83,6%) stated that they need 'Training courses for the integration of computers in the teaching of mathematics'. The second category that was pointed out by ten teachers (18%) was 'lesson plans and worksheets'. Also, ten teachers (18%) expressed that it is essential to become familiar with the software that will be used, before implementing the innovation, and eight teachers revealed their wish to attend courses that will be held by more experienced teachers. Six teachers stated that they need much guidance, three that they considered the co-operation with colleagues important and three that they need the appropriate infrastructure. The last four answers that were reported only by one teacher each, are the following: (a) training courses for the use of computers, (b) more hours devoted to the teaching of mathematics, (c) one coordinator in each school, and (d) adaptation of the books according to the purpose of the curriculum that integrates technology into teaching.

Regarding the second open-ended question, five categories of answers were identified from the 53 answers that were gathered. The majority of teachers (46 teachers-88.7%) stated that they feel that their role would be more like a facilitator during the learning process. Three teachers reported that their role will remain the same and two just mentioned that they will have a decisive role. Lastly, one teacher pointed out that his role will change; he will need to first develop more positive attitudes and knowledge towards the innovation and then transfer them to his students.

## DISCUSSION

The purpose of this study was to examine teachers' beliefs and concerns regarding the expected innovation of integrating the new technology-based curriculum in mathematics at the elementary schools in Cyprus.

The questionnaire was used to provide a description of teachers' concerns and beliefs about the integration of the new technology-based mathematics curriculum, which shows that the great majority of teachers welcome the expected change in mathematics curriculum after the introduction of ICT and they seem to have positive beliefs in general and positive self-efficacy beliefs for teaching mathematics using ICT (Chamblee & Slough, 2002).

The present study showed that in general teachers welcome the introduction of ICT in mathematics education. According to the teachers that participated in the study, however, the majority underlined the importance of in-service and pre-service training on implementing ICT in the mathematics teaching. This is crucial for the successful implementation of the innovation as, according to teachers' answers, teacher role will be changed, new classroom dynamics will appear, and student learning in mathematics will be improved. The results of the study also revealed that teachers believe that this innovation is important and can positively change the way mathematics are taught and student learning can be improved, but this is not an easy task; careful planning is needed and resources like software and lesson plans will help teachers in their new different role (Luehmann, 2002).

The results revealed that differences of beliefs across different groups of teachers in terms of teaching experience existed only for the first factor, namely the 'Beliefs about the nature of the new mathematics curriculum'. Specifically, teachers' beliefs about the nature of the curriculum differed between the inexperienced teachers and teachers with more than five years of experience. As teachers' experience increases, teachers feel that the new curriculum can place sufficient emphasis on the development of pupils' thinking and that the appropriate use of computers can assist students in further developing their mathematical thinking and reasoning skills. These teachers also reported that the integration of ICT in the teaching and learning of mathematics can assist teachers in teaching problem solving skills, an essential and core part of the mathematics curriculum.

The themes emerging from the analysis of teachers' beliefs and concerns about the expected integration of ICT in the mathematics curricula converge to offer a grounded model for the innovation. This model underlines the importance of teachers' training and knowledge on the various aspects that are related with the integration of ICT in mathematics. Furthermore, teachers appeared to be very positive about the innovation and that they expect that the role of ICT will assist the teaching and learning of mathematics. This result is very prominent and encouraging, considering that the majority of these teachers were not well informed about the innovation from educational authorities, but were rather themselves positive and they believe that the role of technology can positively influence the role and impact of school mathematics on student learning and problem solving abilities.

In the future, a longitudinal study could be conducted to examine the development of teachers' beliefs and concerns over the first steps of the innovation. Since teachers appear to have quite strong and positive beliefs and they expressed their willingness to adopt and use the new curriculum, a study on the development of their concerns and beliefs over a long period could provide more useful information for practitioners and researchers. To better examine the research questions that guided the present study, it is recommended that a comparative study could be conducted to examine the differences between pre-service and in-service teachers' concerns and beliefs towards the new technology based mathematics curriculum, and to identify how the more technology experiences pre-service teachers have might influence their concerns and beliefs about the innovation.

Teachers' beliefs and concerns are an important issue for the successful integration of the ICT in the mathematics curricula, and this study examined this issue in relation to elementary school teachers in Cyprus. It is expected that such explorations can suggest good practices for educational authorities and teacher educators. Finally, the findings discussed would provide avenue and references for future studies.

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# SYSTEMIC INNOVATIONS OF MATHEMATICS EDUCATION WITH DYNAMIC WORKSHEETS AS CATALYSTS

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*With reference to theories of cybernetics the paper proposes a general theoretical framework for initiatives aiming at systemic innovations of educational systems. It shows that it is essential to initiate incremental-evolutionary changes on the meta-level of beliefs and attitudes of the agents involved. For the theoretical foundation of concrete activities in mathematics education the didactic concept of learning environments is developed on the basis of constructivist notions of teaching and learning. Such learning environments may integrate dynamic mathematics for educational processes. So technology and especially dynamic worksheets can be considered as means and catalysts for improvements of mathematics education on system level.*

***Keywords:*** *systemic innovation, learning environment, dynamic mathematics*

## INNOVATIONS IN COMPLEX SYSTEMS

There are many efforts to innovate educational systems – on regional, national and international levels – aiming at changes of teaching and learning. For understanding the structure of such initiatives a short glance at theories of cybernetics is useful.

### Innovations

The OECD defines an *innovation* as the implementation of a new or significantly improved product, process or method (OECD, Eurostat, 2005, p. 46). Thus an innovation requires both an *invention* and the *implementation* of the new idea.

In the educational system we are in a situation where lots of concepts, methods and tools have been developed for substantial improvements of teaching and learning. Three examples:

(1) There is a wide range of current pedagogical theories that emphasize self-organised, individual and cooperative inquiry-based learning.

(2) There exists a huge amount of material for teaching and learning in a constructivist manner – available e.g. in electronic data bases or by print media.

(3) A large variety of software and other tools for the integration of ICT in educational processes has been developed.

But for real innovations these promising theories and products have to be implemented in the educational system. Here implementation means a good deal more than diffusion or dissemination of material (papers, guidelines, software tools etc.). And implementation should reach the real agents in the school system, i.e. the teachers and students, their thinking and their working. Let's remember the three examples from above:

(1) Teachers should teach according to current pedagogical concepts.

(2) The proposed new task culture should become standard in everyday lessons.

(3) ICT should be used as a common tool for exploring mathematics.

So for substantial innovations we do not need further material. We need changes in teachers' and students' notions of educational processes, in their attitudes towards mathematics and in their beliefs concerning teaching and learning at school. Hence the crucial question is: How can substantial innovations in the complex system of mathematics education be initiated and maintained successfully?

## Complex Systems

In theories of cybernetics a system is called "complex", if it can potentially be in so many states that nobody can cognitively grasp all possible states of the system and all possible transitions between the states (Malik, 1992; Vester, 1999). Examples are the biosphere, a national park, the economic system, mathematics education in Europe and even mathematics education at a concrete school.

Complex systems usually are networks of multiply connected components. One cannot change a component without influencing the character of the whole system. Furthermore real complex systems are in permanent exchange with their environment.

Maybe this characterization of complex systems seems a bit fuzzy. But, nevertheless, it is of considerable meaning. Let us regard the opposite: If a system is not complex, someone can overview all possible states of the system and all transitions between the states. So this person should be able to steer the system as an omnipotent monarch leading it to "good" states. In contrast, complex systems do not allow this way of steering.

## Steering of Complex Systems

The fundamental problem of mankind dealing with complex systems is how to manage the complexity, how to steer complex systems successfully and how to find ways to sound states.

With reference to theories of cybernetics two dimensions of steering complex systems can be distinguished (Malik, 1992). The first one concerns the manner, the second one the target level of steering activities (see figure 1).

The method of *analytic-constructive* steering needs a controlling and governing authority that defines objectives for the system and determines ways for reaching the aims. Hierarchical-authoritarian systems are founded on this principle. However, fundamental problems are caused just by the complexity of the system. In complex systems no one has the chance to grasp all possible states of the system cognitively.

So the analytic-constructive approach postulates the availability of information about the system that cannot be reached in reality.

In contrast *incremental-evolutionary* steering is based on the assumption that changes in complex systems result from natural growing and developing processes. The steering activities try to influence these systemic processes. They accept the fact that complex systems cannot be steered entirely in all details and they aim at incremental changes in promising directions. The focus on little steps is essential, since revolutionary changes can have unpredictable consequences which may endanger the soundness or even the existence of the whole system.

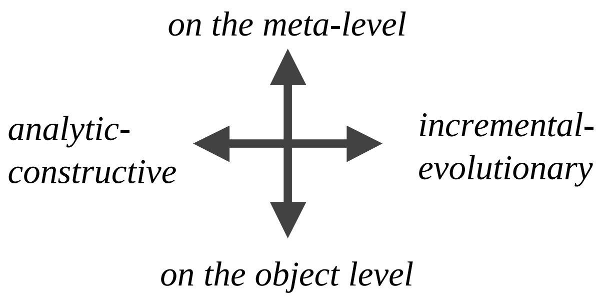


**Figure 1: Steering of complex systems**

The second dimension distinguishes between the object and the meta-level. The *object level* consists of all concrete objects of the system. In the school system such objects are e.g. teachers, students, books, computers, buildings etc. Changes on the object level take place if new books are bought or if a new computer lab is fitted out. Of course such changes are superficial without reaching the substantial structures of the system.

The *meta-level* comprehends e.g. organizational structures, social relationships, notions of the functions of the system etc. In the school system e.g. notions of the nature of the different subjects and beliefs concerning teaching and learning (e.g. Pehkonen, Törner 1996, Leder, Pehkonen, Törner 2002) are included.

## Innovations in Complex Systems

How can substantial innovations in the complex system "mathematics education" be initiated successfully? The theory of cybernetics gives useful hints: Attempts of analytic-constructive steering will fail in the long term, since they ignore the complexity immanent in the system. Changes on the object level do not necessarily cause structural changes of the system. According to the theory of cybernetics it is much more promising to initiate *incremental-evolutionary changes on the meta-level* (see figure 2). They are in accord with the complexity of the system and do not endanger its existence. Nevertheless, they can cause substantial changes within the system by having effects on the meta-level, especially when they work cumulatively.

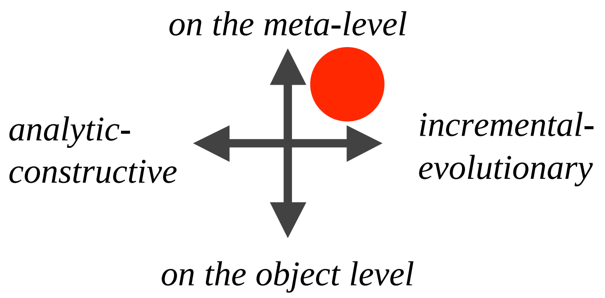


**Figure 2: Innovations in complex systems**

## LEARNING ENVIRONMENTS WITH DYNAMIC WORKSHEETS

### Aspects of Learning

Learning is a very complex phenomenon. Initiatives aiming at the development of mathematics education have to take in account the nature of learning. Let us have a very short glance at some fundamental aspects of learning (e.g. Reinmann-Rothmeier & Mandl, 1998; Haberlandt, 1997) which form a background for the latter:

- Learning is a *constructive* process. Knowledge and understanding cannot be simply transported from teachers to students. Cognitive psychology describes learning as a process of construction and modification of cognitive structures. From the view of neurobiology learning is the construction of neuronal networks. Connections between neurons develop and change.
- Learning is an *individual* process. Learning takes place inside the head of each learner. He creates his own knowledge and understanding by interpreting his personal perceptions on the basis of his individual prior knowledge and prior understanding.
- Learning is an *active* process. Cognitive activity means working with the content in mind, viewing it from different perspectives and relating it to the existing network of knowledge.
- Learning is a *self-organized* process. The learner is at least partially responsible for the organization of his individual learning processes. The degree of responsibility may vary in the phases of planning, realizing or reflecting learning processes.
- Learning is a *situative* process. It is influenced by the learning situation. A meaningful context or a pleasant atmosphere can foster learning processes, fear can hamper them.
- Learning is a *social* process. On the one hand the socio-cultural environment has great impact on educational processes. On the other hand learning in school is based on interpersonal cooperation and communication between students and teachers.

## Concept of Learning Environments

Considering the aspects of learning noted above the following model seem adequate for teaching and learning processes in school:

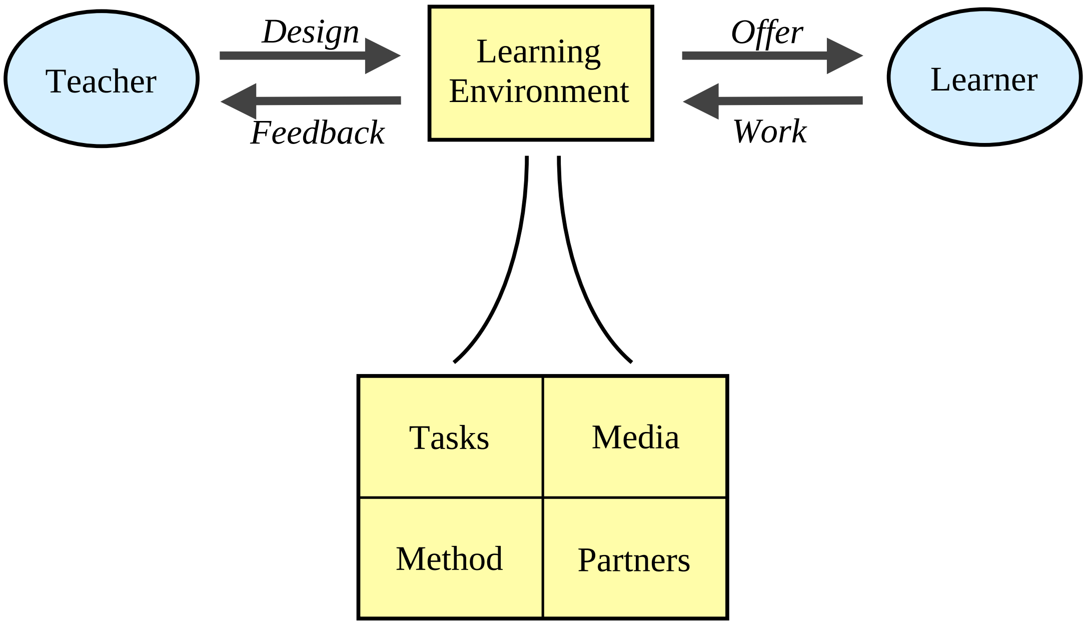


**Figure 3: Working with learning environments, four components of learning environments**

The *learning environment* is the essential link between the teacher and the learner. This notion includes the *tasks* for the learner's activities, the arrangement of *media* and the *method* for teaching and learning as well as the social situation with the teacher and other learners as *partners* for learning. It belongs to the teacher's field of responsibility to design the learning environment. So he offers a basis for the learner's work. This allows the teacher to get feedback about the learner as well as about the learning environment. This model is based on and extends the didactical concepts of "substantial learning environments" by Wittmann (1995, 2001) or "strong learning environments" by Dubs (1995).

The aspects of learning noted above imply fundamental consequences for the design of learning environments: Tasks should be problem-based with necessary openness for learning by discovery. They should offer meaningful contexts and view situations from multiple perspectives. The teaching methods should make the learners work actively, individually and self-organized. But not less important are the learners' communication and cooperation as well as discussions and presentations of ideas and results. Media can have several supporting functions for these processes.

Before we will discuss the relevance of this model for innovations in educational systems, we look at a specific kind of media which may carry general ideas to practice in school and serve as a catalyst for processes of change.

## Dynamic Worksheets

The notion "dynamic mathematics" is currently used for software for dynamic geometry with an integrated computer algebra system, so that geometry, algebra and calculus are connected. When designing learning environments with dynamic mathematics, one faces the necessity to relate dynamic constructions to texts, e.g. for explanations or exercises for the students. For this purpose software for dynamic mathematics – like e.g. GEONExT or Geogebra – can be embedded in HTML-files. So dynamic constructions can be varied on the screen and are combined by the internet browser with texts, pictures, links and other web-elements. This kind of new media for mathematics education is called "dynamic worksheets" (Baptist, 2004; Ehmann, Miller, 2006).

With respect to the model in figure 3 dynamic worksheets are strongly related to all four components of learning environments: Of course they serve as teaching and learning *media*. Since they include text, they may provide *tasks* and instructions for the students. So implicitly they influence the teaching *method* and the cooperation between the learning *partners* (see next section). Hence, when designing learning environments with dynamic worksheets one should carefully take account of all these components and their impact on students' learning.

Figure 4 shows an example: The students are given a mathematical situation leading to an optimization problem. The text is combined with a dynamic construction which helps to understand the context. The rectangular can be moved while fitting exactly in the area between the parabola and the x-axis. The tasks help to structure the lesson according to the methodical concept described in the following section.

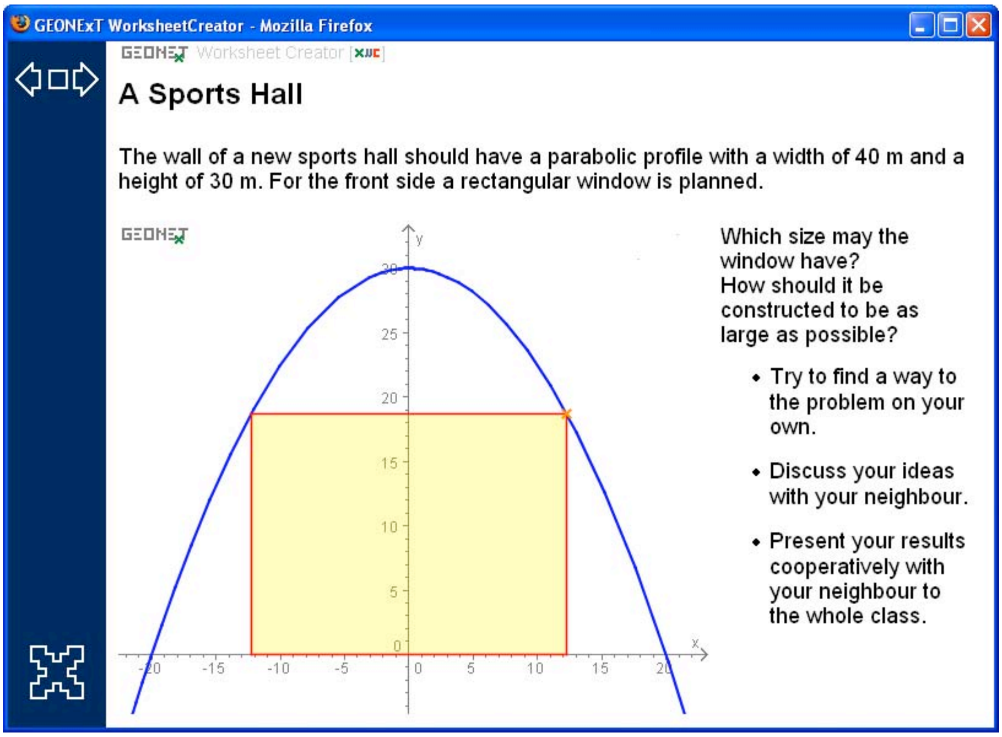


**Figure 4: Screenshot of a dynamic worksheet**

## A Methodical Concept for Learning Environments with Dynamic Worksheets

The use of dynamic worksheets does not automatically improve mathematics education. It is crucial how these media are integrated in teaching and learning processes. If we want to initiate substantial changes on the meta-level of attitudes and beliefs concerning mathematics and mathematics education we have to organize lessons in a way that students work actively, individually, self-organized and cooperatively. They should experience that mathematics is a field for explorations and discoveries. And they should present and discuss their ideas and results cooperatively. Considering the aspects of learning noted above the following four phases structuring lessons with dynamic worksheets methodically are very natural:

| |
|---|
| **1. Individual working:** Learning is an individual, active and self-organized process. So at first the students work on their own. They are faced with the necessity to explore the content, to activate their prior knowledge, to develop ideas and to make discoveries. Learning environments with dynamic worksheets offer a framework for such activities and may support them. |
| **2. Cooperation with partners:** Learning is a social process. It is very natural that the students discuss their ideas with partners in small groups and that they work on the problems cooperatively. This communication helps to order thoughts and to get further ideas. Meanwhile the teacher may remain in the background or turn his attention to individuals. |
| **3. Presentation of ideas:** After having worked individually and in groups the students present their ideas and discuss them in the plenum. The different contributions reveal multiple aspects of the topic and help to view it from varying perspectives. Moreover the students train debating and presentation techniques. |
| **4. Summary of results:** Finally the students' results are summarized and possibly extended by the teacher. It is his task to introduce mathematical conventions and to consider curricular regulations. Since the students have already discovered the new content on their own paths, they can more likely integrate the teacher's explanations into their individual cognitive structures. |

**Table 1: Methodical concept**

This methodical concept combines individual learning with working in small groups as well as in the plenum of the class in a very natural way. It is in close relationship to the methodical concepts “Think – Pair – Share” by Lyman (1981) or “I – You – We” by Gallin and Ruf (1998).

## Learning by Writing: The Study Journal

The call for papers for working group 7 at CERME 6 emphasizes that technology in school should be considered within a wider range of resources for teaching and learning. Students should draw on ICT in combination with more traditional tools.

Accordingly, dynamic worksheets are only one element of rich learning environments. Especially pencil and paper do not lose relevance when student work with the computer. Noting down thoughts helps to order and arrange thoughts. Writing helps to develop understanding for new subject matters. Hence, when using dynamic worksheets students should regularly be asked to draw figures in their exercise book and to write down observations, conjectures, argumentations and personal statements. The exercise book gets the character of a personal "study journal" that accompanies students on their individual learning paths (Gallin, Ruf, 1998).

When designing dynamic worksheets for students' self-responsible learning, one should be aware of the risk that students play with the media as with a computer game quite superficially and do not get to the deeper mathematical content. The regular request of working in the exercise book decelerates the process of clicking through the learning environment. So the students are forced to take their time which is indispensable for individual learning.

Finally, the notes in the study journal ensure that ideas and results are still available when the computer is switched off. They are a basis for further presentations, discussions and summaries in the plenum of class (Baptist, 2004).

## INCREMENTAL-EVOLUTIONARY SYSTEMIC INNOVATIONS WITH DYNAMIC WORKSHEETS AS PARTS OF LEARNING ENVIRONMENTS

In their plenary talks at CERME 5 Ruthven and Artigue observed that current results of activities integrating ICT in school are rather disappointing on system level.

> "Advocacy for new technology is part of a wider reform pattern which has had limited success in changing well established structures of schooling." (Ruthven, 2007) "From the very beginning, digital technologies have been considered as a tool for educational change […]. Unfortunately, the results are far from being those expected" (Artigue, 2007).

For substantial innovations in the educational system there is no lack of general ideas, pedagogical concepts or didactic tools – as discussed above. But there is a wide gap between theoretical knowledge and practice in school. So we have to develop strategies to bridge this gap.

### Conclusion: A Pattern for Innovation Projects

Combining the theory of cybernetics and the concept of learning environments using dynamic worksheets we get a pragmatic, but also theory-based way of initiating innovations in school. Activities are most promising, if they focus on incremental-evolutionary changes on the meta-level of beliefs and attitudes of all agents involved. Learning environments with dynamic worksheets may serve as framework for learning processes of teachers *and* students. How can this be done concretely?

As a conclusion from all reflections above we sketch and propose a pattern for innovation projects for mathematics education. (It is realized e.g. by the current project “InnoMathEd – Innovations in Mathematics Education on European Level”, see http://innomathed.eu).

(1) The key persons for innovations in school are the teachers. Their beliefs, motivation and abilities are crucial for everyday teaching and learning in school. So regional networks of schools are established which form frameworks for teachers’ cooperative learning, exchange of experience and professional development.

(2) Universities are innovation centres for teacher education. They lead the school networks and provide regular and systematic in-service teacher education offers. This teaching and learning is designed according to the aspects of learning and the concept of learning environments described above. So the teachers get acquainted with these theories and concepts by making personal experiences in learning environments designed for them.

(3) Participating schools concentrate on one or a few areas of innovation, e.g. autonomous learning with dynamic worksheets, promoting student cooperation with dynamic worksheets or fostering key competences with dynamic worksheets. It is not promising to aim at total changes of mathematics education – because of the complexity of the system. However, such bounded fields of activity allow teachers to begin with substantial changes without the risk of losing their professional competence in class.

(4) The teachers get acquainted with general ideas and theories of teaching and learning as well as with techniques for constructing learning environments. To bridge the gap between theory and practice the teachers’ project activities are strongly related to their regular work at school. They develop learning environments for their students, they use, test and evaluate them in their classes and finally optimize them on the basis of all experiences. In this process they get guidance and coaching by the University leading the network.

(5) All learning environments which are tested, evaluated and optimized are collected in a data base and made available for public use.

(6) Teachers are given possibilities to exchange experiences with colleagues and to participate in teacher education offers on national and international level. Thus they understand that problems and necessities for development have systemic character and concern the fundaments of mathematics education far beyond their own professional sphere. Moreover, they get ideas for innovation activities from a large community.

(7) Finally, further networks of teachers and schools are essential means for dissemination processes in the long term. Experienced teachers coach colleagues from schools starting with innovation activities.

This approach may be called “theory based and material driven”. On the basis of the theory of cybernetics and the theories of learning the teachers involved make incremental-evolutionary steps on the meta-level of beliefs and attitudes by designing and working with concrete learning environments for their classes.

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# A DIDACTIC ENGINEERING FOR TEACHERS EDUCATION COURSES IN MATHEMATICS USING ICT

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*A first part of our research led us to define a theoretical framework to analyse teachers' education courses and to make hypotheses to explain the lack of efficiency of teachers training (Emprin, 2008). This paper presents the continuation of this work. We use the methodology of didactic engineering, adapted to teachers' education, and a theoretical framework previously built to test our hypothesis. In a first part of this paper we will describe our theoretical framework and hypothesis about teachers training. In a second part we will develop the didactic engineering and its results.*

## TEACHERS EDUCATION COURSES ANALYSIS

The general question guiding this work is the difficulty for mathematics teachers to use ICT in their classrooms. Our choice is to focus on a particular factor explaining this difficulty: teachers' professional education; without denying the existence of other factors such as material problems, resources available etc. Several studies in France or wider as Empirica study of European Commission, TIMSS & PIRL of the Boston College and BECTA's reports indicate this explanatory factor. French political choices since 1970[i] show that a quantitative effort was made, our research thus relates to a qualitative problem of teachers' training.

### A theoretical framework

First we chose to use a framework designed for the analysis of teaching practices and to specify it with teacher educators' practices: the two-fold approach. This framework, defined by Robert (1999), does not take into account specifics of the use of technology. This leads us to use, jointly with the two-fold approach, a framework making it possible to take into account this dimension as described in (Emprin, 2008). The instrumental approach developed by Rabardel (1995) appears to be relevant. This approach, which was already developed in the didactic of Mathematics ,for example in Trouche (2005), leads us to analyse instrumental geneses.

One difficulty is that teacher educators' practices can not be reduced to a teaching activity. A teachers' educator, in France, was most of the time a secondary school teacher, in many instances they keep on teaching to pupils. For this reason, like Abboud Blanchard (1994) specifies, the teacher trainer's previous practices as a teacher intervene in his practices as a teachers' educator.

We borrow the definitions of "activity" and "practices" from Robert & Rogalski (2002) which we must specify on various levels met during a teachers' education course:

This definition of "activity" is nearly similar to Rabardel's notion of "productive activity" (Rabardel, 2005, p. 20). It contains actions but also statements, attitudes and unobservable aspects which influence actions.

The definition of "practices" we use is a reconstitution of the five components described in the two-fold approach. Robert & al. (2007) give the description we have translated here:

> "We developed, taking into account the complexity of the practices, analyses capable of giving an account of what can be observed in class, which results from teacher's homework and the unfolding, and factors which are external to the classroom but which weigh on practices, including those in the classroom, and eventually contribute to the teachers' choices before *and* during the lesson. Indeed, practices in classroom are forced, beyond goals in terms of pupils' acquisitions, by determinants related to teachers' trade: institutional, social… Let us quote programs, timetables, schools, colleagues, class and its composition. Moreover, the practices have a personal anchoring which refers to the teacher as a singular individual, in terms of knowledge, picturing, experiments, trade's idea and also conditions its choices. Our analyses start from class session in which we distinguish components, institutional, social, personal, meditative (related to the unfolding in the classroom and improvisations), cognitive (related to the prepared contents and expected unfolding), closely dependent for a given teacher, and having to be recomposed: it is necessary for us to think of the components together, and to estimate the compensation, balance, the compromises to include/understand and start to explain what is concerned. »

To build our framework of analysis we need to dissociate the various levels of activities and practice but also to see their interactions. Figure 1 makes it possible to describe these various levels.

The first level of activity is the one of the pupil. We note it A0 level. The pupil has a task to realize, and acts accordingly. He uses an instrument belonging to ICT. This level can thus be analyzed with the didactic of mathematics and the instrumental approach. The observation of the process of instrumentation/ instrumentalisation informs us about the instrumental geneses of the pupil and the instruments built.

The second level of activity is the one of the teacher whom we note at level A1. The tasks of the teacher consist of managing and organizing the activity of the pupils. He also organizes the instrumental geneses of the pupil. The two-fold approach enables us to analyze a first level of practices which we note P1 level.

The other two levels of activity are those which exist in teachers' education courses. The activities of the trainees (who are thus teachers) during the training course, are noted as A2. They are organized by those of the teachers' educator noted as A3. The Two-fold approach and the instrumental approach give us access to a second level of practices noted as P2, those of the teachers' educators.

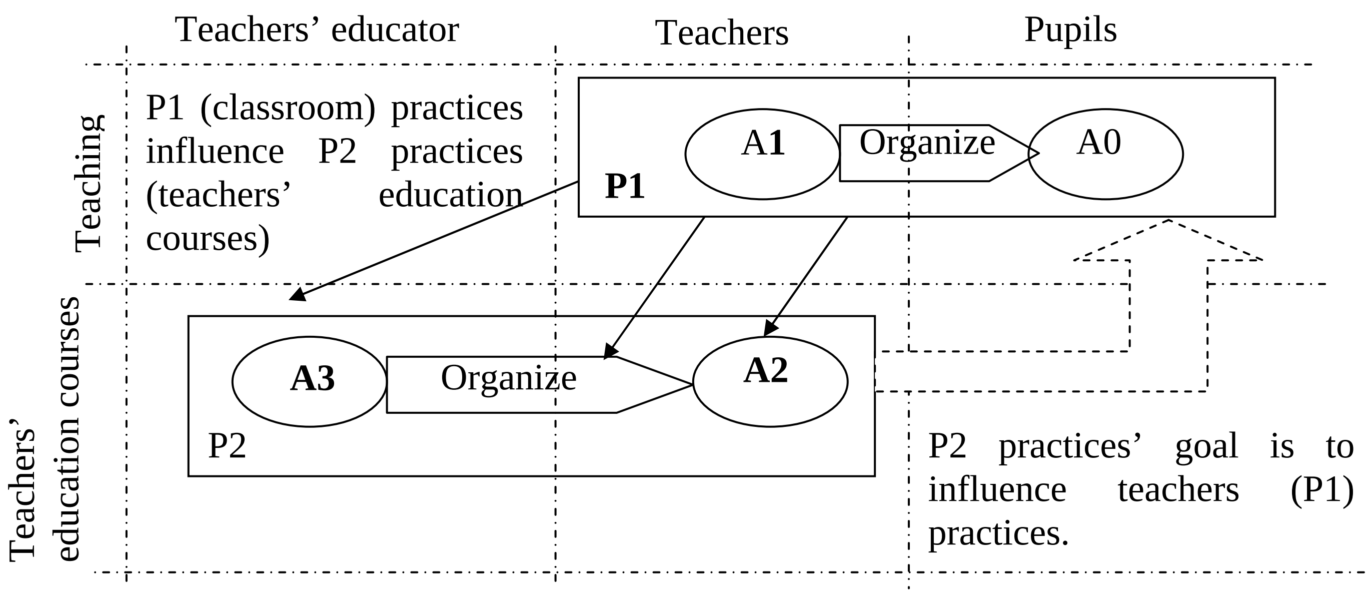


**Figure 1: overlap of the four levels of activity and two levels of practices**

## Use of the theoretical framework

Our work is centred on the analysis of the teachers educators practice, thus we neither directly analyze the practices of P1 level, nor activities of A1 and A2 levels, nevertheless they appear during teachers' education courses as explanatory factors.

Teachers practices (P1) can be seen during teachers education courses in three main ways, through a video: practices are shown, when the teacher's educator narrates a classroom session: practices are narrated through what the teacher's educator asks the trainees to do: the practices are inherent. This last way is linked with a strategy of teachers training which is called homology. This strategy described by Houdement & Kuzniak (1996) shortly consists in doing with teachers (A3→A2) what they will be expected to do when they are back in their classrooms (A1→A0).

The two-fold approach is designed to analyze the real practices; it requires being able to observe the courses and to ask the teacher about the context in which he works. To analyse P1 practises which appear during teacher education session we use two-fold and instrumental approaches as a reading grid to see which part of practices teachers' educator focuses on.

## Hypothesis resulting from the analysis of teachers education courses

We implemented this framework of analysis on a corpus of three teachers' education courses, of fourteen interviews of teachers' educators. The results obtained help us to build the first part of our hypothesis about the lack of effectiveness of teachers trainings.

First we notice that working time is mainly dedicated to a work on computers (more than 50% of the time). When trainees are not in front of computers, the time is devoted to explanations (44 to 62%) and descriptions (35 to 52%) given by the teachers' educator, there is thus very few analysis or debate. In term of two-fold approach social, personal and institutional components of the practices are almost not approached. The mediative component of practices appears in the analysis of video or the narration of courses, but is not questioned. The cognitive dimension remains rather marginal. Our analysis also shows a possible drift of homology strategy: it is likely to introduce confusion between the various instrumental geneses, of pupil and teacher.

## BUILDING OF A DIDACTIC ENGINEERING FOR TEACHERS EDUCATION COURSES

### Hypothesis

We identify two complementary ideas explaining the lack of efficiency pointed previously. The first one results from the work of Ruthven & Hennessy (2002) and Lagrange & Dedeoglu (in press). Theses authors show a gap between teachers' needs and ICT potentialities presented by teachers' trainers. We also observe an absence. In France the "reflexive practitioner" of Schön (1994) and the "analysis of practices" developed by Altet (1994) or Perrenoud (2003) are two important models for teachers' education is thus remarkable that no allusion is made there in teachers' education courses to mathematics with ICT. That leads us to consider the introduction of a reflexive component in ordinary practices' analysis and to formulate four hypotheses taking into account the first part of our work:

- The analysis of real practices would make it possible to initiate a reflexive attitude in teachers (making it possible for the teacher to change their teaching practices)
- Leading trainees to analyze a real professional problem enables them to confront their representations, mobilize their knowledge (resulting from experience) and come to a consensus based on reasoning.
- An analysis of the professional practices taking into account several dimensions of practices (in terms of two-fold approach) and based on the analysis of the relationship between teaching practices and activity of the pupil, makes it possible for the trainees to mobilize their knowledge (resulting from experience and their theoretical knowledge).
- It is necessary to contribute, during teachers' training courses, to the professional instrumental geneses of teachers and to analyze the lessons in terms of instrumental needs and potential instrumental genesis of pupils.

In order to check these hypotheses we use the methodology of didactic engineering that we specify to teachers' education. This methodology defined in Artigue (2002) is based on the verifying of a priori hypothesis. Thus we need to define observable criteria linked to our hypothesis. We decline our four hypotheses in seven criteria:

- The trainees' ability to identify and define a problem.
- The formulation and the use, by the trainee, of knowledge coming from experience associated with theoretical knowledge to analyze the practice
- The implication of trainees' personal practices and of his own experience in the analysis.
- The trainees reach a consensus based on knowledge coming from experience and theory.
- During the session teachers' educator does not give any answers, any explanations. The knowledge is built by trainees and not given by the teachers' educator. We call that an a-didactical lesson referring to theory of didactical situations (Brousseau, 1998)
- The fact that the analysis makes it possible to take into account several dimensions of the practices
- It must then be possible to identify any trace of instrumental genesis making it possible for teachers to consider instrumented actions but also results on pupils' activity.

Our methodology leads us to conceive a scenario for teachers' education whose implementation will be analyzed by means of the theoretical framework built in the first part.

**Scenario and analyzes**

The scenario is inspired from Pouyanne & Robert (2004). It is based on the analysis of teaching practices by means of a video. Four periods are defined: an a priori analysis of the lesson (which has been recorded) where hypothesis about the effects of the teaching practices on pupils' activity are put forward; an analysis of the video and a comparison with the hypothesis; a search for alternatives based on the question "What would you do if you had to do such a lesson?"; and finally a debate around problems emerging during the first three period.

We implemented this scenario twice, in each one, videos show pupils using interactive geometry software (IGS): In the first training course eight grade pupils had to prove that perpendicular bisectors in a triangle converge. The second video show sixth grade pupils solving a problem (which is detailed below). We develop now this second session of teacher education.

In each teacher's education session, the scenario lasts about three hours. This part of the session has been recorded, transcribed and analysed. The analysis takes into account who is speaking, the type of speech (description, explanations, analysis) and its content.

**An example of session**

The lesson recorded for this teachers training is what we call in French "an open problem" referring to Arsac & Mante (2007). This type of problem is called "open" insofar as no specific solution is expected: what matters is pupils' search.

Figure 2 gives the statement of the problem. Pupils are asked to say which one of [EG] or [AC] is longer.

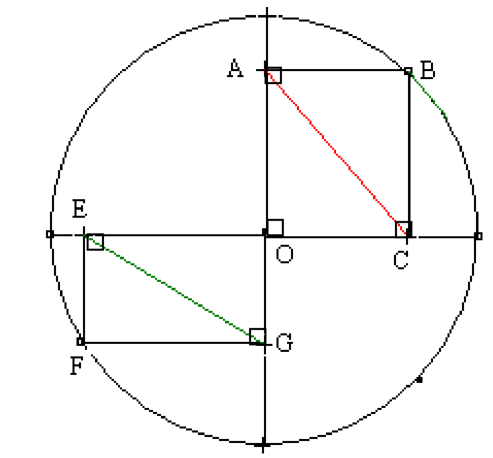

Figure 2 : an « open problem »

During the first part of the work with trainees, the a priori analysis, we had to let them use the IGS. It is a first change in the scenario. It seems to be very difficult for teachers to analyse the problem without having a working time on the computer. This time is not a time of homology even if the trainees do what is expected from pupils.

During the analysis the trainees have a transcription of the discussion with the teacher who is in the video. She specifies what is at stake in this lesson: she wants pupils to develop their critical thinking and to show them not to trust their perception. The trainees identify three stakes: the drawing with the software, the location of the rectangles in the whole geometrical drawing and the property of the diagonals of a rectangle. They specify that they think that the situation cannot be done by the pupils. They propose teaching aids to make the situation feasible. They propose to reveal the radius of the circle, the other two diagonals of the rectangle. Another solution considered is to cut out the problem or to make a preliminary recall of the useful properties. In this stage there is thus an implication of the trainees who adapt the lesson since they try, to some extent, to make it feasible in their classrooms. This implication can be seen in the following example.

> Trainee: that seems difficult to me in 6th grade also because I think that they will see that the diagonals have the same length but that they will not be able to justify it.

The viewing of the film reveals initially the need for dissociating the task of construction in the software from the remainder. Indeed the pupils encounter real difficulties to build the geometrical figure. The trainees realize that pupils need to build uses of the software. It is a part of the instrumental genesis. On the video, once geometrical construction has been carried out, the pupils try to conjecture. The trainees realize that pupils have the necessary knowledge to solve the problem but that they are not able to mobilize it.

In the film, the pooling of pupils' works take place at the end of the lesson, whereas the pupils are still in front of the computers. It is quickly carried out by the teacher. The conclusions of the trainees are that it is necessary to take more time, to move the pupils away from the computers and to let them talk. There is thus a clear evolution in the trainees' mind. In the first part of the analysis they have doubts about the ability of the pupils to solve the problem and in the last part they say it is necessary to devote more time to the pooling of what pupils have found.

The search for alternatives contains the essential components of the analysis. The trainees reaffirm that it is necessary to dissociate the drawing on IGS from conjecture. Some even propose to remove the drawings' work. This work also allows a long discussion about the place of this problem in pupils' training. Before pupils know the property of the diagonals of the rectangle, the problems is centred on research whereas afterwards it acts more as a consolidation of knowledge. This also leads to discuss the place of observations in the geometrical trainings. A trainee proposes to use this problem to introduce the property of equality of the diagonals which disturbs another trainee who believes that observing properties is conflicting with the idea of mathematics. This trainee finally realizes that she does not have tools to give proof of the property to pupils of this level while at the same time the property is in the official programme. During these discussions the teachers' educator scarcely intervenes. Trainees are personally involved in the analysis:

> Trainee: I do think that giving the instructions when the computers are "on" is always rather difficult; it is better to give instructions before turning the computers on.

In this example we can see that this trainee formulates a teaching knowledge, rather simple but which can now be used consciously by other trainees.

Most of the indicators can be observed for "many" trainees. Nevertheless, during a three hours session, a limited number of trainees can speak and consequently the internal evaluation of our methodology is only partial.

Finally, we noticed two changes in our scenario: the time of appropriation of the software was introduced during the analysis of the lesson and the final time of debates was removed. For the first change, the lack of acquaintance of the trainees with the artefact prevents them from making a real analysis. The second change is due to time devoted to debates during the session. The entire subject likely to be alluded to seems to have been discussed before. A last noticeable point is that trainees do not know other pieces of software which could be used in this lesson. The teachers' educator had to show different pieces of software as in the teachers' education courses we analysed in the first part of our work.

## Conclusion on the didactic engineering of formation and continuation

The main results of this didactic engineering are linked with our criteria: it seems to be necessary to let the trainees use and try the artefact. It helps them to analyse the lesson but it also seems to match with trainee expectations. It is possible to take into account several dimensions of the practices but in a smaller number than expected. The analysis of the video helps trainees to make cognitive and mediative components more explicit but the other components are more difficult to reach. The scenario built allows a reflexive analysis of the practices. Experience and theoretical knowledge is used to analyze the problem of introduction of the ICT. Instrumental geneses of the teachers and the pupils are really dissociated. The trainees considered what is necessary to pupil to use ICT in this lesson. They also found different options and they analysed the changes involved by these choices in term of learning or in lesson unfolding. For example ask pupils to draw the figure in the software helps them to use a proper vocabulary (because the software makes it compulsory) but it takes a long time and leads the teacher to reduce the time of conjecture.

Practices, in our didactic engineering, are shown in a video but it is possible to work on other types of practices such as real practices or simulated practices. Simulated practices make it possible for a whole group of trainees to work on the same teaching experience. The construction of such a simulator is the object of a work we initiated in 2007.

To conclude, the fact that teachers use experience knowledge to analyze practices with ICT makes it possible for us to consider the teachers' education course with ICT as a lever for teachers' education generally speaking. It seems to be easier to influence the way of teaching mathematics by influencing the way of teaching mathematics with ICT.

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i This very year the IPT plan began (Informatique Pour Tous) which could be translated in "data computing for everyone". For example in 1985, it allowed the purchase of computers for 33.000 schools and represented 5.500.000

hours of training for teachers. For more information see Archambault, J.-P. (2005), 1985, vingt ans après... Une histoire de l'introduction des TIC dans le système éducatif français. Médialog (54).

# GEOMETERS' SKETCHPAD SOFTWARE FOR NON-THESIS GRADUATE STUDENTS: A CASE STUDY ıN TURKEY

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*The purpose of this paper is to determine mathematics teachers' views about Geometers' Sketchpads Software (GSP) and to analyze the effects of training sessions on prospective teachers' ability to integrate instructional technology in the teaching of geometry. For that purpose, two graduate student teachers were selected; they investigated GSP activities. They followed training sessions about using GSP. The data come from interviews with them and GSP activities improved by them. The results of this study indicate that their awareness level about GSP was increased.*

***Keywords:*** *Teacher Education, Secondary Mathematics Education, Non-thesis Graduate Program, Integrating Technology, Geometers' Sketchpad Software.*

## INTRODUCTION

Today's use of technology as a learning tool supplies the students with gaining the mathematics skills in their lessons. According to Newman (2000), the use of technology in learning arouses curiosity and thinking, and challenges students' intellectual abilities. Kerrigan (2002) state that using mathematics software promote students' higher order thinking skills, develop and maintain their computational skills. For this reason, teacher training is crucial in order to use technology in mathematics education.

Computers could be used in school for teaching geometry, and since then a lot of work has been done that discusses many aspects of using Dynamic Geometry Software (DGS) in education (Kortenkamp, 1999). In this study, it was concerned with DGS activities developed by non-thesis graduate student teachers. Non thesis graduate program is in Turkey was opened for the purpose of educating future teachers. The secondary school (grade 9-11) mathematics teacher training program made up of two different programs. The Five-Year Integrated Programs (3.5+1.5) in Faculty of Education and Non Thesis Graduate Program (4+1.5) in Faculty of Science. Last 1.5 year part is the same for both 3.5+1.5 and 4+1.5 programs. Of these programs 3.5 and 4 year are spent on taking the mathematics courses and remainder years on pedagogical courses. After graduation, they can be secondary school mathematics teacher. This program is described in more detail in YOK (1998). The aim of this study was to investigate whether their views changed after the education process and to determine the outcomes about student teachers' proficiency.

## THEORETICAL FRAMEWORK

In geometry, teachers are expected to provide “well-designed activities, appropriate tools, and teachers’ support, students can make and explore conjectures about geometry and can learn to reason carefully about geometric ideas from the earliest years of schooling” (NCTM, 2000). Mathematics teachers can help students compose their learning by using geometry sketching software. Geometer’s Sketchpad allows younger students to develop the concrete foundation to progress into more advanced levels of study (Key Curriculum Press, 2001).

Reys et al. (2006) point out young learners of mathematics need to

- experience hands-on (concrete) use of manipulative for geometry such as geoboards, pattern blocks and tangrams,
- connect the hands-on to visuals or semi concrete models such as drawings or use the sketching software on a computer,
- comprehend the abstract understanding of the concepts by seeing and operating with the picture or symbol of the mathematical concept (cited in Furner and Marinas, 2007).

GSP is an excellent tool for students to understand the properties of geometric shapes and to model for them mentally manipulating objects. GSP can also provide students to visualize the solid in their mind. In literature, McClintock, Jiang and July (2002) found GSP provides opportunities to have a distinct positive effect on students' learning of three dimensional geometry. In another study, Yu (2004) stated that the students’ concurrent construction of figurative, operative and relational prototypes was facilitated by dynamic geometric environment. That’s why, the knowledge about which DGS and DGS activities how prepared should be given the student teachers.

## METHOD

### Participants

Case study was used in this paper. This research was conducted during the spring term of 2007–2008 academic years in spring term. The study was conducted with two secondary school preservice teachers attending the 4+1.5 Integrated Secondary Mathematics Teacher Education Program at Dokuz Eylul University in Turkey. Of the ten students in this program there were two volunteers. In this process, they took the courses about mathematics content knowledge, pedagogical content knowledge and general pedagogical knowledge. All participants had basic computational skills but none of them knew how to use DGS.

### Data Collection

The data were collected from interviews and the activities which are prepared by the student teachers. The interviews were semi-structured in nature. In the beginning of the research, the opinions of the participants towards GSP software are taken with semi-constructed interview form. Each interview took approximately 15-20 minutes and recorded with a tape. Then the participants attended a six-hour GSP training sessions which is given by the researchers. After the program, it was demanded that the participants developed the GSP activities. Finally, the participants` opinions towards GSP software are taken again.

**The Geometer's Sketchpad Training Sessions**

The training sessions allowed the instructor to prepare the non-thesis graduate student teachers to enter their future mathematics classrooms not only knowledgeable about the capabilities of instructional technology, but also experienced enough to appropriately integrate their selected software. The GSP training sessions' content is given Table 1.

| | Training Sessions | Topics | Duration |
|---|---|---|---|
| *DAY 1* | Introductory *(Guided & Discussed)* | • major concepts of mathematics education<br>• the aim of the involved Software<br>• introduction to dynamic geometry environment with GSP<br>• introduction to tools and menus of the Software | 1 hour |
| | Constructing Geometrical Concepts *(Guided & Discussed)* | • to construct basic concepts of geometry<br>• to transform the rotation, reflection, and dilation of the figures<br>• to construct regular and non-regular polygons, and its interiors<br>• to measure in geometry (length, distance, perimeter, area, circumference, arc angle, arc length, radius, etc.)<br>• to graph various functions and its derivative | 1 hour |
| *DAY 2* | Animation and Presentation (*Guided & Discussed)* | • to use action and hide/show buttons<br>• to tabulate the data<br>• to prepare presentations | 2 hours |
| | Activity Planning *(Guided & Individual)* | • to plan activities and practice it | 2 hours |

**Table 1: Training Sessions**

DAY 1 included two sessions. Each session lasts an hour.

*Introductory Session:* The introductory session contained the major concepts of mathematics education, introduction to dynamic geometry environment with GSP and the aim of the involved Software.

In the beginning of the session, the participants discussed the major concepts - conceptual development, problem solving, modelling verbal problems, creative thinking, analytical thinking etc.- in order to determine their readiness with researcher. Then, they argued the aim of the involved Software. Afterwards, the participants introduced Dynamic Geometry Environment, the menus, sub-menus and tools of the GSP Software. When the participants get information about tool box, text palette, file menu, edit menu, display menu, construct menu etc., the researcher advanced next session.

*Constructing Geometrical Concepts:* In this session, the participants find out how to construct the basic concepts of geometry; such as ray, line, segment, paralel line, perpendicular line, angle bisector, median of triangle, altitude of triangle, arc etc.

When the participants learned how to use the menus, sub-menus and tools, the researcher showed them some operations. The participants learned about constructing regular and non-regular polygons, and its interiors. After that, they learned to change the color and width of the lines and figures.

Then, they transformed the rotation, reflection, and dilation of the figures. Subsequently, they measured length, distance, perimeter, area, circumference, arc angle, arc length, radius, etc. with using GSP.

When they reached the graph menu, they defined coordinate system, chose grid form and they draw some graphs with GSP, such as sinus, cosinus, tangent, etc. Afterwards, they graphed various functions and its derivatives. During this session, the participants discussed the functions of GSP each other if it was necessary or it was forgotten.

DAY 2 comprised two sessions. Each session is made up of two hours.

*Animation and Presentation:* In this session, the participants found out text palette on advanced level. Next they learned motion controller, how to paste picture and then passed animation and hide/show buttons. They learned how to utilize animations and change it's speed. Then they learned to trace points, segments, rays and lines. Afterwards they focused on tabulate the data on tables in order to arrange them regularly.

After they learned animation and presentation clues, they started to organize page set-up and document options in order to prepare excellent presentations.

*Activity Planning:* This session includes all of the applications learned. The researchers wanted the participants to prepare activities. And they also wanted to apply all the operations learned in their activity. In the preparation period, if the participants needed to be supported, the researchers could be guiding them.

## Data Analyses

In the interview, four open-ended questions were asked to the participants and the interview guide was used in this stage. During the interview, the questions like "What are the GSP aims in mathematics learning environment?" "Which students' skills are able to improve by GSP activities?", "What do you take into account while the GSP activities are composed?" and "How can you assess the students with the GSP activities?" were answered by the students.

The evaluating criteria were determined in order to assess the activities improved by the student teachers. These criteria were adapted from Roblyer (2003).

| | |
|---|---|
| 1. | Connection to mathematics standards. |
| 2. | Appropriate approach to mathematics topics with respect to grade, ability. |
| 3. | Presence of conceptual development, problem solving/higher order thinking skills. |
| 4. | Use of practical applications and interdisciplinary connections. |
| 5. | Suitability of activities (interesting, motivating, clear, etc.) |

**Table 2: Evaluation Criteria adapted from Roblyer (2003)**

## RESULT

In this section, the analysis of data obtained from two preservice teachers' view transcripts and activities which they prepared are presented.

### Handan's Case

Handan is working as an assistant teacher in private teaching institution for a year. During the pre-interview, four questions were asked her. She made explanations as follows:

Researcher : What are the GSP aims in mathematics learning environment?
Handan : It supplies the students with learning and visualizing in math lessons and preparing animations.
Researcher : Which students' skills are able to improve by GSP activities?
Handan : The students' spatial thinking skills are improved.
Researcher : What do you take into account while the GSP activities are composed?
Handan : It should be appropriate the students' cognitive level.
Researcher : How can you assess the students with the GSP activities?
Handan : I don't know because of lacking knowledge about GSP.

As can be seen in her statements, although she mentioned that she did not know GSP, she could be able to estimate its aims, skills to be improved and rules taken into account when the activities had done.

After training sessions, the researcher wanted her to prepare GSP activities whatever topics she wished. She chose the congruence as a subject of geometry instruction. Her activity is given Figure 1.

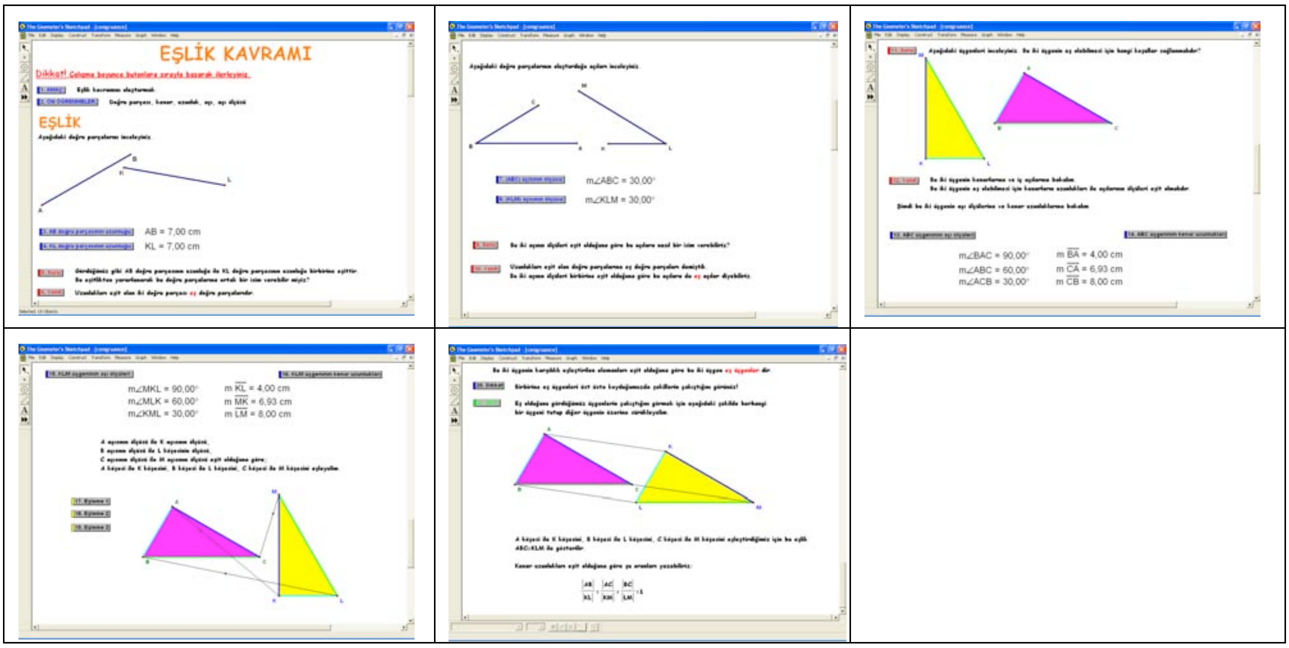

**Figure 1: Handan's Activity**

The content of her activity was about congruence. She decided to plan her activity for constructing the concept of congruence. As regards to the activity, the student knows the aim of the subject (step 1) and the concepts related to the subject (step 2). Handan gave directions to the students in her activity, in general. Therefore the student follows the instructions and carries on step by step. Afterwards, she gave two segments as AB and KL. She demonstrated the length of AB and KL segments (step 3-4). In the next step of the activity, she wanted students to compare the length of AB segment with KL segment. She asked whether the students call a common name to these segments (step 5) and explained it simply (step 6). Subsequently, she gave two angles and its measurements (step 7-8). She told the angles have the same measurement (step 9) and asked what the common name of the angles is (step 10). Later she constructed two triangles (ABC and KLM) and asked the students in what conditions they are congruent (step 11). Later on she showed the conditions of the congruence (step 12) and measurements of the triangles (step 13-14-15-16). In following steps, she paired each corners of the triangles and animated them (step 17-18-19). Finally, she drew the students' attention for the coincidence of triangles and demonstrated this (step 20-21).

When her activity arranged was assessed via the so-called evaluation criteria in Table 2, it was seen that the activity was connected to mathematics standards organized by Ministry of National Education (MNE) in Turkey, suited approach to mathematics topics -to explain congruence of triangle- with respect to 10$^{th}$ grade but it was too simple and like 8$^{th}$ grade level. It was provided conceptual development, also clear but not engaged the students in real life situations and interdisciplinary connections. It is useful for constructing the concept of congruence but not provide satisfactory knowledge. It wasn't prepared for improving the students' problem solving skills also. Handan utilized the mathematical language adequately. In respect of technicality, the activity is good. Each step's button is made as hide/show button. The 17th and 19th steps' button have the same function, so one of them is needless. The activity hasn't got any other technical problem.

Afterwards she had done activity; the post-interview was carried out with her and it was given her comments as follows:

| | |
|---|---|
| Researcher | : What are the GSP aims in mathematics learning environment? |
| Handan | : It provides the students learn geometrical concepts…their problem solving skills are improved and the concepts are visualized. |
| Researcher | : Which students' skills are able to improve by GSP activities? |
| Handan | : The students' spatial thinking…. and problem solving skills are improved. |
| Researcher | : What do you take into account while the GSP activities are composed? |
| Handan | : It should be interesting…. appropriate for the students' cognitive level and the students' opinions can be taken while the activities are prepared. |
| Researcher | : How can you assess the students with the GSP activities? |
| Handan | : The students can be able to do the applications involved in GSP and these are evaluated. |

Considering her statements, it is seen that her views changed after training sessions and her activity. She has primarily information about GSP and she awakes of what taking into account while the GSP activities are composed.

**Mualla's Case**

Mualla is also working as an assistant teacher in private teaching institution for a year. In time of the pre-interview, she gave responses as follows:

| | |
|---|---|
| Researcher | : What are the GSP aims in mathematics learning environment? |
| Mualla | : …It constitutes long lasting learning in math lessons and provides the teachers and the student drawing figures, preparing animations. |
| Researcher | : Which students' skills are able to improve by GSP activities? |
| Mualla | : GSP improves the students' spatial thinking skills. |
| Researcher | : What do you take into account while the GSP activities are composed? |
| Mualla | : It should be interesting… |
| Researcher | : How can you assess the students with the GSP activities? |
| Mualla | : I don't know… |

In the analysis of this interview, she determined which skills improved and what she pays attention during the GSP activities are composed. Besides it is seen that Mualla's responses are similar to the Handan's statements.

After training sessions, the researcher wanted her to prepare GSP activities whatever topics she wished. She chose the similarity as a subject of geometry instruction. The activity involved is given Figure 2.

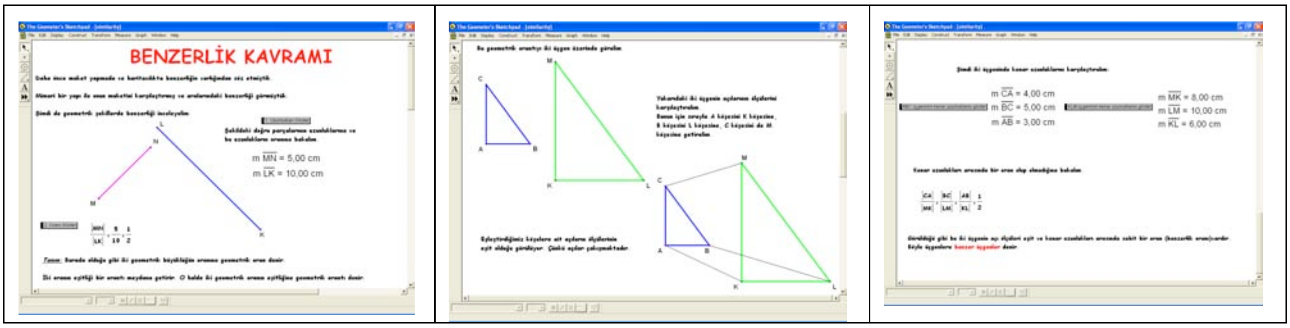


**Figure 2: Mualla's Activity**

Mualla's activity deals with similarity of triangles. She tried to carry out her activity for constructing the concept of similarity. According to her activity, she acknowledged that the students have little knowledge about the subject. Mualla generally gave directions to the students in her activity, as Handan did. However, her activity didn't similar to in terms of following the instructions step by step. In the beginning of the activity, she mentioned few real-life examples to the students about similarity and then she passed the similarity between geometrical concepts. She gave two segments, like Handan, and she compared the length of them under the first button. The second button shows the students the ratio of the lengths of the segments.  After that, the definition -geometrical ratio and geometrical proportion- was given, and demonstrated. Then, she compared the measures of each angle of the triangles and mentioned the coincidence of each angle. Afterwards, she showed and compared the length of sides of the triangle and stated whether the sides of both triangles have a ratio or not. Lastly, she defined a stable ratio, as the ratio of similarity.

When her activity organized was assessed by means of the evaluation criteria in Table 2, it was seen that the activity was overlapped mathematics standards organized by MNE in Turkey, partly suited approach to mathematics topics -to explain similarity of triangle- with respect to 10$^{th}$ grade. It was provided conceptual development, but not connected to the students in real life situations and interdisciplinary connections. Her activity was clear and understandable but it was also towards 8th grade and too simple. It wasn't also provides sufficient knowledge. It wasn't prepared for improving the students' problem solving skills also. Mualla used the mathematical language few adequately. In respect of technicality, the activity is not bad. Each step's button was made as hide/show button, as Handan did. It didn't include enough animation and demonstration. Finally it was said that, the activity hasn't got any technical problem.

After she had done activity; her comments during the post-interview was given as follows:

| | |
|---|---|
| Researcher | : What are the GSP aims in mathematics learning environment? |
| Mualla | : It provides the students learn geometrical concepts and problem solving, proof geometrical theorems. In addition to, it can be long lasting learning. |

Researcher : Which students' skills are able to improve by GSP activities?

Mualla : The students' spatial thinking was improved.

Researcher : What do you take into account while the GSP activities are composed?

Mualla : It should be appropriate the students' cognitive level and the mathematics standards

Researcher : How can you assess the students with the GSP activities?

Mualla : It can be ask some question in GSP aiming at determining whether they learned the geometric concepts. We expect that the students reveal the relationships between geometric concepts.

As her statements, she increases information about GSP. It follows from her responses that her point of view enlarged after training sessions. She encouraged and determined carefully what she does with GSP in mathematics learning environment after she prepared activities herself.

## DıSCUSSıON AND CONCLUSION

In this study, the data indicated that Dynamic Geometry Software (DGS) is important in geometry education. Generally speaking, Handan and Mualla learned some properties of GSP. At the end of the study, they realized how they can use GSP to prepare the activities. Handan gave detailed directives in her activity. She expected that the students to mention the concept of congruence; but this concept was given by her at the beginning of the study. In the other case, Mualla set out the similarity proportion when she prepared her activity. Both of them did not mention the kinds of congruence and similarity. They perhaps fostered the finding of these kinds by the students. As Key Curriculum Press (2001) mentioned, teachers can use GSP to create worksheets, exams, and reports by exporting GSP figures and measurements to spreadsheets, word processors, other drawing programs, and the Web. These results indicate that DGS is important in teacher education and DGS training must be present in non-thesis graduate education.

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# LEADING TEACHERS TO PERCEIVE AND USE TECHNOLOGIES AS RESOURCES FOR THE CONSTRUCTION OF MATHEMATICAL MEANINGS

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*This paper presents the early results of an on-going research project on the use of technology in the mathematics teaching and learning processes. A first aim of this project is to understand how deeply math teachers do perceive the opportunities technologies can bring about for change in pedagogical practice, in order to effectively use them for the students' construction of mathematical meanings. Secondly, the research aims at verify if teachers realise that, in order to successfully deal with perturbation introduced by technologies, they have to keep themselves continuously up-to-date and to acquire not only a specific knowledge about powerful tools, but also a new didactical and professional knowledge emerging from the deep changes in teaching, learning and epistemological phenomena.*

## INTRODUCTION

Due to the continuous spread of technology in the latest years, challenges and expectations in the everyday life, and in education in particular, have dramatically changed. Within this context of rapid technological change the world wide education system is challenged with providing increased educational opportunities. The use of Information and Communication Technology (ICT) in the classroom, however, seems to be, in the majority of cases, still based on a traditional transfer model characterised by a teacher-centred approach (see for example: Midoro, 2005).

But, according to Hoyles *et al.* (2006; p.301):

> «…a learning situation had an economy, that is a specific organization of the many different components intervening in the classroom, and technology brings changes and specificities in this economy. For instance, technological tools have a deep impact on the "didactical contract"…».

That is, the technology-rich classroom is a complex reality that necessitates observation and intervention from a wide range of perspectives and bringing technology in teaching and learning adds complexity to an already complex process (Lagrange et al. 2003).

Moreover, as underlined by Mously et al. (2003; p.427),

> «…technological advances bring about opportunities for change in pedagogical practice, but do not by themselves change essential aspects of teaching and learning ».

As research underlines (Bottino, 2000), indeed, innovative learning environments can result from the integration among educational and cognitive theories, technological opportunities, and teaching and learning needs. However, it is extremely important for teachers to confront themselves with the necessity to understand how the potential offered by technology can help in the overcoming of the everyday didactical practice complex problems.

I believe that for technologies to be effectively used in classroom activities teachers need, not only to “accept” the presence of technologies in their teaching practice but also to see technologies as learning resources and not as ends in themselves. Moreover, learning activities involving technologies should be properly designed to build on and further develop mathematical concepts. Hence, an “adequate” preparation is essential for teachers to cope with technology-rich classrooms, so that using computers not merely consists on a matter of becoming familiar with a software.

This paper presents the early results of an on-going research project on the use of technology in the mathematics teaching and learning processes, investigating mathematics teachers’ perceptions of ICT and of their usefulness in promoting a meaningful learning.

A first aim of this project is to understand how deeply math teachers, both pre-service and in-service, do perceive the opportunities technologies can bring about for change in pedagogical practice in order to effectively use them for the students’ construction of mathematical meanings.

Secondly, the research aims at verify, whether or not, teachers realise that, in order to successfully deal with perturbation introduced by technologies, they have to keep themselves continuously up-to-date and to acquire not only a specific knowledge about powerful tools, but also a new resulting didactical and professional knowledge emerging from the deep changes in teaching, learning and epistemological phenomena.

## THEORETICAL FRAMEWORK AND RELATED LITERATURE

Many researchers in the latest years are answering the challenge to provide educational opportunities by studying teaching and learning mathematics with tools (Lagrange et al., 2003).

Results of both empirical and theoretical studies have also led to the elaboration of the idea of “mathematics laboratory” as reported, for example, in an official Italian document prepared by the UMI (Union of Italian Mathematicians) committee for mathematics education (CIIM):

> «A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects » (UMI-CIIM MIUR, 2004; p.32).

In this sense, a laboratory environment can be seen as a Renaissance workshop, in which the apprentices learned practicing and communicating with each other. In particular in the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students).

According to this approach, and as in Fasano and Casella (2001), I believe that technological tools can assume a crucial role in supporting teaching and learning processes if they allow teachers to create suitable learning environments with the aim to promote the construction of meanings of mathematical objects. Moreover, in agreement with this point of view, I consider important to highlight that, again quoting the UMI-CIIM document (p.32):

> «The meaning cannot be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities.»

Furthermore, as claimed by Laborde (2002; p.285),

> «…whereas the expression integration of technology is used extensively in recommendations, curricula and reports of experimental teaching, the characterisation of this integration is left unelaborated.»

In particular, she underlines the idea that the introduction of technology in the complex teaching system produces a perturbation and, hence, for teacher to ensure a new equilibrium he/she needs to make adequate, non trivial choices. Integrating technology into teaching takes time for teachers because it takes time for them, first of all to understand that, and how, learning might occur in a technology-rich situations and, then, to become able to create appropriate learning situations. This point of view is based on the idea that a computational learning environment could promote the learners' construction of situated abstractions (Noss & Hoyles, 1996; Hölzl, 2001) and on the "instrumental approach" as developed by Vérillon and Rabardel (1995).

Within the instrumental approach, the expression "instrumental genesis" has been coined to indicate the time-consuming process during which a learner elaborates an instrument from an artefact: it is a complex process, at the same time individual and social, linked to the constraints and potential of the artefact and the characteristic of the learner. If, according to the instrumental approach, learners need to acquire non-obvious knowledge and awareness to benefit of a instrument's potential, I firmly believe that teachers need to take into account the student's instrumental genesis (Trouche, 2000).

Finally, I consider worthy of note the concept of "instrumental orchestration" proposed by Trouche (2003) aiming at tackling the didactic management of the instruments systems in order to conceive the integration of artifacts inside teaching institutions. In particular, he underlines that pre-service and in-service teacher training should take in account the complexity of this integration at three levels (Trouche, 2003; p.798):

« - a mathematical one (new environments require a new set of mathematical problems);

- a technological one (to understand the constraints and the potential of artifacts);

- a psychological one (to understand and manage the instrumentation process and their variability). »

## METHODS, CONTEXT AND PROCEDURE

The research I'm going to present consists in two main phases. The first has been carried out with a rather small group of in-service teachers at the University of Bari and a larger group of pre-service teacher at the University of Basilicata. The second involved another small group of pre-service teachers at the University of Bari.

Teachers belonging to the first group at the University of Bari were 16 high-school teachers. Although some of them already taught mathematics, on the whole they were qualified to teach related subject and they were attending a training program in order to get a formal qualification to teach mathematics.

At first, a preliminary anonymous questionnaire was submitted to them with the aim to know if they were able to see technologies as learning resources, as well as if they were available to continuously bring up-to-date in order to properly design and manage with technology-rich classroom activities. Key questions in the questionnaire included the following:

1. Do you think ICT could be useful for your teaching activities? Why?
2. Do you think that the use of ICT can somehow change the learning environment? And the way to teach? And the dynamics among actors in the teaching/learning situations?
3. Which difficulties do you think can be encountered when designing and developing a math lessons using somehow ICT?
4. As a teacher, do you think you need to have some didactical competences in order to properly use ICT? Eventually, which ones? And anyway, why?

Within the training program they attended, a thirty hours course was focused on didactical reflection aiming at helping student teachers to understand how to make the most of the use, in mathematics teaching and learning activities, of general tools such as spreadsheets, multimedia and Internet, as well as mathematics-specific educational software such as Cabri. In order to explain them that the changes produced by the introduction of a technological tool will not necessarily per se bring the students more directly to mathematical thinking, particular attention was devoted to stress the role of the a-didactical milieu in authentic learning situations, as in the known Brousseau's (1997 ) "theory of didactical situations". Furthermore, they were asked to analyse and discuss both successful and questionable examples of teaching/learning mathematics activities in which an important role has been played by the use of ICT.

At the end of the course student teachers designed a teaching/learning activity involving somehow the use of technology: in this way I intended to verify how deeply they have perceived the opportunity to effectively exploit the usage.

A further anonymous questionnaire, free from constraints, was later submitted with the aim to find out any signal for changes in their conceptions to have been occurred. Key questions in this further questionnaire were exactly the same.

Pre-service teachers involved in the research project at the University of Basilicata were a larger number (97). They were only asked to fill in the first questionnaire.

During the second phase, a group of 16 pre-service teachers at the University of Bari, instead, interacted with the researchers/educators in the same way of the first group of in-service teachers: to this further group of teachers a preliminary anonymous questionnaire was submitted; then, they were invited (during a thirty hours course) to reflect on didactical aspects of the use of technologies as well; at the end of the course they were asked to design a teaching/learning activity in which technology played an essential role; finally I analysed the extent of their changes in looking at the integration of technologies in the teaching/learning processes.

According to the results obtained during the first phase (that I'm going to present and discuss in the next paragraph), in the second phase I asked student teachers, not only to design a teaching/learning activity involving the use of technology, but also to put in action the activities they have designed, having as student sample their colleagues: in this way they proved themselves as "actors" in a technology-rich learning "milieu".

## FINDINGS AND DISCUSSION

Findings from the first anonymous questionnaire revealed that in-service student teachers perceived that technology can bring support to their teaching (see Fig.1), but only as much as it is a motivating tool enabling students understanding per se (see Fig. 2).

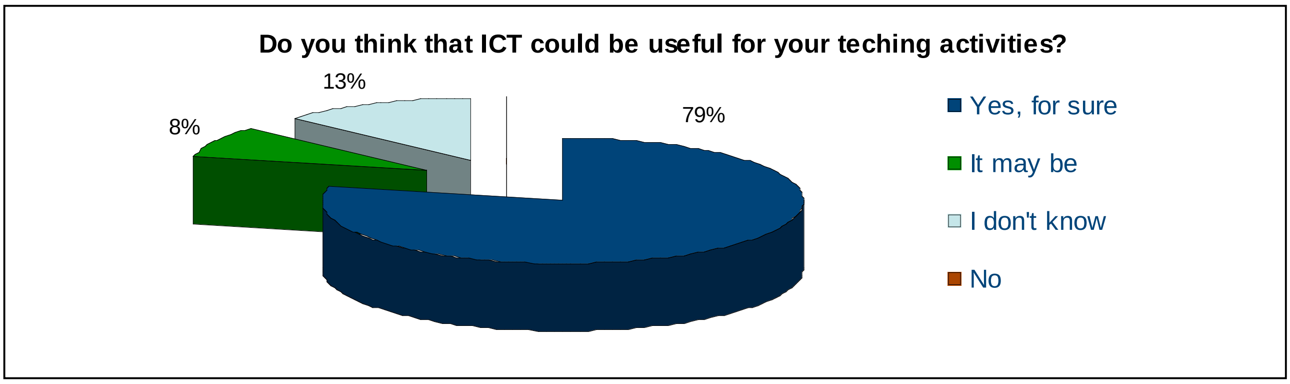



**Figure1: The 79% of the in-service student teachers gave a positive ("Yes, for sure") answer to question 1.**

Sicuramente rende la materia più "accattivante" per gli studenti. La rende anche più dinamica e pratica, più vicina agli alunni e alla loro vita quotidiana.

…math can be more attractive, dynamic, practical

La lezione diventa partecipata ed interattiva; inoltre l'uso del computer utilizza un linguaggio sicuramente più affascinante e motivante per gli studenti rispetto alla più usuale lezione frontale.

…lesson can be more, shared, interactive, fascinating

**Figure2: Some in-service student teachers' answers to question 1: Do you think ICT could be useful for your teaching activities? Why?**

Answers given by the pre-service teachers were, instead, a little bit more didactically oriented: some of them recognise that, if nothing else, the knowledge of the instrument functionality is probably not enough for a teacher to use it in an effective way in terms of construction of meanings by the students (see Fig. 3).

L'insegnante non può semplicemente dare delle consegne ed attendere i risultati. È necessario che oltre a conoscere il software sappia come utilizzarlo per raggiungere i diversi obiettivi didattici, altrimenti si ritornerebbe alla classica lezione frontale con l'unica differenza che è svolta in laboratorio e con il calcolatore al posto di una calcolatrice.

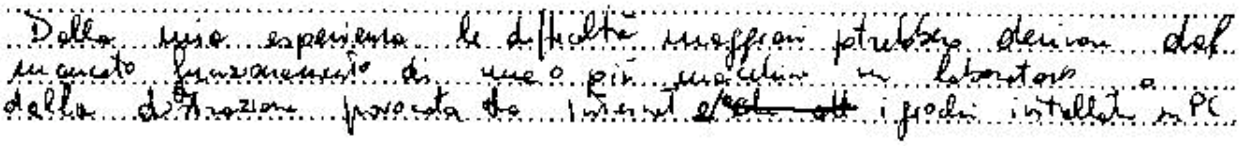
…otherwise the only difference with the classical lesson would be the use of a PC instead of a calculator

**Figure3: A pre-service student teacher's answer to question 1.**

None of the in-service teachers recognised that technology could bring a great support in creating new interesting and attractive learning environments. While, at least some interesting observation could be revealed among answers given (to question 2) by the pre-service teachers: some of them suggested the use of technological tools to allow students "collaboratively solve intriguing problems".

Be aware of the opportunity to create a new "milieu" and change the "economy" of the solving process was, however, extremely far from their perception of the use of technology in mathematics teaching/learning activities, both for in-service and for pre-service teachers.

About the question 3, concerning the difficulties they think can be encountered when designing and developing a math lessons using somehow ICT, they mostly ascribed possible difficulties to the lack of an adequate number of PC and the technical problems that might occur, but also to the natural students' bent for distraction and relaxation, especially when facing a PC (see Fig. 4).



…motivation is needed!

**Figure4: Some student teachers consider new technology as a motivating tool that requires motivation.**

As a consequence they did not feel the need to be skilled in using technology for their teaching and did not usually consider that their lack of skills presents them with any difficulties. And, although the 75% of the student teachers recognised (answering to question 4) the need to have some didactical competences in order to use new technology, what they asked to know about was, in most of the cases, just software functionalities (not potential, nor constrains): only some of the pre-service teachers also asked to know how to effectively integrate their use in the teaching practice.

Even tough some of the activities that in-service teachers prepared at the end of the course revealed the willingness to attempt a new approach to the use of ICT, answers to the second anonymous questionnaire shown they still continued to find difficulty to be aware of the potential offered by ICT (see Fig. 5).

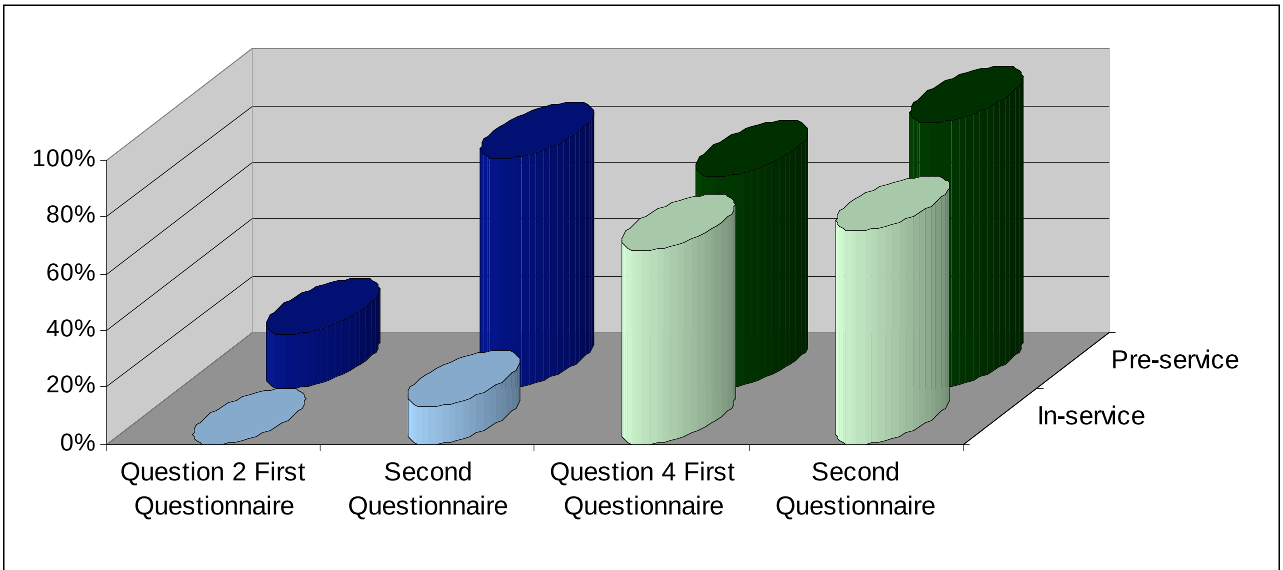



**Figure5: Percentage of positive ("Yes, for sure") answers given by both in-service and pre-service teachers respectively to the first and the second questionnaire to questions 2 and 4.**

For this reasons, for the second phase of the project I planned to pay particular attention to promote teachers' reflections on the opportunities offered by appropriate uses of technological tools in order to create new learning environment and, according to the idea of "mathematics laboratory", to foster the construction of mathematical meanings.

Student teachers were invited not only to design a possible teaching/learning activity involving somehow the use of technology, but they were also involved in a “mise en situation” (as in the known Chevallard’s approach) during which they had the opportunity to assume the roles of the student, the teacher and a researcher/observer.

In this way, they faced with the complexity of the integration of technologies in classroom practice. Their comments at the end of the experience shown that they have developed an awareness of how the students’ instrumental genesis can take shape (psychological level). Moreover, answers to the second anonymous questionnaire revealed that they felt the need to understand the constraint and the potential of technologies (technological level) and to look for new mathematical problems (mathematical level).

## EARLY CONCLUSIONS AND FUTURE WORKS

Discussion suggested by the researches in this field and by the analysis of this on-going experience led me to reflect on and to underline that an adequate preparation is essential for teachers to cope with technology-rich classrooms. In particular I believe that, only if teachers become aware of the potential usefulness and effectiveness of technologies as methodological resources (enable to foster the constructions of meaningful learning environment) they would recognise the need of an effective integration of them in the classroom activities and view new technologies as cultural tools that radically transform teaching and learning.

At the actual stage of this on-going research I can claim that, in my opinion, most of the teachers have difficulty to acquire the awareness of the potential of technology as a methodological resource. Hence, as educators, we also have to deal with the need to lead teachers to develop a more suitable and effective awareness of the usage of new technologies. Furthermore, I believe that the difficulty teachers have to acquire this awareness could be overcome giving teachers the opportunity to be subject of a “mise en situation”. In this way teachers can experience by themselves the difficulties students can encounter and have to overcome, the cognitive processes they can put in action and the attainment they can achieve. They also have the opportunity to understand and manage with the students’ instrumental genesis and to become more skilful and self-confident when deciding to exploit the potentials of technologies in mathematics education.

For future works I think in particular to go on with this idea, promoting further experiences of “mise en situation” according to the following stages:

- let teachers experience the importance of the relationship between the specific knowledge to be acquired by the students and the knowledge teacher possesses of it;

- let teachers experience the importance of the relationship between the specific knowledge to be acquired by the students and whatever students already know;

- let teachers experience the importance of the relationship between their knowledge and the students’ ones.

I suppose, indeed, that through these stages, teachers could experience by themselves the processes that come into play bringing technology in a teaching/learning situations. In particular, according to the early results of this study, I believe that in this way teachers do tackle with the obstacles encountered, the difficulties to be overcome, the cognitive and metacognitive processes carried out and the attainment that can be achieved.

To conclude, in the next future I aim to verify that, thanks to this methodology, not only they can cope with changes they could meet in a technology-rich learning situation but, reflecting on them, they can also become aware of how to better make use of technology as a resource to create an effective and meaningful learning environment.

Finally (also considering the explicit suggestions of the WG7 call for papers), I suppose that an interesting help to foster the development of teacher's instrumental genesis can be given by the use of Geoboards (Bradford, 1987). A Geoboard is a physical board (often used to explore basic concepts in plane geometry) with a certain number of nails half driven in, in a symmetrical square, (for example five-by-five array): stretching rubber bands around pegs, provide a context for a variety of mathematical investigation about concepts and objects such as area, perimeter, fractions, geometric properties of shapes and coordinate graphing.

Thus, I would like to let high school teachers operate with an unusual (at that level) context/tool like a Geoboard, and try to understand if, in this way, they can perceive teaching resources, both digital or not, as methodological resources: when teachers become aware that some resources can be effectively used for the construction of mathematical meanings they can start to successfully design and experiment new interesting learning activities.

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# THE TEACHER'S USE OF ICT TOOLS IN THE CLASSROOM AFTER A SEMIOTIC MEDIATION APPROACH

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*The issue of the teacher's role in exploiting the potentialities of ICT tools in classroom is more and more raising the interest of our community. We approach this issue from the Semiotic Mediation perspective, which assigns a crucial importance to the teacher in using ICT tools in the classroom. In the report we describe a Teaching Sequence centred on the use of the tool Casyopée and inspired by the Theory of Semiotic Mediation. Then we focus on the teachers' use of the tool with respect to the orchestration of collective activities and present an on-going analysis of her actions.*

## INTRODUCTION

Recent research points out a wide-spread sense of dissatisfaction with the degree of integration of technological tools in mathematic classrooms. Kynigos et al. observe that so far one did not succeed to exploit the ICT potential suggested by research in the 80s and the 90s and denounce that "the changes promised by the case study experiences have not really been noticed beyond the empirical evidence given by the studies themselves" (Kynigos et al. 2007, p.1332).

The acknowledgement of the existing gap between the research results on the use of technology in the mathematical learning and the little use of these technologies in the real classroom led recently to reconsider the importance of the teacher in a technology-rich learning environment, and to investigate ways of supporting teachers to use technological tools.

Those "teacher-centred" studies have been developed from different perspective and address different aspects, for instance: teacher education (Wilson, 2005), teachers' ideals and aspirations regarding the use of ICT (Ruthven, 2007), teacher's role in exploiting the potentialities of ICT tools in the classroom.

With that respect, as Trouche underlines, most studies refer to the importance of teachers' guide or assistance to students' activities with the technology (Trouche, 2005). Trouche himself emphasizes the need of taking into account the teacher's actions with ICT. For that purpose he introduces the notion of "instrumental orchestration", that is the intentional systematic organization of both artefacts and humans (students, teachers…) of a learning environment for guiding the instrumental geneses for students (ibidem, p.126).

Within this approach the teacher is taken into account insofar as a guide for the constitution of mathematical instruments.

As we will argue in the next section, guiding the constitution of mathematical instruments does not exhaust the teacher's possible use of ICT. In fact ICT tools can be used by the teacher (a) for developing shared meanings having an explicit formulation, de-contextualized with respect to the ICT tool itself and its actual, recognizable and acceptable in respect to mathematicians' community, and (b) for fostering students' consciousness-raising of those meanings. The Theory of Semiotic Mediation (Bartolini Bussi and Mariotti, 2008) takes charge of that dimension.

In this report, we present an analysis of the teacher's use of an ICT tool within the frame of the Theory of Semiotic Mediation. More precisely we focus on the teacher's promotion and management of collective discussions. But a systematic discussion of the role of the teacher or a classification of her possible actions is out of the goals of the present paper. The context is a teaching sequence, inspired by the Theory of Semiotic Mediation, and centred on the use of the tool Casyopée. Both the teaching sequence and the tool are presented in the next sections, after recalling some basic assumptions of the Theory of Semiotic Mediation.

## THE THEORY OF SEMIOTIC MEDIATION

Assuming a Vygotskijan perspective Bartolini Bussi and Mariotti put into evidence that the use of an artefact for accomplishing a (mathematical) task in a social context may lead to the production of signs, which, on the one hand, are related to the actual use of the artefact (the so called **artefact-signs**), and, on the other one, may be related to the (mathematical) knowledge relevant to the use of the artefact and to the task. As obvious, this knowledge is expressed through a shared system of signs, the mathematical signs. The complex of relationships among use of the artefact, accomplishment of the task, artefact-signs and mathematical signs, is called the **semiotic potential** of the artefact with respect to the given task.

Hence, in a mathematics class context, when using an artefact for accomplishing a task, students can be led to produce signs which can be put in relationship with mathematical signs. But, as the authors clearly state, the construction of such relationship is not a spontaneous process for students. On the contrary it should be assumed as an explicit educational aim by the teacher. In fact the teacher can intentionally orient her/his own action towards the promotion of the evolution of signs expressing the relationship between the artefact and tasks into signs expressing the relationship between the artefact and knowledge.

According to the Theory of Semiotic Mediation, the evolution of students' personal signs towards the desired mathematical signs is fostered by iteration of **didactic cycles** (Fig.1) encompassing the following semiotic activities:


**Fig. 1. Didactical Cycle**

- activities with the artefact for accomplishing given tasks: students work in pair or small groups and are asked to produce common solutions. That entails the production of shared signs;
- students' individual production of reports on the class activity which entails personal and delayed rethinking about the activity with the artefact and individual production of signs;
- classroom collective discussion orchestrated by the teacher

The action of the teacher is crucial at each step of the didactic cycle. In fact the teacher has to design tasks which could favour the unfolding of the semiotic potential of the artefact, observe students' activity with the artefact, collect and analyse students' written solutions and home reports in particular posing attention to the signs which emerge in the solution, then, basing on her analysis of students written productions, she has to design and manage the classroom discussion in a way to foster the evolution towards the desired mathematical signs.

The Theory of Semiotic Mediation offers not only a frame for designing teaching interventions based on the use of ICT, but also a lens through which semiotic processes, which take place in the classroom, can be analysed (for a more exhaustive view, see Bartolini Bussi and Mariotti, 2008).

## CASYOPÉE

Casyopée (Lagrange and Gelis 2008) is constituted by two main environments which can "communicate" and "interact" between them: an Algebraic Environment and a Dynamic Geometry Environment (though the designers' objective was not to develop a complete CAS or a complete DGE). Possible interactions between the two environments are supported through a third environment, the so called "Geometric Calculation". Without entering the details of Casyopée functioning, we can illustrate it through the following example.

If one has two variable geometrical objects in the DGE linked through a functional relationship (e.g. the side of a square and the square itself), Casyopée supports the user in associating algebraic variables to the geometrical variables and building an algebraic expression for the function (e.g. the function linking the measure of the length of the side, as independent variable, and the measure of the area of the square, as dependent variable). The generated algebraic variables and functions can be exported in the Algebraic Environment, and then explored and manipulated.

## DESCRIPTION OF THE TEACHING EXPERIMENT

The Theory of Semiotic Mediation shaped both the design and the analysis of the teaching experiment carried out. In this chapter, we briefly describe the design.

### Educational Goals of the designed teaching sequence.

The design of the teaching intervention started from the analysis of the semiotic potential of the tools of Casyopée. That analysis led us to identify two main educational goals: fostering the evolution of students' personal signs towards

1. the mathematical signs of function as co-variation and thus consolidate (or enrich) the meanings of function they have already appropriated, that entails also the notions of variable, domain of a variables…;

2. the mathematical meanings related to the processes characterizing the algebraic modelling of geometrical situation.

### Description of the teaching sequence

According to our planning the whole teaching sequence is composed of 7 sessions which could be realized over 11 school hours.

The whole teaching sequence is structured in didactical cycles: activities with Casyopée alternate with class discussions, and at the end of each session students are required to produce reports on the class activity for homework.

The familiarization session is designed as a set of tasks and aims at providing students with an overview of Cayopée features and guiding students to observe and reflect upon the "effects" of their interaction with the tool itself, e.g.:

> Could you choose a variable acceptable by Casyopée and click on the "validate" button? Describe how the window "Geometric Calculation" change did after clicking on the button. Which new button appeared?

Besides familiarization, the designed activities with Casyopée consist of coping with "complex" optimization problems formulated in a geometrical setting and posed in generic terms, e.g.:

> Given a triangle, what is the maximum value of the area of a rectangle inscribed in the triangle? Find a rectangle whose area has the maximum possible value.

The aim is to elaborate on those problems so to reveal and unravel the complexity and put into evidence step by step the specific mathematical meanings at stake.

The diagram (Fig. 2) depicts the structure of the teaching sequence: the cyclic nature of the process, which develops in spirals, is rendered through the boxing of the cycles themselves.

### Implementation and data collection

With some differences, the teaching sequence was implemented in 4 different classes (3 different teachers): two 13 grade classes and a 12 grade class of two Scientific High Schools, and a 13 grade class of Technical School with Scientific Curriculum.

Different kinds of data were collected: students' written productions; screen, audio and video recordings, and Casyopée log files. The analysis presented below is based on the verbatim transcription of the video recordings of the classroom discussions.

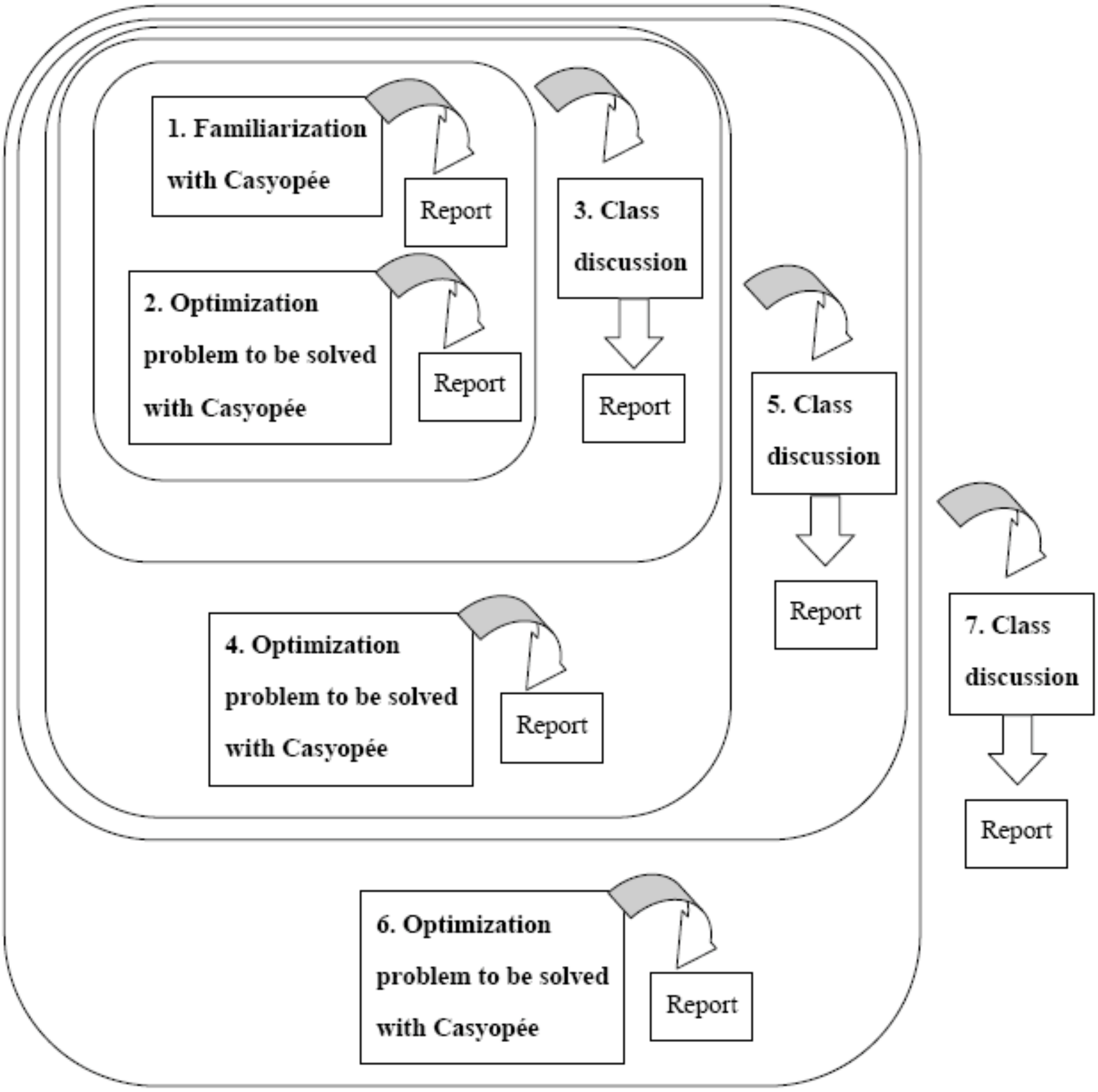


**Fig. 2. Outline of the Teaching S**

## ANALYSIS OF THE TEACHER'S ACTIONS

According to the theory of Semiotic Mediation, the teacher's action should aim at promoting the evolution of students' personal signs towards mathematical signs. Such evolution can be described in terms of *semiotic chains*, or chains of signification to use Walkerdine's terminology, that is:

> "particular chain of relations of signification, in which the external reference is suppressed and yet held there by its place in a gradually shifting signifying chain." (Walkerdine, 1990, p.121).

The following excerpt is drawn from the transcript of the class discussion held in the 5th session. It shows an example of how artefacts signs are produced in relation to the use of the artefact, and how they may evolve during the discussion. We first go quickly through the excerpt showing the evolution of signs, then we will analyse how the teacher contributes to this evolution.

| | |
|---|---|
| 1. Teacher A: | "Which are the main points to approach this kind of problem? Which kind of problem did we deal with? […] What is an important thing you should do now? To see the general aspects and apply them for solving possible more problems with or without the software, […] the software guided you proposing specific points to focus on.[…]" |
| 2. Cor: | "[…] First of all we had to choose the triangle by giving coordinates" |

[Students recall the steps to represent the geometrical situation within Casyopée DGE]

| | |
|---|---|
| 5. Luc: | "But you have to choose a mobile point, first […]" |
| 6. Teacher A: | "Does everybody agree?[…]How would you label this first part? […]" |
| 7. Students: | "Setting up" |
| 8. Teacher A: | "Luc has just highlighted something […] do you see anything similar between the two problems?" |
| 9. Sam: | "One has always to take a free point which varies, in this case, the areas considered […]" |
| 10. Teacher A: | "Then we have a figure which is…" |
| 11. Students: | "Mobile." |
| 12. Teacher A: | "Mobile, dynamical. Let us pass to the second phase. Andrea, which is the next phase? […]" |
| 13. And: | "The observation of the figure would let us see… we need to study that figure and observe what the shift of the variable causes…" |
| 14. Teacher A: | "Ok, then? Everybody did that, isn't it?" |
| 15. Sil: | "We computed the area of the triangle and of the parallelogram, we summed them, and by shifting the mobile point one observed as [the sum of the areas] varied […]" |

Focusing on students' signs, one can notice:

- Elements of a collectively constructed *semiotic chain*, in which a connection is established between artefact signs ("mobile point") and mathematical signs ("variable"). The elements of this semiotic chain are: "movable point" (item 5), "free point" (item 9), "variable" (item 13), and "movable point" (item 15). It is worth noticing the two directions: from the artefact sign ("mobile point") to the mathematical sign ("variable") and vice versa. That semiotic chain shows: (a) students' recognition that geometrical objects can be considered (can be treated, can act) as variables, and (b) the enrichment of students' meanings of variable to include meanings related to "movement".

- Elements of a collectively constructed *semiotic chain,* in which the meaning of function as a relation of co-variation of two variables emerges. The elements of this semiotic chain are: "a free point which varies […] the areas" (item 9), "the shift of the variable causes" (item 13), "by shifting the movable point, one observed as [the sum of the areas] varied" (item 15).

**Analysis of the Teachers' orchestration of the discussion.**

We reconsider the excerpt previously analysed form the point of view of the signs produced and used by students. Here we focus on how the teacher's actions fuel the discussion, foster the production of artefacts signs in relation to the use of the artefact, and create the conditions for their evolution during the discussion.

| | | |
|---|---|---|
| 1. | Teacher A: | "Which are <u>the main points to approach</u> this kind of problem? Which <u>kind of problem</u> did we deal with? […] What is an important thing you should do now? <u>To see the general aspects</u> and apply them for solving possible more problems with or <u>without the software</u>, […] <u>the software guided you proposing specific points to focus on</u>.[…]" |

The teacher starts the discussion by making explicit its objectives: to arrive at a shared and de-contextualized formulation of the different mathematical notions at stake ("to see the general aspects and apply them for solving possible more problems with or without the software").

In order to do that, the teacher asks students to recall the problem dealt with in the previous section and to report on the solutions they produced. She explicitly orients the discussion towards the specification of the main phases of the solution of the problem, asking students to look for similarities between the two problems addressed so far and between the strategies enacted to solve them.

While asking students to do that, the teacher suggests to refer to (or to remind) the use of the DDA. The suggestion to explicitly refer to the use of Casyopée facilitates the production and use of artefact-signs and the unfolding of the semiotic potential.

| | | |
|---|---|---|
| 5. | Luc: | "But you have to <u>choose a mobile point</u>, first […]" |
| … | | |
| 8. | Teacher A: | "<u>Luc has just highlighted something</u> […] do <u>you see anything similar between the two problems</u>?" |
| 9. | Sam: | "One has always to take a <u>free point</u> <u>which varies</u>, in this case, <u>the areas</u> considered […]" |

Following the teacher's request, students collectively report on their work with Casyopée. That leads to the production of the artefact sign "mobile point" (out of the others) (item 5). The sign “mobile point” is clearly related to the task and the use of Casyopée for accomplishing it. At the same time it may be related to the mathematical knowledge at stake: the notion of variable. There are several possibilities for the subsequent development of the discussion: one could orient the discussion towards the distinction between mobile and variable, towards the specification of other variable elements, discussion towards the distinction between algebraic or numerical variable and geometrical variable, towards the recognition of the aspects of co-variation between the variable elements of the geometrical figure, towards the distinction between independent and dependent variable.

Certainly, the teacher’s intervention is needed both to drive the attention of the class towards the sign introduced by Luc and to orient the discussion. The teacher is aware of that and intentionally emphasizes Luc’s contribution to the discussion (item 8). At one time, she requires to generalize so to foster a de-contextualization from the specific problems faced and strategies enacted, and to provide the possibilities for the evolution of personal signs to initiate.

After the teacher’s intervention, Sam (item 9) echoes Luc’s words. But she uses the sign “free point” instead of “mobile point”, and introduces the consideration of other variable elements (“areas”) also emphasizing the existence of a link between them (“free point which varies […] the areas”). Those are the first elements of the two semiotic chains described in the previous section.

10. Teacher A: “Then we have a <u>figure</u> which is…”
11. Students: “<u>Mobile</u>.”
12. Teacher A: “<u>Mobile, dynamical</u>. Let us pass to the second phase. Andrea, which is the next phase? […]”
13. And: “The <u>observation of the figure</u> would let us see… we need to study that <u>figure and observe what the shift of the variable causes</u>…”

Sam’s contribution (item 9) ends with the reference to variable areas. That could prematurely move the discussion towards the consideration of algebraic or numerical aspects, without giving time to elaborate on variable and variation in the geometric setting. In order to contrast this risk, the teacher introduces the term “figure” (item 10) which has the effect of keeping students’ attention still on the geometrical objects. In addition the teacher fuels the discussion echoing students and, thus, emphasizing the reference to the dynamical aspects (item 12), which nurtures the construction of the semiotic chains on variation and co-variation.

And, whose intervention is stimulated by the teacher, echoes the use of the sign “figure” and makes explicit exactly the co-variation between the geometrical objects in focus. She also introduces the sign “variable” so establishing a connection between the artefact sign “mobile point” and the sign “variable”.

We are not claiming that the evolution towards the target mathematical signs is completed: a shared and de-contextualized formulation of the different mathematical notions at stake is not reached yet, as witnessed by Sil's words (item 15), who still makes reference to the use of the artefact in her speech.

14. Teacher A: "Ok, then? Everybody did that, isn't it?"
15. Sil: "We computed the area of the triangle and of the parallelogram, we summed them, and by shifting the mobile point one observed as [the sum of the areas] varied […]"

The above analysis puts into evidence a number of interventions of the teachers who succeeds in exploiting the semiotic potential of Casyopée, and thus in making the class progress towards the achievement of the designed educational goals.

One can find also episodes in which the teacher' action is not so efficient. The following excerpt is drawn from a discussion held in another class and orchestrated by a different teacher, and it shows an episode in which the teacher does not succeed to exploit the potentialities of the students' interventions. Chi countered the sign "variable" with the sign "variable point" so offering the possibility to dwell on the relationship between not measurable geometrical variables and measurable geometrical variables. The specification of this distinction was considered a key aspect of algebraic modeling, and as such highly pertinent to the designed educational goals. The teacher does not seize the occasion and does not take any action to fuel the discussion on that, she was probably aiming at orienting the discussion along a different direction.

184. Chi: "we put CD as <u>variable</u>, and not by chance CD, in fact we used a fixed point, C, and a <u>variable point</u> on the segment, D"

185. Teacher B**:** "well, the underpinning idea is to link numbers, and, […] having observed a link between the position of the point D and […] the area of the rectangle […] a link is established between a geometrical world and an algebraic world"

That witnesses the difficulty of mobilizing strategies to foster the evolution of students' signs. One has to constantly keep the finger on the pulse of the discussion and of its possible development. In fact the evolution of students' signs depends on extemporary stimuli asking for a number of decisions on the spot.

## CONCLUSIONS

The analysis carried out in the paper confirms the crucial role of the teacher in technology-rich learning environments. In particular, such role may (and should from our perspective) go beyond that of assistant or guide for students' instrumental genesis process. In fact through her interventions the teacher promotes and guides the development of the class discussion, so to foster the production and the evolution of students' signs towards the target mathematical signs, and to facilitate students' consciousness-raising of the mathematical meanings at stake.

Certainly we are aware that the analysis presented is still at a phenomenological level. There is an emerging need for elaborating a more specific model for analysing the teacher' semiotic actions. But there is not only the need of developing tools for finer analysis. We showed an episode witnessing the difficulty of mobilizing strategies to foster the evolution of students' signs. Currently, the Theory of Semiotic Mediation does not equally support analysis and planning. Due to the richness of a class discussion and the number of extemporary stimuli which could emerge, one cannot foresee the exact development of the discussion. That makes the teacher's role still more crucial. Nevertheless there is the need of an effort for elaborating more specific theoretical tools for supporting the a-priori design of classroom discussion. All this is also relevant to the more generic issue of teacher's formation.

## NOTES

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# ESTABLISHING DIDACTICAL PRAXEOLOGIES: TEACHERS USING DIGITAL TOOLS IN UPPER SECONDARY MATHEMATICS CLASSROOMS

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*This paper discusses elements of the didactical work of ordinary mathematics teachers using digital tools. The upper secondary school in Norway where the data was collected has run an internal project to integrate the Personal Computer into the mathematics classroom. Using the Instrumental Approach as a framework this paper seeks to describe and interpret elements of teacher practice exploring also the notion of instrumental genesis from a teacher perspective. From the analysis of classroom observations, interviews, meetings, and study of documents three main didactical practices were found to be linked to the introduction of the digital tools: the digital notebook, the digital textbook, and the phenomenon of weaving between tools/instruments in the classroom.*

## INTRODUCTION

The recent school reform in Norway, Knowledge Promotion 2006, formally acknowledges digital competence as one of the five basic skills students should acquire and develop in their formal schooling[1]. This places on schools and individual teachers a responsibility to integrate these tools into classroom practice. This study looks at the practice of two teachers in a comprehensive upper secondary school in Norway who have been using digital tools over a period of five years. In 2007 the school joined the project "Learning Better Mathematics", hereafter LBM[2], a developmental project initiated by school authorities through a co-operation with University of Agder. Data used in this paper was collected at the school's point of entry to the project. The classrooms observed were equipped with a blackboard and a projector with screen and set up as "paperless" environments where all students had their own laptop PC and when observed rarely used paper and pencil in their mathematics lessons: all student work was done on the computer.

## THEORETICAL FRAMEWORK

The theoretical approach employed emerged in the mid-nineties in France when researchers became aware that traditional constructivist frameworks were inadequate in the analysis of CAS environments (Artigue, 2002). Artigue claims that this approach is less student centred but provides a wider systemic view also giving the instrumental dimension of teaching and learning more focus (Artigue, 2007). The

[1] Knowledge Promotion (Kunnskapsløftet 2006). These basic skills are given as the ability: to express oneself orally to read, to do arithmetic, to express oneself in writing, to make use of information and communication technology

[2] The project is supported by the Research Council of Norway

approach uses notions both from the Theory of Instrumentation from the field of Cognitive Ergonomy, and from the Anthropological Theory of Didactics (ATD hereafter) in the field of Mathematics Education (Laborde, 2007).

Cognitive Ergonomy considers all situations where human activity is instrumented by some sort of technology. The theory of instrumentation employs the notion of "instrument" and the notion of "instrumental genesis" (Artigue, 2002). The instrument has a mixed identity, made up of part artefact and part cognitive scheme. It is seen as a mediator between subject and object but also as made up of both psychological structures, called schema which organise the activity, and physical artefact structures such as pencil, paper, or digital tools (Béguin & Rabardel, 2000). For the individual user, the artefact becomes an instrument through a process of instrumental genesis which involves the construction of personal schema or the appropriation of socially pre-existing schemes (Artigue, 2002). This process of instrumental genesis has two elements, *instrumentalisation* the process whereby the user acts on the tool shaping and personalising the tool, and *instrumentation* the process whereby the tool acts on the user shaping the psychological schema (Rabardel, 2003). Instrumental genesis is a process occurring through the user's activity through participation at the social plane. Guin and Trouche (1999) applied the Theory of Instrumentation in research in mathematics classrooms, studying the process by which the graphic calculator becomes an instrument for the students to learn mathematics. They term the teachers' role in guiding the students' instrumental genesis *instrumental orchestration*. This is defined as a plan of action having four components: a set of individuals, a set of objectives, a didactic configuration and a set of exploitation of this configuration (Guin & Trouche, 2002, p. 208).

ATD on the other hand aims at the construction of models of mathematical activity to study phenomena related to the diffusion of mathematics in social institutions, see for example (Barbé, Bosch, Espinoza, & Gascón, 2005). The theory analyses human action including mathematical activity by studying *praxeologies*:

> But what I shall call a praxeology is, in some way, the basic unit into which one can analyse human action at large. (Chevallard, 2005, p. 23)

Any human praxeology is constituted of a practical element (praxis) and a theoretical element (logos). The praxis has two components, the task and the technique to solve the task. The logos also has two components, the technology (or discourse) and the theory which provide a justification for the praxis.

Mathematical knowledge in an educational institution can be described in terms of two types of praxeologies: mathematical praxelologies and didactical praxeologies. The object of the didactical praxeologies is the setting up of and construction of the the mathematical praxeologies. It is these didactic praxeologies, representing teacher practice, that are of interest to me in my study. Questions arising are: What constitutes or defines the didactical task, technique, discourse and theory? How are the mathematical praxeology and the didactical praxeology entwined? How do the existing didactical praxeologies change when digital tools are introduced into the mathematics classroom? Laborde's conclusion that, "A tool is not transparent. It affects the way a user solves a task and thinks" (Laborde, 2007, p. 142) should apply equally to both teacher and student.

Research indicates that the interventions of the teacher are critical in relation to student learning of mathematical knowledge when digital tools are introduced (Guin & Trouche, 1999). The teacher's instrumental orchestration is part of the didactical praxeology. As new tools are introduced, the teacher must develop new didactical praxeologies to support the students' instrumental genesis for the particular tool (Trouche, 2004, p. 296). The teacher must also incorporate the new tool into an existing repertoire of tools and didactical techniques. Practically in the classroom, this involves for the teacher: (1) Organisation of space and time, (2) the choosing of the mathematical tasks and the techniques to solve these tasks, and (3) the steering of the mathematical activity in the classroom by discourse.

### Aim and research question

This paper aims to identify features of didactical praxeologies that have been established in relation to the introduction of the digital tool and also to describe the process of introduction of the digital tool and changes to practice from the teacher perspective. The research questions are: What features of the teachers' didactical praxeologies can be identified as pertaining to/originating specifically from the introduction and use of the digital tool? Can these features be seen as evidence of a process of instrumental genesis for the teachers in relation to the digital tool? What factors influence this process?

This short paper allows for in depth discussion of only some of the features indicated above. I have therefore selected features that appear to be of significant importance to the teachers when they describe the changing practice in relation to the tool. The paper also seeks to describe only commonalities in teacher practice.

## THE EMPIRICAL STUDY

### The teachers, their classes and classrooms

The two teachers in this study very generously opened their classrooms and gave of their time to this researcher. Both were active in initiating the ICT project at the school. The ICT project had been established and operated entirely within the school and was not part of any external research, design or development project. It is therefore claimed that it is the practice of two "ordinary" teachers that is described in this paper. In 2005, the school was the only school in the country to conduct final examinations in mathematics entirely on the portable PC.

This part of the study involved classroom visits to two classes of approximately twenty five students. The students were studying the subject "Theoretical Mathematics 1" (1T), which is allocated three double lessons a week, each of 90 minutes duration. These two classes were two of five classes at the school studying this subject. Each classroom was equipped with a blackboard and a projector with screen. The screen covered part of the blackboard but it was still possible to use the blackboard. The technical features of the environment functioned without difficulties in the observation period. The classrooms observed presented as “paperless” environments as all students had their own laptop PC, leased from the school, and when observed rarely used paper and pencil in their mathematics lessons though this was permitted. All student work including exercises, notes, rough work was done on the computer. I have chosen to refer to this practice as the “digital notebook”. Standard paper textbooks were no longer in use as the teachers have developed their own digital textbooks, which are made available to the students through a Learning Management System (LMS). This practice I refer to as the “digital textbook”. The classrooms appeared very orderly as there were no books, papers, rulers or other items littering the desks. Each student had a PC and perhaps a bag placed on the floor under the desk. The students started work quickly plugging in and turning on the PC, contrasting sharply with “normal” classrooms where students take some minutes to find notebooks, textbooks, pencils and so on. In the observed lessons only the teachers used the projector. Student work was not displayed using the projector.

## Data collection and analysis

Data collection over a period of four months involved: audio recording of an introductory meeting between the school and the university where the two teachers, a school leader, two researchers and a project leader from the university were present; lesson observation with video recording of eight lessons; audio recording of three semi-structured interviews before and after lessons with the teachers; audio-recording of seven structured interviews with students (Billington, 2008); and audio data from LBM project meetings where the teachers were present and took part in discussions. The writer was present at all events, taking field notes. In the classroom observations, researchers were present as observers, taking no active part in the planning or carrying through of the lesson. Shortly after each event a preliminary data reduction using the notes and recordings was made. Passages were also transcribed. Later all data was again reviewed, coded and further transcribed. Each data episode renders different information helping to build a picture of teacher practice identifying didactical praxeologies that would not be there without the digital tool. The meetings and interviews tell of the temporal dimension and of the changing nature of the didactical praxeologies from the teacher perspective and also reveal the institutional influences. Classroom episodes record teacher activity in the classroom revealing techniques of instrumental orchestration. Student interviews tell of the students’ instrumental genesis and the teachers’ orchestration from the students’ perspective.

## Analysis of data from meetings and interviews

The teachers were very keen to discuss the introduction of the digital tool and there were clear indications in the data that the teachers saw a process of development in their teaching practice.  Examples of such comments were as follows:

Teacher 1: … and it, it has been, been of course, a long process to come this far, this software …

Teacher 2: But …there is, as such, a remarkable difference from when we started, now…

Reviewing the data from the meetings and interviews, reoccurring themes emerged. These were first categorised under three headings, justification, implementation and evaluation. I then attempted to interpret these themes in the light of the theory as presented in the table below. In a didactical praxeology, implementation would pertain to the praxis while justification and evaluation would pertain to the logos.

| **Justification** | **Implementation** | **Evaluation** |
|---|---|---|
| Teachers explained why “we do what we do and continue to do what we do ” | Teachers explained how they organised and carried out the project | Teachers talked about what they identified as affordances and constraints of the tool |
| **Didactical theory – justification of practice** | **Didactical tasks and techniques** | **Didactical technology (discourse) – relating theory to tasks and techniques** |

**Table 1: Interview Themes**

The most common reoccurring themes under implementation were: the digital textbook, the digital notebook and teaching techniques in the classroom. There was also some discussion input from a school leader, which is relevant to the discussion on orchestration.

## Results and discussion of data from meetings and interviews

As stated above, the teachers referred constantly to the introduction of the digital textbook and the digital notebook. Discussion of these two innovative features of the implementation occupied much of meeting and interview time. The teachers referred to the digital textbook as “Learning Book”[3]. This digital textbook has replaced the usual paper textbook that students would normally buy. It is made available through the functioning LMS. Commenting on the digital textbook, the teachers explained that as the project progressed they found that the students preferred to read the notes that they had made rather than read the paper textbook. As a new syllabus came into force this year they decided to make their own digital textbook from scratch.

Teacher 1: Yes. Totally from scratch, just from the syllabus. Not from any textbook ….We have taken the syllabus point by point …

Teacher 2: Now we use the syllabus, and it has been extremely useful to go thoroughly into the plans and now we have to make the right choices … we feel we have to make a good deal of choices … that we make for the students …

[3] Here literally translated from the Norwegian “læringsbok”

The teachers have been provoked to return to the mathematical goals in the syllabus and build from these. This development is in line with that described by Monaghan (2004). The students save this textbook to their own PC and can write in memos, and notes. All problems and exercises are also made available through the LMS for the students. According to the teachers giving out solutions on the LMS saves time that can be used to other things, for example, “we can go around and help”. The students also retain these files from year to year whereas previously they sold the textbook at the end of the year. In terms of the theoretical framework of ATD this could be interpreted as a transposition of mathematical knowledge (Balacheff & Kaput, 1996) from the syllabus to a form usable on the PC.

The second innovation, the digital notebook, a notebook kept by each individual student where s/he writes and stores all notes, exercises, and rough work on his/her own PC, was also clearly important to the teachers. In fact one teacher gave this aspect some credit for the increased enrolment of girls in these maths courses.

> Teacher 1: ...and they (girls) sat on the fence for a year or so. And then a few girls signalled to the others, see here, and then the girls joined in force, ….That was when the girls saw that this was not about playing games, but this was a way to make it very nice. They got everything very systematic, got a way to keep all their notes in order, and very, very nice presentation, and this, the girls thought was very ok, and the boys too, now they have all their notes from last year and can build on this.

Choosing supporting materials for the student is a didactical task for the teacher. In this case the production of a digital textbook and the promotion of a digital notebook are clearly identifiable as innovations in relation to normal practice and could be interpreted as an instance of instrumentalisation where the user shapes the tool to his/her purpose. Data from the student interviews confirmed that these two innovations were important in the students’ instrumental genesis (Billington, 2008).

This leads to the reoccurring third theme in the meetings and interviews: reflection over teaching practice in the classroom. The teachers expanded on the teaching philosophy on which they have based the project claiming that they tried to avoid the standard structure of theory, example, exercises, and method.

> Teacher 2: We have had a main principle since we started with this. These textbooks are always alike, theory, examples, and then exercises exactly like the example, and then examples that are almost the same. As far as possible we try to avoid this. Our philosophy is fewer exercises and they can rather sit and struggle with the same exercise and if it takes the whole lesson that does not matter.

Interestingly the teachers did not expose on the wonders of the digital tool per se, but rather talked of the teaching possibilities with the tool as illustrated by these quotes.

> Teacher 1: I have much more influence on my own teaching before...

> Teacher 1: The role of the teacher is a bit …you have greater possibilities, that is what we have seen …

> Teacher 2: But, I must say, for my sake, that I have opportunities that I would never had had without the PC.

These possibilities can be interpreted as new didactical techniques. One teacher claimed that his teaching had changed since the students have now chosen not to use the standard paper textbooks. They discussed the need to focus on understanding rather than the reproduction of algorithms. They saw the creation of the digital textbook as allowing them more freedom to steer the activities of the classroom in line with their philosophy. These reflections I interpret as discourse justifying the praxis element of the didactical praxeologies.

Choosing for students the mathematical tasks, and the techniques and tools to solve these tasks, is a didactical task for the teacher. These tools include the textbook as well as the digital tools, the software and the hardware. The nature of this didactical task has changed for these teachers in the course of the project. They have explained how previously they just followed the book, a routine, but now because of the new situation they have been forced to make new choices. They now worked together to select mathematical tasks themselves rather than following a set up in a book.

**Analysis of data from classroom observation**

In looking at the data from classroom observations I attempted to identify didactical praxeologies that were a result of the introduction of the digital tool. In the classroom observation data I looked at the teachers' (1) Organisation of space and time, (2) Choice of mathematical tasks and mathematical techniques and physical tools, and (3) Steering of activity through discourse, considering these to be three practical moments of the didactical praxeologies.

In the lessons observed, neither the organisation of space or time nor the choice of mathematical tasks seemed to be dependent on or unique for a classroom where the digital tool of the PC has been introduced. For example, analysis of the time disposition in lessons showed a script with recapping, homework correction, new theory, and then exercises with approximately 50 – 60 % of the lesson time spent with students working alone or in pairs on exercises. Some time however was given to the explanation of the technical aspects of performing the mathematical techniques with the digital tool. This time allocation varied from lesson to lesson.

Deviation from a standard classroom environment without digital tools was observed in the type of tools used by the students and by the teacher and also in the public discourse of the teacher. Choosing the tools for use in the lesson, for the teacher and for the students to carry out mathematical tasks is a didactical task. This is an ongoing task as choices are made in the planning but also in the conduct of the lesson. Two aspects that stood out in the observations were the manner in which the teacher used both the digital tool and the blackboard to support his/her public discourse and the manner, which the teacher referred to and talked about using the digital tool when describing the mathematical techniques to solve the mathematical tasks. This second aspect involves a too broad discussion to take in this paper but will be discussed in the thesis of which this work forms a part.

**Results and discussion of classroom observation**

In the classroom observations the teachers used both the blackboard and the screen, which was connected to the PC to support their public discourse. One feature that emerged frequently in each observed lesson, I term “weaving”. Weaving describes the manner in which the teacher moved between the available tools. Three physical tools were noted to be in use when the teacher was holding public discourse: the blackboard, the PC+screen, and gestures with own body such as tracing out a curve in the air. Each of these tools is used in conjunction with the voice and schemas (cognitive apparatus). It appeared that in prepared sequences of the lessons the digital tool was used but in spontaneous situations, for example when pressed for further explanation, the teacher turned to the blackboard or to gestures.

Discussing this weaving with the teachers, one teacher explained, that “we use what is appropriate in the situation”. Teachers seemed to identify affordances and constraints of each tool. It appeared that an affordance of the blackboard was that it allowed more personal and spontaneous expression by the teacher. It may also be the case that such unplanned use of the digital tool requires a high level of skill and familiarity with the tool and as such this is a constraint of the tool. In a later instance one teacher began to draw a circle on the blackboard freehand but suddenly stopped saying; “I have an excellent tool to do this”, and then drew the circle using the dynamic geometry software on PC screen instead. Also the mathematical tasks in use were standard tasks, which could be solved without the digital tool. Had these tasks been more complex or tasks that required the use of digital tools perhaps the response of the teacher would have been different.

## CONCLUSIONS AND FURTHER DISCUSSION

Returning to the research questions, three features of the didactical praxeologies as specifically pertaining to and “provoked” by the introduction of the digital tool have been identified and discussed: the digital notebook, the digital textbook, and the phenomena of weaving between tools/instruments in the classroom. The two features that are seen as particularly important by the teachers are the digital textbook and the digital notebook. These could be interpreted as examples of instrumentalisation whereby the teacher as user has adapted the tool to his/her usage. In the classroom, the observation of patterns of inter-dependent mediation between physical tools that have been adapted by the teachers, where they weave between blackboard and the digital tool in response to the situation, could be interpreted as observations of schema or expression of instrumentation as in these cases the tool which is thought to be the most appropriate is used.

Can the project implementation described above be modelled as a process of instrumental genesis for the teachers and is such a modelling helpful in gaining an understanding of the situation? Further examination of teacher discourse will provide more information about this possible instrumental genesis process though tentative findings in this report seem to lead in this direction. Some issues to be discussed in relation to such a process are for example: the temporal dimension; if instrumental genesis is a process how is it possible to identify the different stages of this process for the teacher; and also as to which observations would indicate the formation of schema. The notion of instrumental orchestration has been discussed earlier. Is the process of instrumental genesis for the teacher also influenced by some constraining factors? Comments by the teachers indicated that, for the teachers, the process is steered in part on an organisational level by the schooling authorities at school, region, and national levels. Financial and policy support from schooling authorities is necessary for the survival of the ICT project. In the meetings, the school leader was highly supportive of the project and expressed the opinion that when students think it (mathematics) is fun, then they use more time on mathematics and so become better at it. Enrolment in mathematics has also increased dramatically. However, more important to the teachers seemed to be the response of the students. In the categories of justification and evaluation the majority of comments by the teachers concerned student learning and engagement as illustrated by the comment below.

> Teacher 1: Need to give students a challenge. Students are not educated to work in this way. Now they think it is fun. Looking for methods …

For the teachers in this study, the students' response to the new situation appears to influence the teachers' use and adaptation of the digital tool. Such comments as above also indicate that the teachers are aware of their role in orchestrating their teaching to support the instrumental genesis of the student.

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# DYNAMIC GEOMETRY SOFTWARE: THE TEACHER'S ROLE IN FACILITATING INSTRUMENTAL GENESIS

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*In the UK, use of dynamic geometry software (DGS) in classrooms has remained limited. Whilst the importance of the teacher's role is often stated in dynamic geometry research, it has been seldom elaborated. This study aims to address the apparent deficiency in research. By analysing teacher/pupil interactions in a DGS context, the intention is to identify situations and dialogue that are helpful in promoting mathematical thinking. The analysis draws on an instrumental approach to describe such interactions. Elements of instrumental genesis are distinguished in pupils' dialogue and written work which suggest techniques that teachers can employ to facilitate this process.*

***Keywords****: teacher's role, dynamic geometry, instrumental genesis*

## INTRODUCTION

This study aims to elicit teaching techniques that teachers might employ in their classrooms to help pupils engage constructively with dynamic geometry software. Currently DGS has made little impact on classrooms in the UK. Research has tended to focus on elaborating situations of innovative use and student/machine interaction. This study hopes to re-focus on "the teacher dimension" (Lagrange et al, 2003). The author carried out this study in the role of a practitioner-researcher with a high ability year 8 class. Whilst the class cannot be deemed to be representative, nevertheless it is an 'ordinary classroom' and therefore this study can claim to respond to the need for research into how dynamic geometry software is integrated into 'the *regular* classroom' (Gawlick, 2002).

## DGS – A CLASSROOM FAILURE?

Dynamic geometry software (DGS) appears to be following the cycle of oversell and high expectations, ending in limited classroom use identified by Cuban (2001) as a general pattern for technological innovation in education. Research in mathematics education generally presents DGS as a potentially important and effective tool in the teaching and learning of geometry (see for example Holzl, 1996; Marrades and Gutierrez, 2000; Mariotti, 2000). In their survey of geometry curricula, Hoyles et al (2001) state that although most countries seek to integrate ICT into teaching geometry, there is little explicit influence of ICT in classrooms. In the UK, despite recommendations in the Key Stage 3 Mathematics Framework (DfEE, 2001) for using DGS to develop geometrical reasoning, classroom use has remained limited (Ofsted, 2004). Syntheses of research findings generally conclude by favouring the strong potential of ICT but give few explanations for the contrasting poor reality of classroom use (Lagrange et al, 2003).

## THE ABSENT TEACHER

A criticism of educational policy and discourse on ICT is that the predominant focus has been on technology rather than education (Selwyn, 1999). The picture painted by Lagrange et al (2003) of research on ICT within mathematics education is of a field dominated by “publications about innovative use or new tools and applications” where issues of the integration of technology into ordinary classrooms have been largely neglected. In particular, the voice and role of the teacher has been notably absent. DGS is no exception: in his review of research on dynamic geometry software, Jones (2002) suggests that future research could usefully focus on teacher input and its impact, amongst other issues. Although research has begun to examine the role of the teacher in DGS integration, the practices of ordinary teachers in ordinary classrooms remains an area requiring further investigation (Lagrange 2008).

This study was designed with these issues in mind. The instrumental approach, described in the next section, was used to analyse teacher/pupil interactions in order to elicit teaching techniques which might facilitate pupils’ instrumental genesis.

## THEORETICAL BACKGROUND

Instrumental genesis is described as the process by which an artefact is transformed into an instrument by the subject or user (Guin and Trouche, 1999). An artefact is a material or abstract object, given to a subject. An instrument is a psychological construct built from the artefact by the subject internalising its constraints, resources and procedures (Guin and Trouche, 1999). Once the user has achieved instrumentalisation, he is able to reinterpret or reflect on the activity he is engaged in. Drijvers and Gravemeijer (2005) describe instrumental genesis as the “emergence and evolution of utilisation schemes”. A utilisation scheme is a “stable mental organisation” including both technical skills and supporting concepts as a method of using the artefact for a given class of tasks (Drijvers and Gravemeijer, 2005). The interrelation between machine techniques and concepts seems important since Drijvers and Gravemeijer (2005) found that the apparent technical difficulties that students had often had a conceptual background.

The instrumental approach has been mainly developed and applied within the context of computer algebra software (Drijvers and Gravemeijer, 2005) and there remains a question over how general its applicability is. Drijvers and Gravemeijer (2005) cite two examples where the instrumental approach has been applied to DGS. Thus it seems instrumental genesis may be an appropriate tool to analyse observations of student behaviour within a dynamic geometry environment.

## RESEARCH CONTEXT AND METHODOLOGY

This study was conducted as part of a Best Practice Research Scholarship-funded project on using DGS as a resource for teaching geometrical proof. Much of the previous research on DGS has focused on pupils in upper secondary school (Jones, 2000). It has been suggested that more research is needed on the impact of dynamic geometry software on students in lower secondary school (Marrades and Gutiérrez, 2000). The decision to conduct the research with the researcher's year 8 class was partly influenced by this consideration. Since the pupils were in year 8, there was an added advantage that they were not subject to public examinations, the curriculum is less pressurised and therefore ethical considerations about deviating from schemes of work were somewhat reduced. The school in which the research was conducted is a private day school for girls. The research was conducted with the highest attaining set in year 8, containing 23 pupils, with girls expected to achieve levels 7 or 8 at Key Stage 3 [1]. In common with several other research studies, this was seen as an advantage since students judged to be above average in mathematical ability are most likely to be able to engage with proving processes and therefore allow meaningful data collection to take place (Jones, 2000; Marrades and Gutiérrez, 2000).

In this paper, I consider data drawn from a sequence of 5 lessons in which pupils were engaged in investigating a series of construction problems in pairs using Cabri Geometre. The tasks were based upon the Phase 1 and 2 tasks developed by Jones (2000) and were intended to progress in difficulty. Each task consisted of a figure which the pupils were to construct in Cabri so that it remained constant under drag. The methods for constructing a figure were linked and developed from previous problems to encourage the pupils to examine how additional constraints might affect the resultant shape. They were prompted to say what the resultant shape was and, importantly, how did they know? The point of the teaching sequence was to encourage pupils to justify or prove these assertions.

The pupils were asked to choose a construction of their choice and produce a Power-point presentation on why their construction had worked which was presented to the class. Printouts of the pupils' Power-point presentations and audiotape recordings of their presentations to the class form one part of the data collected. During the lessons, the researcher carried an audiotape so that any teacher/pupil interactions would be recorded: these recordings form another part of the data collected. After the lessons, brief field-notes were made on the major events in the lesson.

The initial stage of data analysis concerned the transcription of tape-recordings made during lessons. Using field notes, the tapes were broken down into major events or "episodes" (Bliss et al, 1996). In the sense described by Bliss et al (1996) these episodes had "an internal coherence"; they were complete conversations which allowed the researcher to "interrupt momentarily, for the purpose of analysis, the 'relentless flow of the lesson'". A second stage of analysis involved going through the transcripts and pupils' work making notes, identifying critical incidents that build towards detailed accounts of practices. The final analysis was based on a grounded approach using narrative techniques (Kvale, 1996) which moved back and forth between the theoretical viewpoint developed in the review of literature and the pupils' work and transcribed episodes. Each step in this process eased the transition from emotionally involved participant towards objective observer. Using the concept of instrumental genesis to achieve a "rich and vivid description of events" (Hitchcock and Hughes, 1995), this study hopes to tease out the threads of a tapestry of complex social interactions to see if techniques for promoting mathematical thinking can be discerned in the weave.

## ANALYSIS

From the analysis of data, three teaching techniques emerged for facilitating pupils' instrumental genesis in Cabri. Using excerpts from teacher/pupil dialogue, these techniques are described below, where T represents the teacher throughout.

### Unravelling functional dependency in DGS

In common with other students, Pupils H and C experienced difficulty with specifying where they wanted objects to intersect when attempting to construct two circles sharing the same radius. They constructed the first circle successfully and correctly placed the centre of the second circle on its edge. The difficulty arose when they tried to adjust the size of the second circle so that its edge would pass through the centre of the first circle, thus ensuring that they would share a radius. The problem was that they made it *look* like the edge of the second circle passed through the centre of the first circle rather than specifying to Cabri that the circle should go "By this point" – as the Cabri pop-up phrase suggests if you hover over the required centre point. Although their Cabri drawing looked successful, when it was subjected to a drag-test, the circles changed size in relation to each other instead of maintaining their pattern:

T: Yeahhh. That's it because you see this computer program will only do exactly what you tell it so if you just make it look like it… sort of, yeah. I'm going to be able to change the shape of your circle so if you tell it, look….

*crackle: teacher using the computer to show how the circle can still be messed up. Then creates a new one "by this point" method to show the difference*

T: Ok now try and mess it up, you try and mess it up now mess up one of the other circles yeah… ok so…

*There follows some unintelligible comments and crackling then…*

H: You think a computer's smart but it's not, you can't just sit there and watch it do it for you, you have to know what to do and you have to <u>tell</u> it to do it so it's like a something…. like it's like a lightswitch.

The difficulties that students have in coming to terms with the concept of functional dependency in geometry exemplifies Drijvers and Gravemeijer's (2005) conception of utilisation schemes in which the technical and conceptual elements co-evolve. Pupil H articulates this point very clearly: "you have to know what to do and you have to tell it to do it". Mathematical knowledge is knowing "what to do" and technical knowledge is required in order to tell the computer to do it. The *gap* in H and C's knowledge was an appreciation of the functional dependencies inherent in Cabri: on the one hand, a conceptual gap of the *necessity* of specifying the required geometrical relationship and, on the other hand, a gap in the technical knowledge of *how* to specify the relationship using Cabri. The teacher explains the *need* to specify the geometrical relationship: the "computer program will only do exactly what you tell it". The teacher goes on to illustrate the technical knowledge of *how* to specify the relationship by contrasting the construction 'by eye', which could still be messed-up, to the "by this point" version in which the geometrical relationships remained intact.

Pupil K had similar difficulties to H and C: although she seemed to be clear about how the circle should be positioned, she appeared unaware of the necessity to specify to Cabri that the circle should go "By this point". Again the teacher makes the technical elements explicit:

> Ok. Keep your hand …[K: uhuh] yeah? So if you actually put it on the point and say I want it "by this point" that's how the comp… that's the only bit of IT you're using. [K: But that's…] That's the only knowledge…IT knowledge you've used. And really then you've had to tell it to do that haven't you?

In this case, the teacher is more direct in making the functional dependencies explicit, by guiding the pupil's construction and referring to the software language "by this point". The teacher even describes this technical knowledge of how to specify the relationship as "IT knowledge", unravelling it from the mathematical knowledge of the geometrical relationship. The teacher again refers to the conceptual necessity of specifying the relationship: "you've had to tell it to do that". Drijvers and Gravemeijer (2005) describe instrumental genesis as the "emergence and evolution of utilisation schemes, in which technical and conceptual elements co-evolve". The role of the teacher in supporting instrumental genesis is partly in making the technical and conceptual elements explicit. In the case of dynamic geometry software such as Cabri, the role of the teacher is to unravel the notion of functional dependency by highlighting the *necessity* of specifying the required geometrical relationship and the technical knowledge of *how* to specify the relationship.

## Exploiting dynamic variation to highlight geometric invariance

All the figures presented to the pupils for construction were based on the initial construction of a line which was apparently horizontal. Of course, there is no geometrical reason for the line to be horizontal, the figures had been presented in this way purely for neatness and it had not been given a second thought, until the teacher noticed that all students appeared to be constructing *intentionally* horizontal lines. The pupils had discovered that by pressing the "shift" key whilst constructing a line, the line would snap to the horizontal. According to the pupils, a similar feature of "snapping to a grid" occurs in a piece of completely unrelated software, which was how the discovery was made. Pupil K was insistent that the line should be horizontal:

T: Why do you always insist on that being horizontal? Does it matter if it….

The teacher draws attention to the pupil's misconception and, by dragging, attempts to convey that the horizontal constraint is artificial, that it can be broken without disturbing the figure under construction. As the pupils were presenting their work to the class, it became clear that all groups had produced figures with horizontal lines. The teacher again attempted to question this feature of their constructions but this time in a whole class context. Pupil MC was asked to reconstruct her solution to Problem 2 (a perpendicular bisector) without starting from a horizontal line. She did this successfully on an interactive whiteboard so that the whole class could see. She then dragged the figure, directed by the teacher, changing its orientation to show its invariance, including the situation with the initial line being horizontal. The teacher exploits dynamic variation to highlight the geometric invariance of the construction in order to help pupils differentiate between geometrical relationships which were or were not crucial.

A similar situation occurred when a pair of pupils, MC and ML, successfully completed the construction leading to a square (Problem 4). They both excitedly told the teacher that the shape they had produced was a diamond. The teacher dragged their construction so that the base of the shape was horizontal, at which point they both concurred that the shape was a square. Upon dragging it back to the original position, ML in particular returned to her previous statement that it was a diamond. Repeated dragging, more and more slowly to emphasise the continuous 'transformation' of the shape, convinced both students that the shape was, in fact, always a square. Again the teacher's strategy is to demonstrate the potential of the software, by exploiting dynamic variation to demonstrate the invariance of the constructed shape. Recognising the potential of the software and making its affordances explicit to pupils is a key element in supporting instrumental genesis.

## Making connections between DGS and pencil-and-paper

Pupil N had constructed a rhombus but, as in the examples in the previous paragraph, had difficulty identifying the shape due to its unfamiliar orientation. The teacher employs dynamic variation to convince pupil N that the shape is indeed a rhombus but then continues the explanation on paper:

N: Is this a rhombus? But a rhombus supposed to be like tilted so…?

*Teacher manipulating the diagram on screen*

N: Oh so it can be, it can be any way up and it [T: Oh!] would still be a rhombus.

T: Well yeah… [N to another pupil: Well it is a rhombus.] it's like, look, this is a well no that's not. This a rectangle isn't it? Ok, it's still a rectangle. It's still a rectangle. However much I turn it, it's still a rectangle. Yeah, ok?

*Diagram of rectangle drawn on paper and then the paper twisted and turned as a demonstration that orientation doesn't alter the shape.*

Guin and Trouche (1999) suggest that teachers should highlight the constraints and limitations of the software to students in order to support their instrumental genesis. In these cases, the teacher is in fact using the dynamic nature of the Cabri software to highlight the constraints and limitations of the paper-and-pencil environment, exposing a misconception and thereby supporting the pupils' instrumental genesis in the more traditional medium. In the case of the tilted rhombus, the teacher sketched a rectangle on paper in order to further illustrate the concept that orientation does not affect the nature of the shape. This sketch was done on paper at the time mainly because it was quicker than constructing the shape on Cabri. The teacher's return to the paper-and-pencil environment is important because it makes a connection between the two environments: although dynamic variation makes it easier to appreciate that orientation does not affect the shape, the concept still holds in a paper-and-pencil environment. The return to paper-and-pencil is thus an attempt by the teacher to "build connections with the official mathematics outside the microworld", a responsibility which Guin and Trouche (1999) identify as being a crucial part of the teacher's role.

## DISCUSSION

From the sequence of lessons, three teaching techniques have been distilled that serve to facilitate pupils' instrumental genesis in a DGS context. These techniques are clearly not exhaustive: exploiting anomalies of measurement in Cabri such as rounding errors might be another way to promote mathematical thinking, for example. These techniques are specific to DGS in general and Cabri Geometre. They are also analogous to teaching techniques used in other contexts. Guin and Trouche (1999) suggest that teachers should highlight the constraints and limitations of the software to students: in the case of Derive, the discrete and finite nature of the software. Similarly, a dynamic geometry environment such as Cabri is only a discrete model of Euclidean geometry, despite its continuous appearance. All tools and resources have constraints and limitations. In the case of paper and pencil, a limitation is the static nature of the environment. Thus techniques such as those identified in this paper may apply to any teaching resource. In a sense, the teaching techniques mentioned here essentially highlight general principles of mathematics teaching applied to a specific context, in this case DGS. The resource provides a context for learning but cannot teach. The focus of research needs to shift away from the context, towards teachers and the teaching techniques they may employ in order to aid pupils' instrumental genesis. In this way research on ICT may avoid the criticism that the predominant focus has been on technology rather than education.

## NOTES

1. Key Stage 3 covers the first three years of secondary schooling in England: Year 7 (age 11-12), Year 8 (age 12-13) and Year 9 (age 13-14). Average attainment at the end of KS3 is at level 5/6. Level 8 is the highest level possible in maths at KS3.

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# INSTRUMENTAL ORCHESTRATION: THEORY AND PRACTICE

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*The paper concerns the way teachers use technological tools in their mathematics lessons. The aim is to investigate the explanatory power of the theory of instrumental orchestration through its confrontation with a teaching episode. An instrumental orchestration is defined through a didactical configuration, an exploitation mode and a didactical performance. This model is applied to a teaching episode on the concept of function, using an applet embedded in an electronic learning environment. The results suggest that the instrumental orchestration model is fruitful for analysing teacher behaviour, particularly in combination with additional theoretical perspectives.*

## INTRODUCTION

The integration of technological tools into mathematics education is a non-trivial issue. More and more, teachers, educators and researchers are aware of the complexity of the use of ICT, which affects all aspects of education, including the didactical contract, the working formats, the paper-and-pencil skills and the individual and whole-class conceptual development.

A theoretical framework that acknowledges this complexity is the instrumental approach (Artigue, 2002). According to this perspective, the use of a technological tool involves a process of *instrumental genesis*, during which the object or artefact is turned into an instrument. The instrument, then, is the psychological construct of the artefact together with the mental schemes the user develops for specific types of tasks. In such schemes, technical knowledge and domain-specific knowledge (in our case mathematical knowledge) are intertwined. Instrumental genesis, in short, involves the co-emergence of mental schemes and techniques for using the artefact, in which mathematical meanings and understandings are embedded.

Many studies focus on the students' instrumental genesis and its possible benefits for learning (e.g., see Kieran & Drijvers, 2006). However, it was acknowledged that instrumental genesis needs to be monitored by the teacher through the orchestration of mathematical situations. In order to describe the management by the teacher of the individual instruments in the collective learning process, Trouche (2004) introduced the metaphorical theory of instrumental orchestration.

Until today, however, the number of elaborated examples of instrumental orchestrations is limited. Therefore, the aim of this paper is to investigate the explanatory power of the theory of instrumental orchestration through its confrontation with a teaching episode. As such, this contribution can be situated in the intersection of themes 2 and 3 of Cerme6 WG7: it concerns the interaction between resources or artefacts and teachers' professional practice, in which students use tools in their mathematical activity.

In the following, we first define instrumental orchestration. Then a description of a classroom teaching episode in which a technological tool plays an important role is provided. The episode is analysed in terms of the theory. This is followed by a reflection on the application and the conclusions which we have drawn.

## INSTRUMENTAL ORCHESTRATION: A THEORETICAL MODEL

The theory of instrumental orchestration is meant to answer the question of how the teacher can fine-tune the students' instruments and compose coherent sets of instruments, thus enhancing both individual and collective instrumental genesis.

An *instrumental orchestration* is defined as the intentional and systematic organisation and use of the various artefacts available in an – in our case computerised – learning environment by the teacher in a given mathematical task situation, in order to guide students' instrumental genesis. An instrumental orchestration in our view consists of three elements: a didactic configuration, an exploitation mode and a didactical performance.

1. *A didactical configuration* is an arrangement of artefacts in the environment, or, in other words, a configuration of the teaching setting and the artefacts involved in it. These artefacts can be technological tools, but the tasks students work can be seen as artefacts as well.

   In the musical metaphor of orchestration, setting up the didactical configuration can be compared with choosing musical instruments to be included in the orchestra, and arranging them in space so that the different sounds result in the most beautiful harmony.

2. *An exploitation mode* of a didactical configuration is the way the teacher decides to exploit it for the benefit of his didactical intentions. This includes decisions on the way a task is introduced and is worked on, on the possible roles of the artefacts to be played, and on the schemes and techniques to be developed and established by the students.

   In the musical metaphor of orchestration, setting up the exploitation mode can be compared with determining the partition for each of the musical instruments involved, bearing in mind the anticipated harmonies to emerge.

3. *A didactical performance* involves the ad hoc decisions taken while teaching on how to actually perform the enacted teaching in the chosen didactic configuration and exploitation mode: what question to raise now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool?

> In the musical metaphor of orchestration, the didactical performance can be compared with a musical performance, in which the actual inspiration and the interplay between conductor and musicians reveal the feasibility of the intentions and the success of their realization.

The model for instrumental orchestration initially was developed by Trouche (Trouche 2004) and included the first and the second points above, i.e. the didactical configuration and the exploitation mode. As an instrumental orchestration is partially prepared beforehand and partially created 'on the spot' while teaching, we felt the need for a third component reflecting the actual performance. Establishing the didactical configuration has a strong preparatory aspect: often, didactical configurations need to be thought of before the lesson and cannot easily be changed during the teaching. Exploitation modes may be more flexible, whereas didactical performance has a strong ad hoc aspect. Our threefold model thus has a time dimension.

The model also has a structural dimension: an instrumental orchestration on the one hand has a structural, global component in that it is part of the teacher's repertoire of teaching techniques (in the sense of Sensevy *et al*. 2005) and can be reflected in operational invariants of teacher behaviour. On the other hand, an instrumental orchestration has an incidental, local actualisation appropriate for the specific didactical context and adapted to the target group and the didactical intentions.

The instrumental orchestration model brings about a double-layered view on instrumental genesis. At the first level, instrumental orchestration aims at enhancing the students' instrumental genesis. At the second level, the orchestration is instrumented by artefacts for the teachers, which may not necessarily be the same artefacts as the students use. As such, the teacher himself is also involved in a process of instrumental genesis for accomplishing his teaching tasks (Bueno-Ravel & Gueudet, 2007).

In literature, the number of elaborated examples of instrumental orchestrations is limited. Trouche (2004) and Drijvers & Trouche (2008) describe a so-called Sherpa orchestration. Kieran & Drijvers (2006), without mentioning this orchestration explicitly, describe an instrumental orchestration of short cycles of individual work with the artefact and whole-class discussion of results.

## THE CASE OF TWO VERTICALLY ALIGNED POINTS

The case we describe here stems from a research project on an innovative technology-rich learning arrangement for the concept of function[1]. In this project, a learning arrangement for students in grade 8 was developed, aiming at the development of a rich function concept. This includes viewing functions as input-output assignments, as dynamic processes of co-variation and as mathematical

[1] For further information on the project see Drijvers, Doorman, Boon, Van Gisbergen & Gravemeijer (2007) and the project website www.fi.uu.nl/tooluse/en/.

objects with different representations. The main technological artefact is an applet called AlgebraArrows embedded in an electronic learning environment (ELO). The applet allows for the construction and use of chains of operations, and options for creating tables, graphs and formulae and for scrolling and tracing. A hypothetical learning trajectory, in which the expected instrumental genesis is sketched, guided the design of the student materials. An accompanying teacher guide contained suggestions for orchestrations.

After group work on diverse problem situations involving dependency and co-variation, the notion of arrow chains is introduced to the students. In the third and fourth lessons, students work with arrow chains in the ELO. One of the tasks of the fourth lesson, which some of the students did at home, is task 8, shown in Figure 1.

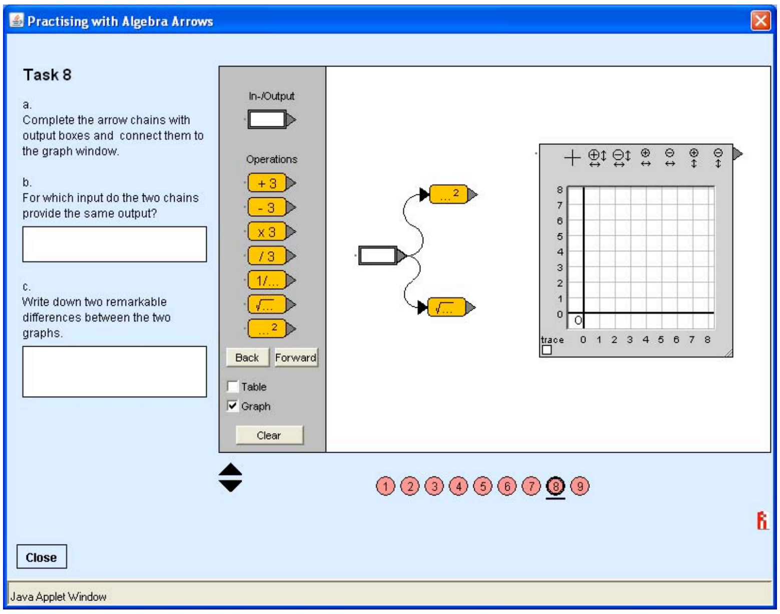


**Figure 1 Computer task 8**

At the right of Figure 1 is the applet window, which in this task contains the start of the square and the square root chain, and an empty graph window. At the left you see the tasks and two boxes in which the students type their answers. The numbered circles at the bottom allow for navigation through the tasks.

The following verbatim extract describes the way the teacher discusses this task during the fifth lesson.

Using a data projector, the ELO with the list of student pairs is projected on the wall above the blackboard. The teacher T navigates within this list to Tim and Kay's solution for task 8.

T: It says here [*referring to question c*]: what do you notice? Oh yes, I actually wanted to see quite a different one, because they had …

T navigates to Florence and her classmate's work. The Table option is checked. That leads to 'point graphs' on the screen. The students' answer to question c reads:

"For the square they are all whole numbers, and for the square root they are whole numbers and fractions. And the square of a number is always right above the root. ?"

| | |
|---|---|
| T: | Look here, what this says. [*indicates the students' answer of question c on the screen with the mouse*] For the square they are all whole numbers, okay, and for the square root they aren't whole numbers, we agree with that too, and the square of a number is always right above the square root. |
| F(lorence): | Was that right? |
| T: | I'm not saying. |
| St1[2]: | Yes, I had that too. |
| T: | What they say, then, is that every time there is…if I've got something *here*, there is something above it, and if I've got something *there*, there is also something above it. [*points vertically in the graph with the mouse*] Why is that, that these things are right above each other? |
| F: | Well, because it…the square root is just…no the square is just, um, twice the root, or something. |
| St2: | No. |
| T: | Kay? |
| Kay: | That's because the line underneath, that's got a number on it, which you take the square root of and square, so on the same line anyway. |
| T: | What are those numbers called that are on the horizontal line then? |
| St3: | The input numbers. |
| T: | The input numbers. |
| T: | Ehm, Florence, did you follow what Kay said? |
| F: | No, but I […]. It was about numbers and about square roots and about… |
| Sts: | [*laughs*] |
| St: | It was about numbers! |
| T: | Kay said: these are the input numbers, here on the horizontal line. [*indicates the points on the horizontal axis with the mouse*] And for an input number you get an output number. And that is right above it. So if you take the same input number for two functions… [*indicates the two arrow chains with the mouse*] |
| F: | Oh yes. |

[2] St1, St2, .. stands for one of the students

| | |
|---|---|
| T: | … then you also get…then you get points above it. So that's got nothing at all to do with the functions. It's just got to do with from which number you are going to calculate the output value. Now, if for both of them you calculate what the output value is for 10, they both get a point above the 10 [indicates on the screen with the mouse]. Do you understand that? |
| F: | Oh yes, I didn't know that. |
| T navigates back to the list of student pairs. | |

Figure 2 shows the work of Florence and her classmate on this task in Dutch at the end of the teaching sequence. They changed their answer to question c into: "for the square they are always whole numbers, and for the square root they are whole numbers and fractions. The squares get higher with much bigger steps."

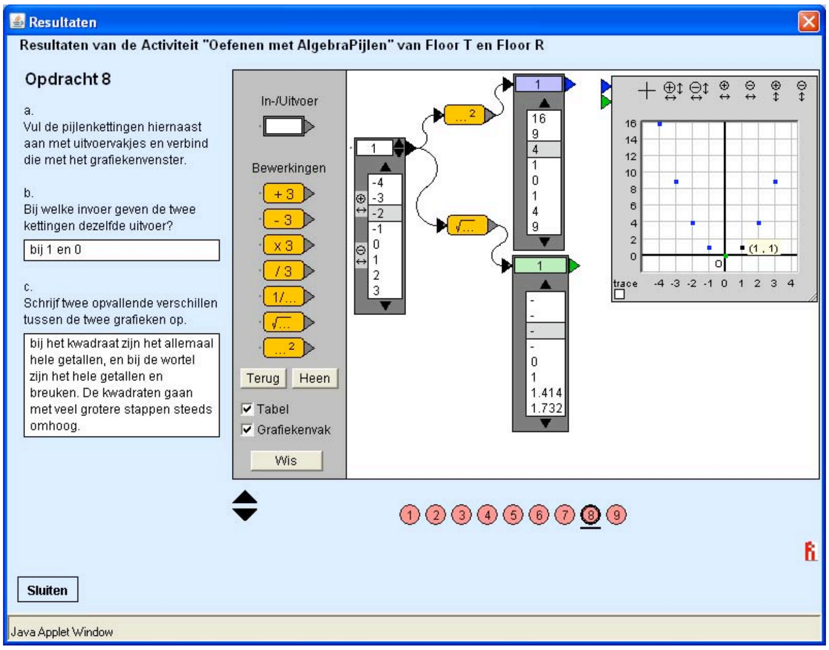


**Figure 2 Revision of the answer after whole class discussion**

## APPLYING THEORY TO PRACTICE

In this section we apply the theory of instrumental orchestration to the above teaching episode, which essentially reflects the teacher's way to treat a misconception of (at least) one of the students, whose use of the Table-Graph technique leads to thinking it is 'special' that two points reflecting function values for the same input value are vertically aligned.

Let us call the instrumental orchestration the teacher puts into action the 'spot and show orchestration'. By 'spot' we mean that the teacher, while preparing the lesson, spotted the students' work in the ELO and thus came across Florence's misconception. The 'show' refers to the teacher's decision to display Florence's results as a starting point for the whole-class discussion of item 8c. The teacher's phrase “Oh yes, I actually wanted to see quite a different one” and her straight navigation to Florence’s work reveal her deliberate intention to act the way she does.

The *didactical configuration* for the preparatory phase consists of the ELO’s option for teachers to look at the students’ work at any time. As a result, the teacher notices the misconception and decides to deal with it in her lesson. This preparation is instrumented by ELO-facilities that are not available for students. In this sense, the teacher’s artefact is different from the students’ artefact. For the classroom teaching, the configuration includes a regular classroom with a PC with ELO access, connected to a data projector. Apparently, the teacher finds the computer lab not appropriate for whole-class teaching. The configuration includes putting the computer with the data projector in the centre of the classroom. This choice is driven by the constraints of one of the artefacts: if the projector was at the front, the projection would get too small for the students to read. The screen is projected on the wall above the blackboard, thus enabling the teacher to write on the blackboard, which she regularly does – though not in the episode presented here. Both the way of preparing the lessons and the setting in the classroom are observed more often in this teacher’s lessons.

The *exploitation mode* of this configuration includes that teacher’s choice to operate the PC herself. These two aspects of the exploitation mode result in the teacher standing in the centre of the classroom, with the students closely around her, all focused on the screen on the wall. From these and other observations, we conjecture that this exploitation mode enhances classroom discussion and student involvement. Observations of another teacher using the same orchestration in a less convenient setting support this conjecture.

The *didactical performance* starts with the teacher reading the student’s answer with some minor comments (“Look here, …”). Then she reformulates the answer and asks Florence for an explanation (“What they say…”). When the explanation turns out to be inappropriate, she makes Kay give his explanation, and checks whether Florence understands it. When this is not the case, the teacher rephrases Kay’s explanation and once more checks it with Florence, who now says she understands. Of course, this didactical performance might be different a next time. For example, Florence could be asked to explain her understanding in her own words.

Now how about the link between instrumental orchestration and instrumental genesis? As the episode does not show students using the artefact, we do not see direct traces of the students’ instrumental genesis. We do claim, however, that Florence’s idea of two vertically aligned points being special is part of her scheme of using the TableGraph technique to produce point graphs. Even though this is a misconception, the episode shows that the teacher can exploit the students’ experiences, and those of Florence and Kay in particular, for the purpose of attaching mathematical meaning to the technique they used, which leads to a convergence in a shared function conception in class. We see the development of mathematical meanings of techniques as an important aspect of instrumental genesis.

This ‘spot&show’ orchestration was one of the options suggested in the teacher guide accompanying the teaching sequence. This teacher used it quite often, whereas she felt free to neglect other suggestions made in the teacher guide. In the post-experiment interview, she indicated to really appreciate the possibility to get an overview of students’ results while preparing the lesson: “The ELO is practical to see what students do, you can adapt your lesson to that.” She seemed to see this ‘spot&show’ orchestration as a means to enhance student involvement and discussion, which she believed to be relevant and seem to be part of her operational invariants . We do not have data, however, that confirm such operational invariants across other teaching settings.

Finally, an interesting aspect of the teacher’s own instrumental genesis is worth discussing. The teacher points with her mouse on the screen, but does not really make changes in the students’ work. Other observations suggest that she doesn’t do so because she is afraid that such changes will be saved and thus affect the students’ work. When she learns that this is not the case as long she uses her teacher login, she benefits from the freedom to demonstrate other options and to investigate the consequences of changes. This behaviour is instrumented by the facilities of the artefact that she initially was not aware of.

## REFLECTION ON THE THEORY AND THE CASE

Let us briefly reflect on the application of the theory of instrumental orchestration to the data presented above. A first remark is that the three elements of the model – didactical configuration, exploitation mode and didactical performance – allow for a distinction and an analysis of the relevant issues within the orchestration, and their interplay. As such, the model offers a useful framework for describing the orchestration by the teacher.

As a second remark, however, we notice that it is not always easy to decide in which category something that is considered relevant should be placed. For example, does the fact that the teacher operates the computer herself belong to the didactical configuration or to the exploitation mode? This probably is a matter of granularity: if we study the ‘spot & show’ orchestration, this is part of the exploitation mode. If the focus of the analysis is on students’ activity, it might be identified as a didactical configuration issue.

A third reflection is that the model has the advantage of fitting with the instrumental approach of students learning while using tools. This has proved to be a powerful approach (Artigue, 2002; Kieran & Drijvers, 2006), and it is therefore of great value having a framework for analysing teaching practices that is consistent with it. As such, instrumentation and orchestration form a coherent pair. In terms of instrumentation, we notice that the teachers’ tasks, artefacts and techniques are not the same as those of the students; still, we can use a similar framework for analysis and interpretation.

The time dimension in the model – ranging from the didactical configuration having a strong preparatory character to the didactical performance with its strong ad hoc character – comes out clearly in the model. For the structural dimension, this is not as straightforward. As a fourth remark, therefore, we notice that operational invariants of the teacher are not limited to the preparatory phases, but also emerge in the performance. For example, the wish to have students explain their reasoning to each other appears as an operational invariant for this teacher, which is more explicit in the performance than in the configuration or in the exploitation mode. As an aside, we are aware that the data presented here do not allow for full identification of the teacher's operational invariants. More observations over time need to be included.

## CONCLUSION

As far as this is possible from the one single, specific exemplary case study presented in this paper, we conclude that the model of instrumental orchestration can be a fruitful framework for analysing teachers' practices when teaching mathematics with technological tools. As it is important for teachers to develop a repertoire of instrumental orchestrations, more elaborated examples are needed. Such examples could not only help us to better understand teaching practices, but could also enhance teachers' professional development.

In addition to the need to find and elaborate exemplary orchestrations, a second challenge is to link the theory of instrumental orchestration with complementary approaches. Lagrange (2008), for example, uses additional models provided by Saxe (1991) and Ruthven and Hennessey (2002) to identify and understand teaching techniques. Another interesting perspective concerns the alternation of teacher guidance and student construction, as described by Sherin (2002). In short, the instrumental orchestration approach is promising, but needs elaboration and integration with other perspectives. For the moment, its descriptive power seems to be more important than its explanatory power.

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# TEACHING RESOURCES AND TEACHERS' PROFESSIONAL DEVELOPMENT:

# TOWARDS A DOCUMENTATIONAL APPROACH OF DIDACTICS

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*In this paper we propose a theoretical approach of teachers' professional development, focusing on teachers' interactions with resources, digital resources in particular. Documents, entailing resources and schemes of utilization of these resources, are developed through documentational geneses occurring along teachers' documentation work (selecting resources, adapting, combining, refining them). The study of teachers' documentation systems permits to seize the changes brought by digital resources; it also constitutes a way to capture teachers' professional change.*

## INTRODUCTION

We present in this paper the first elements of a theoretical approach elaborated for the study of teachers' development, and in particular teachers ICT integration.

Technology integration, and the way teachers work in technology-rich environments, have been extensively researched, and discussed at previous CERME conferences (Drijvers *et al.*, 2005, Kynigos *et al.*, 2007). Ruthven's presentation at CERME 5 drew attention on the structuring context of the classroom practice, and on its five key features: *working environment, resource system, activity format, curriculum script, time economy* (Ruthven, 2007). This leads in particular to consider ICT as part of a wider range of available teaching resources. This view also fits technological evolutions: most of paper material is now at some point in digital format; teachers exchange digital files by e-mail, use digital textbooks, draw on resources found on websites etc. Considering ICT amongst other resources raises the question of connections between research on ICT and resources-oriented research.

Many research works address the use of *curriculum material* (Ball & Cohen, 1996; Remillard, 2005). They observe the influence of such material on the enacted curriculum, but also highlight the way teachers shape the material they draw on, introducing a vision of "curriculum use as participation with the text" (Remillard, 2005, p.121). Other authors consider more general resources involved in teaching: material and human, but also mathematical, cultural and social resources (Adler, 2000). They analyze the way teachers interpret and use the available resources, and the consequences of these processes on teachers' professional evolution.

Such statements sound familiar for researchers interested in ICT, who "consider not only the ways in which digital technologies shape mathematical learning through novel infrastructures, but also how it is shaped by its incorporation into mathematical learning and teaching contexts" (Hoyles & Noss, 2008, p. 89). Conceptualization of these processes is provided by the *instrumental approach* (Guin *et al.*, 2005) and by the work of Rabardel (1995) grounding it; this theoretical frame has contributed to set many insightful results about the way students learn mathematics with ICT. Further refinements of this theory have led to take into account the role of the teacher and her intervention on students instrumental geneses, introducing the notion of *orchestration* (Trouche, 2004). Considering instrumental geneses for teachers has been proposed in the context of spreadsheets (Haspekian, 2008) and e-exercises bases (Bueno-Ravel & Gueudet, 2007). These refinements can be considered as first steps towards the introduction of concepts coming from the instrumental approach and illuminating the interactions between teachers and ICT.

Thus connections between studies about the use of teaching resources, and studies about the way in which teachers work in a technology-rich environment exist; however, elaborating a theoretical frame encompassing both perspectives requires a specific care. We present here an approach designed for this purpose, and aiming at studying teachers' documentation work: looking for resources, selecting, designing mathematical tasks, planning their order, carrying them out in class, managing the available artefacts, etc. We take into account teachers' work in class, but also their (too often neglected) work out of class.

We draw on the theoretical elements evoked above, but also on field data. Some of these data come from previous research in which we were engaged: particularly about use of e-exercises bases (Bueno-Ravel & Gueudet, 2007), and about an in-service training design, the SFoDEM (Guin & Trouche, 2005). Other data were specifically collected: we have set up a series of interviews with nine secondary school teachers. We chose teachers with different collective involvements, different institutional contexts and responsibilities, and different ICT integration degrees (Assude, 2007). We met them at their homes (where, in France and for secondary teachers, most of their documentation work takes place), and asked them about their uses of resources, and the evolution of these ways of use. We observed the organization of their workplaces at home, of their files (both paper and digital), and collected materials they designed or used. The analyses of these data contributed to shape the concepts; in this paper we only use them to display illustrations of the theory. All the interviews took place in France; thus the national context certainly influences the results we display. We hypothesize nevertheless that the concepts exposed are likely to illuminate documentation work in diverse situations.

We present here the elementary concepts of this theory, introducing in particular a distinction between *resources* and *documents*, and the notion of *documentational genesis*. We also expose the specific view of professional evolutions it entails.

## RESOURCES, DOCUMENTS, DOCUMENTATIONAL GENESES

The instrumental approach (Rabardel, 1995, Guin *et al.*, 2005) proposes a distinction between *artefact* and *instrument.* An artefact is a cultural and social means provided by human activity, offered to mediate another human activity. An instrument comes from a process, named *instrumental genesis*, along which the subject builds a *scheme* of utilization of the artefact, for a given class of situations. A scheme, as Vergnaud (1998) defined it from Piaget, is an *invariant organization of activity* for a given class of situations, comprising in particular rules of action, and structured by *operational invariants*, which consist of implicit knowledge built through various contexts of utilization of the artefact. Instrumental geneses have a dual nature. On the one hand, the subject guides the way the artefact is used and, in a sense, *shapes* the artefact: this process is called *instrumentalization*. On the other hand, the affordances and constraints of the artefact influence the subject's activity: this process is called *instrumentation*. We propose here a theoretical approach of teaching resources, inspired by this instrumental approach, with distinctive features that we detail hereafter, and a specific vocabulary.

We use the term *resources* to emphasize the variety of the artefacts we consider: a textbook, software, a student's sheet, a discussion with a colleague etc. A resource is never isolated: it belongs to a set of resources. The subjects we study are teachers. A teacher draws on resources sets for her documentation work. A genesis process takes place, bearing what we call *a document*. The teacher builds schemes of utilization of a set of resources, for the same class of situations, across a variety of contexts. The formula we retain here is:

*Document = Resources + Scheme of Utilization.*

A document entails, in particular, operational invariants, which consist of implicit knowledge built through various contexts of utilization of the artefact, and can be inferred from the observation of invariant behaviors of the teacher for the same class of situations across different contexts.

Figure 1 represents a *documentational genesis*. The instrumentalization process conceptualizes teacher appropriating and reshaping resources, and the instrumentation process captures the influence, on the teacher's activity, of the resources she draws on.

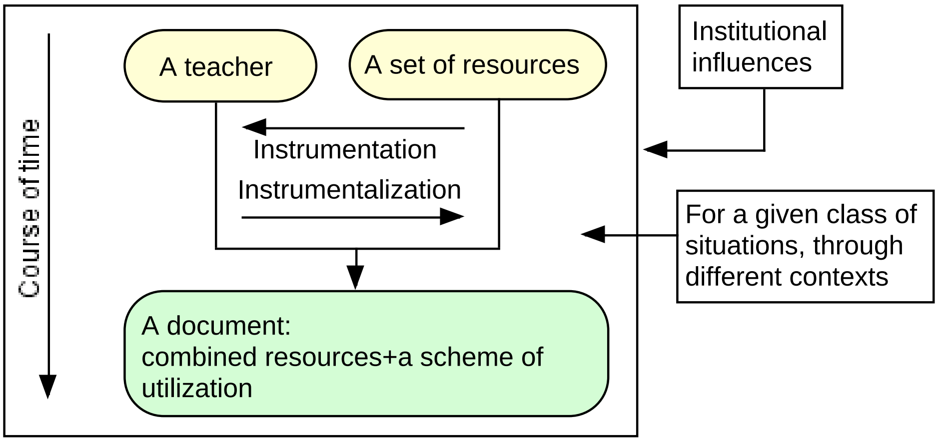



**Figure 1. Schematic representation of a documentational genesis**

## DOCUMENTATIONAL GENESES: TWO ILLUSTRATIVE EXAMPLES

We use a first case study (figure 2) coming from our interviews to illustrate the distinction between a set of resources and a document.

Marie-Pierre (aged 40, involved in collective work within an IREM[1] group; no institutional responsibilities, strong degree of ICT integration) is teaching at secondary school for 14 years, from grade 6 to 9. She uses dynamic geometry systems, spreadsheets, and many online resources (e-exercises and mathematics history websites in particular). She has a digital version of the class textbook. Marie-Pierre has an interactive whiteboard in her classroom for three years, and uses it in each of her courses. For the introduction of the circle's area in grade 7, she starts in class by using a website comprising historical references (Archimedes using circular sections to link the perimeter and the area of a circle) and displaying an animation of the circle unfolding and transforming into a triangle (roughly, but that point is not discussed). Then she presents her own course, based on an extract of the class digital textbook. She complements as usual the files displayed on the whiteboard by writing additional comments and explanations, highlighting important expressions etc.

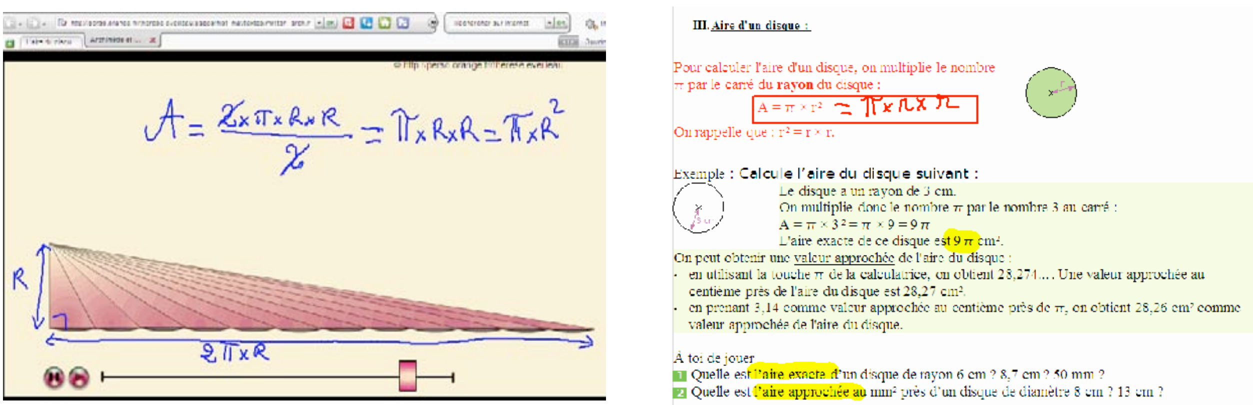



**Figure 2. Marie-Pierre, example of a lesson introducing the circle's area**

For the class of situations: "design and implement the introduction of the circle's area in grade 7" (figure 2), Marie-Pierre draws on a set of resources comprising the

[1] Institute for Research on Mathematics Teaching.

interactive whiteboard, a website[2], a digital textbook, and a hard copy of it. The official curriculum texts, about the circle area, only state that “an inquiry-based approach permits to check the area formula”, with no more details. The digital textbook proposes an introductory activity with a digital geometry software: drawing circles, and displaying their areas. Several radius are tested, the radius square and the corresponding area are noted by the students in a table, and they are asked to observe that they obtain an (approximate) ratio table. But Marie-Pierre prefers to draw on the website animated picture (both choices correspond more to an observation activity for the students than to an inquiry-based approach, but we will not discuss this aspect here). So, we claim that she has developed a scheme of utilization of this set of resources, structured by several operational invariants. These invariants are professional beliefs that we infer from our data:

-“A new area formula must be justified by an animation showing a cutting and recombining of the pieces to form a figure whose area is known”. This operational invariant concerns all the areas introduced, it also intervenes in the document corresponding to the introduction of the triangle’s area for example;

-“The circle’s area must be linked with a previously known area: the triangle”; “The circle’s area must be linked with the circle’s perimeter”. These operational invariants are related with the precise mathematical content of the lesson, they were built along the years, with different grade 7 classes (Marie-Pierre uses this website’s animation for three years, with two grade 7 classes each year).

We do not assert that these operational invariants were not present among Marie-Pierre’s professional knowledge before her integration of the interactive whiteboard. But the possibility to display an animation on a website, to complement it by writing additional explanations, to go back to a previous state of the board to link the “official” formula with what has been observed, yielded a document integrating these operational invariants. And we claim that the development of this document is likely to reinforce, in particular, the above presented beliefs. The operational invariants are both driving forces and outcomes of the teacher’s activity.

Documentational geneses are ongoing processes; we use a second case study (figure 3) to emphasize this important aspect. Rabardel & Bourmaud (2005) claim that the design *continues in usage*. We consider here accordingly that a document developed from a set of resources provides new resources, which can be involved in a new set of resources, which will lead to a new document etc. Because of this process, we speak of a *dialectical* relationship between resources and documents.

[2] http://pagesperso-orange.fr/therese.eveilleau/pages/hist_mat/textes/mirliton.htm

Marie-Françoise (aged 55; involved in collective work within an IREM group, institutional responsibilities as in-service teacher trainer, strong degree of ICT integration) works with students from grade 10 to 12. She organizes for them 'research narratives': problem solving sessions, where students work in groups on a problem and write down their own 'research narratives' (both solutions and research processes). Thus one class of situations, for Marie-Françoise is 'elaborating open problems for research narratives sessions'. For this class of situations, she draws on a set of resources comprising various websites, but also personal existing resources, colleagues' ideas, etc. Students' ideas constitute a major resource for Marie-Françoise, as she told us: "There is the problem and the way you enact it, because students are free to invent things, and afterwards we benefit from the richness of all these ideas, and you can build on it". The design clearly goes on in class. Moreover, the class sessions provide new resources: the students' research narrative, that Marie-Françoise collects, and saves in a new binder, aiming to enrich the next document built on the same open problem.

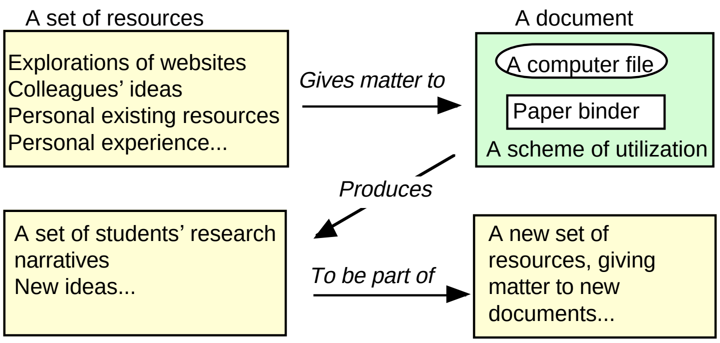



**Figure 3. An illustration of the resources/document dialectical relationship**

The resources evolve, are modified, combined; documents develop along geneses and bear new resources (figure 3) etc. We consider that these processes are part of teachers' professional evolutions, and play a crucial role in them.

## DOCUMENTATION SYSTEMS AND PROFESSIONAL DEVELOPMENT

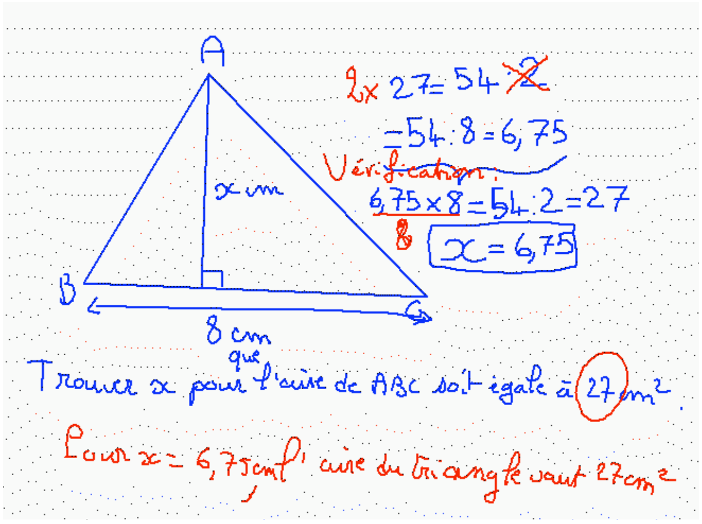
According to Rabardel (2005), professional activity has a double dimension. Obviously a productive dimension: the outcome of the work done. But the activity also entails a modification of the subject's professional practice and beliefs, within a constructive dimension. Naturally, this modification influences further production processes: the productive/constructive relationship has a dialectical nature.

Teachers' documentation work is the driving force behind documentational geneses, thus it yields productive and constructive professional changes. Literature about teachers' professional change raises the question of the articulation between change of practice and change of knowledge and beliefs. We consider that both are strongly intertwined (e.g., Cooney, 2001). The documentational geneses provide a specific view of this relationship. Working with resources, for the same class of situations across different contexts, leads to the development of a scheme, and in particular of rules of action (professional practice features) and of operational invariants (professional implicit knowledge or beliefs). And naturally these schemes influence the subsequent documentation work. All kinds of professional knowledge are concerned by these processes, the evolutions they generate are not curtailed to curricular knowledge (Schulman, 1986). Thus, studying teachers' documents can be considered as a specific way to study teachers' professional development.

According to Rabardel and Bourmaud (2005), the instruments developed by a subject in his/her professional activity constitute a system, whose structure corresponds to the structure of the subject's professional activity. We hypothesize here similarly that a given teacher develops a structured documentation system.

Let us go back to the example of Marie-Pierre evoked above.

Marie-Pierre keeps all her "paperboards" (digital files with images corresponding to the successive states of the board). She uses these paperboards at the beginning of a new session, to recall what has been written, by herself or by her students, during the preceding session. On her laptop, Marie-Pierre has one folder for each class level. Each of these folders contains one file with the whole year's schedule, and lessons folders for each mathematical theme. The paperboards are inside the lessons folders. The interactive whiteboard screen below corresponds to the introduction of equations in grade 7, in the context of triangles areas.




(*Translation:* Find *x* such that ABC area equals 27 cm$^2$. For $x$ = 6.75cm, the triangle's area is 27 cm$^2$).

**Figure 4. A view on Marie-Pierre's documents**

Marie-Pierre's files organization on her computer (figure 4), and her statements during the interviews, clearly indicate articulations between her documents. The document whose material component is the year schedule naturally influenced her lesson preparations; but on the opposite, the documents she developed for lessons preparations during previous years certainly intervened in the schedule design. Documents corresponding to connected mathematical themes are also connected. For a given lesson, the students' interventions can contribute to generate operational invariants that will intervene in preparations about other related topics.

A teacher's documents constitute a system, whose organization matches the organization of her professional activity. The evolutions of this documentation system correspond to professional evolutions. Integration of new materials is a visible of the professional practice, and of the documentation system (in the approach we propose, this integration means that a new material is inserted in a set of resources involved in the development of a document). When Marie-Pierre integrates the interactive whiteboard in her courses, it entails a productive dimension: she now teaches with this whiteboard. But it also yields other changes of her practice: she makes more links with previous sessions, in particular recalling students productions is now present in her orchestration choices. And it even generates changes in her professional beliefs, for example about the possible participation of students to her teaching. She seems to have developed an operational invariant like: “a good way to launch a lesson is to recall students’ interventions done during the preceding lesson”.

The integration of new material is always connected with professional practice and professional beliefs evolutions. But professional evolutions do not always correspond to integration of new material, and the same is true for documentation systems evolutions. For example, Arnaud (47 years old, no collective involvement, institutional responsibilities as in-service teacher trainer, low degree of ICT integration) presented during his interview “help sheets”, that he designed years ago for students encountering specific difficulties. He now uses the same sheets as exercises for the whole class; thus while no changes can be observed in the material, the action rules associated evolved.

Integration of new material remains an important issue, especially when the focus is on ICT. The study of a given teacher’s documentation system also provides insights in the reasons for the integration or non-integration of a given material. The integration depends indeed on the possibility for this material to be involved in the development of a document, that will be articulated with others within the documentation system. For many years Marie-Pierre prepares her courses as digital files, she uses dynamic geometry software and online resources; the interactive whiteboard articulates with this material. Moreover, Marie-Pierre is convinced of the necessity of fostering students’ interventions, and even of including these in the written courses, and the interactive whiteboard matches this conviction. Possible material articulations are important; but other types of articulations must be taken into account, and the integration of new material also strongly depends on operational invariants, thus on teachers’ professional knowledge and beliefs.

## CONCLUSION

This paper is related with the second theme of WG7: *Interaction between resources and teachers’ professional practice*. It introduces a conceptualization of teachers’ interactions with resources and of the associated professional development. Here we just presented the first concepts of a theory whose elaboration is still in progress. Studying teachers’ documentation work requires to set specific methodologies, permitting to capture their work in and out of class, to precise their professional beliefs, and to follow long-term processes: it is the main goal of our research. We did not discuss here the very important issue of collective documentation work, which causes particular processes: its study raises the question of collective documentational genesis and documentation systems, and raises new theoretical needs. The documentational approach we propose also needs to be confronted with other teaching contexts: primary school, tertiary level; diverse countries; and also outside the field of mathematics. Further research is clearly needed; the present evolutions of digital resources make it a major challenge for the studies of teachers' professional evolutions.

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# AN INVESTIGATIVE LESSON WITH DYNAMIC GEOMETRY: A CASE STUDY OF KEY STRUCTURING FEATURES OF TECHNOLOGY INTEGRATION IN CLASSROOM PRACTICE

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*The research literatures concerning the classroom practice of mathematics teaching and technology integration in school mathematics point to key structuring features – working environment, resource system, activity format, curriculum script, time economy – that shape patterns of technology integration into classroom practice and require teachers to develop their craft knowledge accordingly. This conceptual framework is applied to an investigative lesson incorporating dynamic geometry use, employing evidence from classroom observation and teacher interview. This illuminates the many aspects of professional adaptation and development on which successful technology integration into classroom practice depends.*

## INTRODUCTION TO THE STUDY

From synthesis of relevant research literatures, I argued at CERME-5 that successful integration of computer-based tools and resources into school mathematics depends on coordinating working environment, resource system, activity format and curriculum script to underpin classroom practice which is viable within the time economy (Ruthven, 2007). This paper will illustrate –and test– that conceptual framework by using it to analyse the practitioner thinking and professional learning surrounding a lesson incorporating the use of dynamic geometry.

The lesson was one of four cases investigated in a study of classroom practice incorporating dynamic geometry use (Ruthven, Hennessy & Deaney, 2008). In the original study, this specific case was followed up because the teacher concerned talked lucidly about his experience of teaching such a lesson for the first time, and displayed particular awareness of the potential of dynamic geometry for developing visuo-spatial and linguistic aspects of students' geometrical thinking.

This case has been chosen for further analysis because the teacher was unusually expansive in all his interviews, illuminating a range of aspects of practitioner thinking and professional learning. While an exhaustive case analysis in terms of the conceptual framework would require data to be collected with its use specifically in mind, the richness of the evidence from this case provides a convenient interim means of exploring its application to a concrete example.

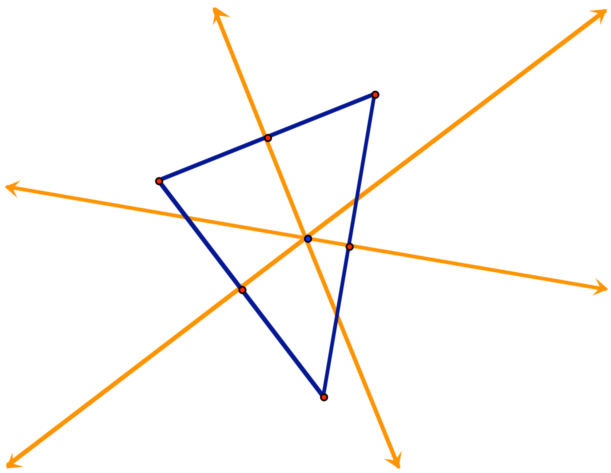
## ORIENTATION TO THE LESSON

As the teacher explained when nominating the lesson, it had recently been developed in response to improved technology provision in the mathematics department prompting him to "*to explore some geometry*":

> So we'd done some very rough work on constructions with compasses and bisecting triangles and then I extended that to Geometer's Sketchpad… on the interactive whiteboard using it in front of the class.

He reported that the lesson (with a class in the early stages of secondary education) had started with him constructing a triangle, and then the perpendicular bisectors of its edges. The focus of the investigation which ensued had been on the idea that this construction might identify the 'centre' of a triangle:

> And we drew a triangle and bisected the sides of a triangle and they noted that they all met at a point. And then I said: "Well let's have a look, is that the centre of a triangle?" And we moved it around and it wasn't the centre of the triangle, sometimes it was inside the triangle and sometimes outside.



According to the teacher, one particularly successful aspect of the lesson had been the extent to which students actively participated in the investigation:

> *And they were all exploring; sometimes they were coming up and actually sort of playing with the board themselves… I was really pleased because lots of people were taking part and people wanted to come and have a go at the constructions.*

Indeed, because of the interest and engagement shown by students, the teacher had decided to extend the lesson into a second session, held in a computer room to allow students to work individually at a computer:

> *And it was clear they all wanted to have a go so we went into the computer room for the next lesson so they could just continue it individually on a computer… I was expecting them all to arrive in the computer room and say: "How do you do this? What do I have to do again?"… But virtually everyone… could get just straight down and do it. I was really surprised. And the constructions, remembering all the constructions as well.*

For the teacher, then, this recall by students of ideas from the earlier session was another aspect of the lesson's success. In terms of the specific contribution of dynamic geometry to this success, the teacher noted how the software supported exploration of different cases, and overcome the practical difficulties which students encountered in using classical tools to attempt such an investigation by hand:

> *You can move it around and see that it's always the case and not just that one off example. But I also think they get bogged down with the technicalities of drawing the things and getting their compasses right, and [with] their pencils broken.*

But the teacher saw the contribution of the software as going beyond ease and accuracy; using it required properties to be formulated precisely in geometrical terms:

> *And it's the precision of realising that the compass construction… is about the definition of what the perpendicular bisector is… And Geometer's Sketchpad forces you to use the geometry and know the actual properties that you can explore.*

These, then, were the terms in which the original lesson was nominated as an example of successful practice. This nomination was followed up by studying a later lesson along similar lines through classroom observations and teacher interviews. The observed lesson was conducted over two 45-minute sessions on consecutive days with a Year 7 class of students (aged 11-12) in their first year of secondary education.

## WORKING ENVIRONMENT

The use of ICT in teaching often involves changes in the working environment of lessons: change of room location and physical layout, change in class organisation and classroom procedures.

Each session of the observed lesson started in the normal classroom and then moved to a nearby computer suite, a modification of the pattern originally reported. This movement between rooms allowed the teacher to follow a particular activity cycle common to each session, shifting working environment to match changing activity format. The classroom was equipped with a single computer linked to a ceiling-mounted projector directed towards a whiteboard at the front: this supported use of computer-based resources within whole-class activity formats. However, only in the computer suite was it possible for students to work individually at a machine.

Even though the suite was also equipped with a projectable computer, starting sessions in the teacher's own classroom was expedient for several reasons. Doing so avoided disruption to the established routines underpinning the smooth launch of lessons. Moreover, the classroom provided an environment more conducive to sustaining effective communication during whole-class activity and to maintaining the attention of students. Whereas in the computer suite each student was seated behind a sizeable monitor perched on a desktop computer unit, so blocking lines of sight and placing diversion at students' fingertips, in the classroom the teacher could introduce the lesson *"without the distraction of computers in front of each of them"*.

It was only recently that the classroom had been refurbished and equipped, and a neighbouring computer suite established for the exclusive use of the mathematics department. The teacher contrasted this new arrangement favourably in terms of the easier and more regular access to technology that it afforded, and the consequent increase in the fluency of students' use:

*Before… you'd book a computer suite, you'd go in and then… you'[d] just not get anywhere, because the whole lesson's been sorting out logging on, sorting out how to use [the software]… And [now] having the access to it so easily and readily just makes a huge difference.*

New routines were being introduced to students for opening a workstation, including logging on to the school network, using shortcuts to access resources, and maximising the document window. Likewise, routines were being developed for closing sessions in the computer suite. Towards the end of each session, the teacher prompted students to plan to save their files and print out their work, advising them that he'd *"rather have a small amount that you understand well than loads and loads of pages printed out that you haven't even read"*. He asked students to avoid rushing to print their work at the end of the lesson, and explained how they could adjust their output to try to fit it onto a single page; he reminded them to give their file a name that indicated its contents, and to put their name on their document to make it easy to identify amongst all the output from the single shared printer.

## RESOURCE SYSTEM

New technologies have broadened the types of resource available to support school mathematics. Nevertheless, there is a great difference between a collection of resources and a coherent system.

The department maintained its own schemes of work under continuous development, with teachers encouraged to explore new possibilities and report to colleagues. This meant that they were accustomed to integrating material from different sources into a common scheme. However, so wide was the range of computer-based resources currently being trialled that our informant (who was head of department) expressed concern about incorporating them effectively into departmental schemes:

*At the moment we're just dabbling in [a variety of technologies and resources] when people feel like it, but we're moving towards integrating [them] into schemes of work now… I'm slightly worried that we've got so much… It's getting everybody familiar with it all.*

In terms of coordinating use of old and new technologies, work with dynamic geometry was seen as complementing established work on construction by hand, by strengthening attention to the related geometric properties:

*I thought of Geometer's Sketchpad [because] I wanted to balance the being able to actually draw [a figure] with pencil and compasses and straight edges, with also seeing the geometrical facts about it as well. And sometimes [students] don't draw it accurately enough to get things like that all the [perpendicular bisectors] meet at the orthocentre[1] of the circle.*

The accuracy, speed and manipulative ease of dynamic geometry facilitated geometrical investigations which were difficult to undertake by hand:

*[It] takes hours and hours if you try and do that by pencil and paper… So just that power of Geometer's Sketchpad to move the triangle around and try different triangles within seconds was fantastic. Ideal for this sort of exploration.*

Nevertheless, the teacher felt that old and new tools lacked congruence, because certain manual techniques appeared to lack computer counterparts. Accordingly, old and new were seen as involving different methods and having distinct functions:

*When you do compasses, you use circles and arcs, and you keep your compasses the same. And I say to them: "Never move your compasses once you've started drawing."… Well Geometer's Sketchpad doesn't use that notion at all… So it's a different method.… I don't think there's a great deal of connection. I don't think it's a way of teaching constructions, it's a way of exploring the geometry.*

Equally, some features of computer tools were not wholly welcome: students could be deflected from the mathematical focus of a task by overconcern with presentation. During this lesson the teacher had tried out a new technique for managing this, by briefly projecting a prepared example to show students the kind of document that they were expected to produce, and illustrating appropriate use of colour coding:

*They spend about three quarters of the lesson making the font look nice and making it all look pretty [but] getting away from the maths.… I've never tried it before, but that showing at the end roughly what I wanted them to have would help. Because it showed that I did want them to think about the presentation, I did want them to slightly adjust the font and change the colours a little bit, to emphasise the maths, not to make it just look pretty.*

Here we see the development of sociomathematical norms for using new technologies, and classroom strategies for establishing and maintaining these norms. Likewise, the way in which dynamic geometry required clear instructions to be given in precise mathematical terms was conveyed as being its key characteristic:

*I always introduce Geometer's Sketchpad by saying: "It's very specific, you've got to tell it. It's not just drawing, it's drawing using mathematical rules."… They're quite happy with that notion of… the computer only following certain clear instructions.*

## ACTIVITY FORMAT

Classroom activity is organised around formats for action and interaction which frame the contributions of teacher and students to particular lesson segments (Burns & Anderson, 1987). The crafting of lessons around familiar activity formats and their supporting classroom routines helps to make them flow smoothly in a focused, predictable and fluid way (Leinhardt, Weidman & Hammond, 1987). This leads to the creation of prototypical activity structures or cycles for particular styles of lesson.

Each session of the observed lesson followed a similar activity cycle, starting with teacher-led activity in the normal classroom, followed by student activity at individual computers in the nearby computer suite, and with change of rooms during sessions serving to match working environment to activity format. Indeed, when the teacher had first nominated this lesson, he had remarked on how it combined a range of classroom activity formats to create a promising lesson structure:

> *There was a bit of whole class, a bit of individual work and some exploration, so it's a model that I'd like to pursue because it was the first time I'd done something that involved quite all those different aspects.*

In discussing the observed lesson, however, the teacher highlighted one aspect of the model which had not functioned as well as he would have liked: the fostering of discussion during individual student work. He identified a need for further consideration of the balance between opportunities for individual exploration and productive discussion, through exploring having students work in pairs:

> *There was not as much discussion as I would have liked. I'm not sure really how combine working with computers with discussing. You can put two or three [students] on a computer, which is what you might have done in the days when we didn't have enough computers, but that takes away the opportunity for everybody to explore things for themselves. Perhaps in other lessons… as I develop the use of the computer room I might decide… [to] work in pairs. That's something I'll have to explore.*

At the same time, the teacher noted a number of ways in which the computer environment helped to support his own interactions with students within an activity format of individual working. Such opportunities arose from helping students to identify and resolve bugs in their dynamic geometry constructions:

> *[Named student] had a mid point of one line selected and the line of another, so he had a perpendicular line to another, and he didn't actually notice which is worrying… And that's what I was trying to do when I was going round to individuals. They were saying: "Oh, something's wrong." So I was: "Which line is perpendicular to that one?"*

Equally, the teacher was developing ideas about the pedagogical affordances of text-boxes, realising that they created conditions under which students might be more willing to consider revising their written comments:

> *And also the fact that they had a text box… and they could change it and edit it. They could actually then think about what they were writing, how they describe, I could have those discussions. With handwritten, if someone writes a whole sentence next to a neat diagram, and you say: "Well actually, what about that word? Can you add this in?" You've just ruined their work. But with technology you can just change it, highlight it and add on an extra bit, and they don't mind.*

This was helping him to achieve his goal of developing students' capacity to express themselves clearly in geometrical terms:

> *I was focusing on getting them to write a rule clearly. I mean there were a lot writing "They all meet" or even, someone said "They all have a centre."… So we were trying to discuss what "all" meant, and a girl at the back had "The perpendicular bisectors meet", but I think she'd heard me say that to someone else, and changed it herself. "Meet at a point": having that sort of sentence there.*

## CURRICULUM SCRIPT

In planning and conducting lessons on a topic, teachers draw on a loosely ordered model of relevant goals and actions that guides their teaching. This forms what has been termed a 'curriculum script' – where 'script' is used in the psychological sense of a form of event-structured cognitive organisation, which includes variant expectancies of a situation and alternative courses of action (Leinhardt, Putnam, Stein & Baxter, 1991). This script includes tasks to be undertaken, representations to be employed, activity formats to be used, and student difficulties to be anticipated.

The observed lesson followed on from earlier ones in which the class had undertaken simple constructions with classical tools: in particular, using compasses to construct the perpendicular bisector of a line segment. Further evidence that the teacher's script for this topic originated prior to the availability of dynamic geometry was his reference to the practical difficulties which students encountered in working by hand to accurately construct the perpendicular bisectors of a triangle. His evolving script now included knowledge of how software operation might likewise derail students' attempts to construct perpendicular bisectors, and of how such difficulties might be turned to advantage in reinforcing the mathematical focus of the task:

> *Understanding the idea of perpendicular bisector… you select the line and the [mid]point… There's a few people that missed that and drew random lines… And I think they just misunderstood, because one of the awkward things about it is the selection tool. If you select on something and then you select another thing, it adds to the selection, which is quite unusual for any Windows package… So you have to click away and de-select things, and that caused a bit of confusion, even though I had told them a lot. But… quite a few discussions I had with them emphasised which line is perpendicular to that edge… So sometimes the mistakes actually helped.*

Equally, the teacher's curriculum script anticipated that students might not appreciate the geometrical significance of the concurrence of perpendicular bisectors, and incorporated strategies for addressing this:

> *They didn't spot that [the perpendicular bisectors] all met at a point as easily… I don't think anybody got that without some sort of prompting. It's not that they didn't notice it, but they didn't see it as a significant thing to look for… even though there were a few hints in the worksheet that that's what they were supposed to be looking at, because I thought that they might not spot it. So I was quite surprised… that they didn't seem to think that three lines all meeting at a point was particularly exceptional circumstances. I tried to get them to see that… three random lines, what was the chance of them all meeting at a point.*

The line of argument alluded to here was one already applicable in a pencil and paper environment. Later in the interview, however, the teacher made reference to another strategy which brought the distinctive affordances of dragging the dynamic figure to bear on this issue:

*When I talked about meeting at a point, they were able to move it around, and I think there's more potential to do that on the screen.*

Likewise, his extended curriculum script depended on exploiting the distinctive affordance of the dynamic tool to explore how dragging the triangle affected the position of the 'centre'.

This suggests that the teacher's curriculum script was evolving through experience of teaching the lesson with dynamic geometry, incorporating new mathematical knowledge specifically linked to mediation by the software. Indeed, he drew attention to a striking example of this which had arisen from his question to the class about the position of the 'centre' when the triangle was dragged to become right angled:

*Teacher: What's happening to the [centre] point as I drag towards 90 degrees? What do you think is going to happen to the point when it's at 90?*

*Student: The centre's going to be on the same point as the midpoint of the line.*

*Teacher [with surprise]: Does it always have to be at the midpoint?*

*[Dragging the figure] Yes, it is! Look at that! It's always going to be on the midpoint of that side.… Brilliant!*

Reviewing the lesson, the teacher commented on this episode, linking it to distinctive features of the mediation of the task by the dynamic figure:

*I don't know why it hadn't occurred to me, but it wasn't something I'd focused on in terms of the learning idea, but the point would actually be on the mid point.… As soon as I'd said it I thought "Of course!" But you know, in maths there's things that you just don't really notice because you're not focusing on them. And… I was just expecting them to say it was on the line. Because when you've got a compass point, you don't actually see the point, it's just a little hole in the paper… But because the point is actually there and quite clear, a big red blob, then I saw it was exactly on that centre point, and that was good when they came up with that.*

In effect, his available curriculum script did not attune the teacher to this property. One can reasonably hazard that this changed as a direct result of this episode.

## TIME ECONOMY

Assude (2005) examines how teachers seek to improve the 'rate' at which the physical time available for classroom activity is converted into a didactic time measured in terms of advance of knowledge. The adaptation and coordination of working environment, resource system, activity format and curriculum script are very important in improving this didactic 'return' on time 'investment'.

In respect of this time economy, a basic consideration of physical time for the teacher in this study was the proximity of the new computer suite to his normal classroom:

*I'm particularly lucky being next door… If I was upstairs or something like that, it would be much harder; it would take five minutes to move down.*

However, a more fundamental feature of this case was the degree to which the teacher measured didactic time in terms of progression towards securing student learning rather than pace in covering a curriculum. At the end of the first session, he linked his management of time to what he considered to be key learning processes:

*It's really important that we do have that discussion next lesson. Because they've seen it. Whether they've learned it yet, I don't know… They're probably vaguely aware of different properties and they've explored it, so it now needs to be brought out through a discussion, and then they can go and focus on writing things for themselves. So the process of exploring something, then discussing it in a quite focused way, as a group, and then writing it up… They've got to actually write down what they think they've learned. Because at the moment, I suspect… they've got vague notions of what they've learned but nothing concrete in their heads.*

A further crucial consideration within the time economy is instrumental investment. The larger study from which this case has been derived showed that the ways in which teachers incorporated dynamic geometry into classroom activity were influenced by their assessments of costs and benefits. Essentially, teachers were willing to invest time in developing students' instrumental knowledge of dynamic geometry to the extent that they saw this as promoting students' mathematical learning. As already noted, this teacher saw working with the software as engaging students in disciplined interaction with a geometric system. Consequently, he was willing to spend time to make them aware of the construction process underlying the dynamic figures used in lessons:

*I very rarely use Geometer's Sketchpad from anything other than a blank page. Even when I'm doing something in demonstration… I always like to start with a blank page and actually put it together in front of the students so they can see where it's coming from.*

Equally, this perspective underpinned his willingness to invest time in familiarising students with the software, capitalising on earlier investment in using classical tools:

*That getting them used to the program beforehand, giving a lesson where the aim wasn't to do that particular maths, but just for them to get familiar with it… was very helpful. And also they're doing the constructions by hand first, to see, getting all the words, the key words, out of the way.*

As this recognition of a productive interaction between learning to use old and new technologies indicates, this teacher also took an integrative perspective on the 'double instrumentation' entailed. Indeed, this was demonstrated earlier in his concern with the complementarity of old and new as components of a coherent resource system.

## CONCLUSION

This analysis of a lesson incorporating dynamic geometry illuminates the influence of the key structuring features of working environment, resource system, activity format, curriculum script and time economy on technology use. Although only employing a dataset conveniently available from earlier research, it starts to show the complex character of the professional adaptation on which technology integration into the classroom practice of school mathematics depends. This points to the value of conducting further studies in which data collection (as well as analysis) is guided by the conceptual framework developed in this paper and its predecessor. Such studies might profitably focus not just on the teacher/classroom level, but on the school/departmental level, and the systemic/institutional level.

## NOTES

[1] The point at which the perpendicular bisectors of the sides of a triangle meet is the ‘circumcentre’. However, in the course of the interview, the teacher referred to this centre as the ‘orthocentre’. Note that it is now many years since reference to these (and other) terms – which distinguish the different ‘centres’ of a triangle – was removed from the school mathematics curriculum in England.

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# METHODS AND TOOLS TO FACE RESEARCH FRAGMENTATION IN TECHNOLOGY ENHANCHED MATHEMATICS EDUCATION

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*This paper, addresses the issue of how to successfully bring into school practices the results in technology enhanced mathematics learning obtained at research level. The distance among different research teams and between researchers and teachers is addressed in terms of fragmentation of the research field. A methodology is presented to reduce such fragmentation illustrating a pathway followed at the European level in the EC co-funded projects TELMA and ReMath.*

## INTRODUCTION

In the CERME 5 conference two plenary sessions (Ruthven, 2007; Artigue, 2007), drawing from the discussions developed in different working groups, highlighted key issues concerning Technology Enhanced Learning (TEL) in mathematics.

According to Ruthven (ibid pp. 52), despite of a generalized advocacy for new technologies in education, these have had a limited success in school. As a matter of fact, he observes that, even if technologies had some positive impact on the instruction of teachers, they remain marginal in classroom practice. This is true, in particular, for mathematics, even if, from the beginning, a wide number of researchers have been concerned with the study of the opportunities brought about by new technologies to the teaching and learning of this discipline (Lagrange, Artigue, Laborde, & Trouche, 2003). As a matter of fact, despite the positive results produced in a number of experimental settings and the budget invested by many governments for equipping schools, actual use of ICT tools in real school environments is still having a limited impact. Recent studies witness difficulties encountered by teachers in implementing teaching and learning activities mediated by technologies due to variables such as working environment, resource system, activity format, curriculum script and time economy (Cuban, 2001; Sutherland, 2004). The coordination of such variables is necessary in order to develop a coherent use of technological tools and to form an effective system. According to Ruthven (ibid pp. 64), this challenge “involves moving from idealised aspiration to effective realisation through the development of practical theories and craft knowledge”. Drawing from our own experience, we identify as a crucial issue the necessity to establish effective interactions among the different actors involved in the process, that is researchers, teachers, policy makers, curriculum developers, software designers, etc.

Such a view is coherent with what is reported in (Pratt, Winters, Cerulli & Leemkuil, in press) from the perspective of educational technology designers. Authors, making reference to the specific field of games for mathematics education, speak of the necessity of a multi-disciplinary approach to design and deployment of technologies as opposed to the frequently experienced design fragmentation. Such fragmentation is often due to the fact that the different communities involved are not fully cognisant of the structuring forces that impinge on each other's activities. From one hand, discontinuities between design and deployment of technological tools impede the effective use of such tools in school practice and, on the other hand, the development of isolated projects that often do not go beyond experimental settings, do not contribute to cumulative knowledge about the design process that could inform future work. Pratt et al. advocate the need to integrate key stakeholders in the creation of technology enhanced learning tools, as "the problem of design fragmentation remains a real impediment to widespread innovation in the field". They thus state the opportunity of creating multidisciplinary teams focusing on the design and deployment of educational technology that bring together the perspective of different stakeholders: designers, educators, researchers, etc.

Fragmentation, however, is not only a problem experienced among different communities of stakeholders, but it is a problem often experienced also within each community. In particular, as highlighted by Artigue (2007) during her plenary speech at CERME 6, this is one of the key issues of concern within the community of the researchers in mathematics education, and, in particular, within the community of researchers focusing on technology enhanced learning in mathematics. Such a fragmentation is rooted at theoretical level, as witnessed also by the work of the working group 11 of ERME that has been established to discuss such specific issue (Prediger, Arzarello, Bosch & Lenfant, 2008). As a matter of fact the theoretical background of a research team has an important bearing on the epistemological assumptions, the research methodologies, the way in which tools, and, in particular, technology enhanced tools, are perceived and used.

At the European level, where a great variety of different approaches and background is present, there is a specific sensibility to the problem of fragmentation and to the necessity to find feasible ways to overcome it, since, as observed in (Arzarello, Bosch, Gascón & Sabena, 2008) a too wide variety of poorly connected conceptual and methodological tools does not encourage consideration of the results obtained as convincing and valuable. Moreover, in the specific area of TEL, there is the need of designing and implementing tools and methodologies that have a wide scope of application and that are not restricted to a particular community or context. For these reasons, following the impulse given by projects funded by the European Community, efforts have been made to try to overcome such fragmentation.

Our Institute has been involved in European research projects concerned with Information Society Technologies (IST) for several years and, in particular in Networks of Excellence (NoE) and Specific Targeted Research Projects (STREPs). These are two instruments of the European Community $6^{th}$ and $7^{th}$ Research Framework Programmes that aim at promoting research integration and collaboration in several fields including technology enhanced learning.

This paper presents some methods and tools, developed within the context of such European projects, which have been developed and tested to address the fragmentation issues discussed above.

Firstly we report on the work performed within the TELMA (Technology Enhanced Learning in Mathematics) initiative that explored the conditions for sharing experience and knowledge among different research teams interested in analysing mathematics learning environments integrating technologies, in spite of the differences in the theoretical frameworks and in the methodological approaches adopted. For this purpose, the notions of "*didactical functionality*" (Cerulli, Pedemonte & Robotti, 2006) and of "*key concerns*" - issues functionally important (Artigue, Haspékian, Cazes, Bottino, Cerulli, Kynigos, Lagrange & Mariotti, 2006) - together with a methodology based on the idea of a "*cross experiments*" (Bottino, Artigue & Noss, in press) were defined and conceptualized as concrete methods to address the problem of fragmentation.

Secondly, we give account of some of the outcomes of the ReMath project that, building on the results of the TELMA project, has addressed the fragmentation problem from the perspective of the design, implementation, and in-depth experimentation of ICT-based interactive learning environments for mathematics, thus involving not only researchers but teachers and technology designers as well. In particular, within the ReMath project, the problem of how to effectively support collaboration in pedagogical planning has been faced. Efforts have been made to provide a solid basis for accommodating the different perspectives adopted, for analysing the factors at play, and also for understanding the initial assumptions and theoretical frameworks embraced. A web-based system, the Pedagogical Plan Manager (PPM), was developed to support researchers, tool designers and teachers to jointly design and/or deploy mathematics pedagogical plans involving the use of technological tools (Bottino, Earp, Olimpo, Ott, Pozzi & Tavella, 2008).

Summing up, in the following sections, we delineate the process that has brought us to afford the problem of the fragmentation of approaches and frameworks, in the field of mathematics teaching and learning mediated by technologies, from different but complementary perspectives.

## A COLLABORATIVE METHODOLOGY FOR NETWORKING RESEARCH TEAMS IN TECHNOLOGY ENHANCED LEARNING IN MATHEMATICS

NoEs have been established by the European Commission within the last Framework Research Programmes as instruments to promote integration and collaborative work of key European research teams and stakeholders in given fields. In particular, the network of Excellence Kaleidoscope was established and funded with the aim of shaping the scientific evolution of technology enhanced learning (http://www.noe-kaleidoscope.org, accessed March 2009). Since each knowledge domain raises specific issues either for learning or for the design of learning environments, within Kaleidoscope a number of different joint research initiatives, covering a wide range of domains, have been carried out. Among these, TELMA was specifically focused on Technology Enhanced Learning in Mathematics. It involved six European teams[1] and had as its main aim that of building a shared view of key research topics in the area of digital technologies and mathematics education, proposing related research activities, and developing common research methodologies.

In TELMA, each team brought to the project particular focuses and theoretical frameworks, adopted and developed over a period of time. Most of these teams have also designed, implemented and experimented, in different classroom settings, computer-based systems for supporting teaching and learning processes in mathematics. It was clear from the beginning that, to connect the work of groups that have different traditions and frameworks it was necessary to develop a better mutual understanding and to find some common perspectives from which to look at the different approaches adopted. It was also necessary to develop a common language since the same words were sometime used with different meanings by each team, causing misunderstanding and hindering productive collaboration. Moreover, it became clear that the theoretical assumptions made by each team, were often implicit and thus not accessible to the others.

*The notion of didactical functionalities*

In order to overcome these difficulties it was decided to focus the work of TELMA on the theoretical frameworks within which the different research teams face research in mathematics education with technology. A first level of integration has been then pursued through the definition of the notion of *didactical functionality* for interpreting and comparing different research studies (Cerulli et al., 2006). Such notion has been used as a way to develop a common perspective among teams linking theoretical reflections to the real tasks that one has to face when designing or analysing effective uses of digital technologies in given contexts. The notion of didactical functionality is structured by three inter-related components:

- a set of features/characteristics of the considered ICT-tool;
- an educational aim;
- the modalities of employing the ICT-tool in a teaching/learning process to achieve the chosen educational aim.

The different didactical functionalities designed and experimented by each team have been compared trying to delineate how different theoretical backgrounds can influence the design of an ICT-based tool, the definition of the educational goals to be pursued, and the modalities of use of the tool to achieve such goals. At the beginning, this analysis was conducted on the basis of a selection of papers published by each team. This approach, even if useful, was considered not sufficient to enter the less explicit aspects of the research work of each team. Thus, TELMA researchers decided to move toward a strategy that could allow them to gain more intimate insights into their respective research and design practices. This strategy relies on the idea of 'cross-experiments' and on the development of a methodological tool for systematic exploration of the role played by theoretical frames.

*The cross-experiments methodology*

The idea of cross-experiments was developed in order to provide a systematic way of gaining insight into theoretical and methodological similarities and differences in the work of the various TELMA teams. This is a new approach to collaboration that seeks to facilitate common understanding across teams with diverse practices and cultures and to elaborate integrated views that transcend individual team cultures. There are two principal characteristics of the cross-experiments project implemented within TELMA that distinguish it from other forms of collaborative research:

- the design and implementation by each research team of a teaching experiment making use of a ICT-based tool developed by one of the other team involved;
- the joint construction of a common set of questions to be answered by each team in order to frame the process of cross-team communication.

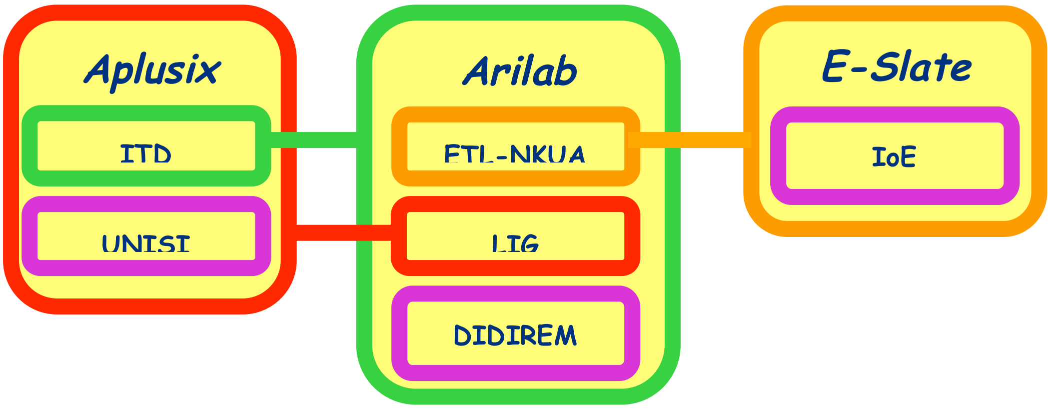


**Figure 1: *Aplusix*, developed by Metah, was experimented by ITD and UNISI. *Arilab*, developed by ITD, was experimented by LIG, ETL-NKUA and DIDIREM. *E-Slate*, developed by ETL-NKUA was experimented by IoE.**

Each team was asked to select an ICT-tool among those developed by the other TELMA teams (Figure 1). This decision was expected to induce deep exchanges between the teams and to make visible the influence of theoretical frames through comparison of the didactical functionalities developed by the designers of given tools and those implemented by the teams experimenting the tools. Moreover, in order to facilitate the comparison between the different experimental settings, it was also agreed to address common knowledge domains (fractions and introduction to algebra), to carry out the teaching experiments with students between the 5th to 8th grade, and to perform them for about the same amount of time (one month).

Guidelines (Cerulli, Pedemonte & Robotti, 2007) were collectively built for monitoring the whole process: from the design and the a priori analysis of the experiments to their implementation, the collection of data and the a posteriori analysis. Beyond that, reflective interviews (using the technique of "interview for explicitation" (Vermesch & Maurel, 1997)) were a-posteriori organized in order to make clear the exact role that theoretical frames and contextual characteristics had played in the different phases of the experimental work, either explicitly or in a more naturalized and implicit way.

It was hypothesized that, for each team, the use of a non familiar (alien) tool would have made problematic, thus visible, design decisions and practices that generally remain implicit when one uses tools developed within his/her research and educational culture, and that this visibility would have been increased by the guidelines' request of making explicit the choices performed.

The cross experiments provided interesting insights on the complexities involved in designing and implementing mathematics learning environments integrating technology and allowed to make some reflections (Bottino et al., in press; Cerulli, Trgalova, Maracci, Psycharis & Georget, 2008).

The first reflection was on the conditions that can facilitate the sharing of experience and knowledge among researchers in spite of the differences in the theoretical frameworks adopted. Theoretical frameworks, while influencing design and analysis of a teaching experiment, were far from playing the role they are usually given in the literature. As a matter of fact, in the design of the cross-experiments, theoretical frameworks acted mainly as implicit and naturalized frames, and more in terms of general principles than of operational constructs. Even if some variations could be noticed, all the teams experienced a gap between the support offered by theoretical frames and the decisions to be taken in the design process. The acknowledgment of such a gap can be a starting point for establishing a better communication channel not only among researchers but also with teachers. As a matter of fact, a marked emphasis on theoretical assumptions is often too far from the practical needs of teachers. For this reason it is important to establish the exact role that theoretical frameworks play in the planning of an effective teaching experiment. In particular, it was found that researchers tend to overestimate such role, thus making the distance with teachers' needs even bigger. A methodology for making explicit, and justifying, the choices made, proved a useful tool for reducing communication disparities.

A second observation concerns the understanding of what it means to adapt an ICT based tool to a context different from the one it was designed for. In our work this was accomplished by experimenting in each country tools developed in other countries by different teams. Thanks to the adopted methodology and to the request of making explicit assumptions, choices and decisions taken, it was possible to individuate some variables that strongly affect the development of teaching experiments involving the use of technologies. For instance, the attention paid to different *research priorities* (e.g. the detailed organization of the milieu; the social construction of knowledge; the teacher's role) and to *local constrains* (e.g. curricular; institutional; cultural) appeared to be crucial. Such variables are to be deeply considered and made explicit in the communication with teachers to effectively support them to adapt research experiments to their teaching contexts. In other words, researchers should find ways to make explicit all the key assumptions at the basis of their experiments. Of course, this is not enough, since, as suggested in (Pratt et al., in press), it is also necessary to promote a more strict collaboration between researchers, tool designers and teachers also at the level of the design and the implementation of ICT based tools, and in the planning of the experiments.

Taking into account these needs, and on the basis of the results obtained in TELMA, a new European project was thus developed, involving the same research teams: the ReMath project (IST - 4 – 26751 - STP). In this project the issue of collaboration between different stakeholders was addressed by developing a specific tool to be used to design teaching experiments involving ICT based tools.

## A TOOL TO SUPPORT THE COMMUNICATION OF DIFFERENT STAKEHOLDERS IN THE PLANNING OF LEARNING ACTIVITIES INVOLVING TECHNOLOGY

The TELMA project provided a strategy for reducing the difficulties of communication among researchers; this strategy proved to be quite effective, thus it was decided to adapt it to the needs of the ReMath project where communication in a wider community, including software designers, researchers and teachers, has been addressed. The Remath project has two main goals: the development of ICT-based tools for mathematics education at secondary school level and the design and experimentation, in different contexts, of learning activities for classroom practice involving the use of such tools (see: http://remath.cti.gr/default_remath.asp; accessed March 2009). In order to pursue this last goal, a cross-experiment methodology, widening the one developed by TELMA, was adopted. A tool, the Pedagogical Plan Manager (PPM), was, thus, developed to support communication between researchers and teachers when planning learning activities involving ICT tools. The idea was originated by the analysis of some the difficulties, pointed out by researchers in the wide field of learning design (Koper & Olivier, 2004), concerning dialogue and transfer between teachers, researchers and designers. To overcome such difficulties the PPM was realized, relying on the concept of *pedagogical plan*, as a specific system for supporting the process of pedagogical design, namely the description of learning activities to be enacted during cross-experiments (thus also enabling and fostering their reusability).

*Pedagogical plans* are conceived as descriptions of pedagogical activities to be carried out in real contexts (e.g. a class, a laboratory, etc.) where a number of different indicators could be made explicit, at different level of details (Bottino et al., 2008): educational target (What learning outcomes? What learning contexts? Who are the target learners?); pedagogical rationale (Why those learning outcomes? Why applying a certain strategy? Why using a give tool?); specifications (Which activities are to be carried out? Which roles are to be assumed by the different actors? Which resources and tools are to be used? etc.).

The PPM is a web environment, organized as a flexible structure allowing a three-alike representation of *pedagogical plans* as hierarchical entities which can be built and red at different levels of detail. This structure supports both “authors” of pedagogical plans, providing them with the possibility to work with a top-down structure, and “readers”, who in top-down organization have a facilitating factor for navigating from the general to the particular and vice versa.

In other words the PPM presents a flexible structure that tries to respond to the different needs of both researchers and teachers; the first, in fact, were mainly interested in sharing ideas about aspects such as the theoretical frameworks and the pedagogical rationale behind each educational intervention, while teachers were mainly interested in retrieving suitable information about the most suitable ways to carry out educational activities in their classes (Earp & Pozzi, 2006).

For space reason, we cannot provide here a detailed description of the model adopted and of the prototype implemented (more details can be found in Bottino et al., 2008). Outputs of its use are currently under examination and will be further analysed at the end of the ReMath project (May 2009).

## CONCLUSIONS

Software designers, researchers and teachers may have different needs, different constrains, and different perspectives. This can be an obstacle for the effectiveness of technology enhanced learning in mathematics, also in terms of impact in school practice. The projects briefly presented tried to develop a coherent methodology for reducing the distance between the different stakeholders. In TELMA it was addressed the problem of networking research teams with different backgrounds and approaches by means of a specific collaborative methodology. In ReMath such methodology was extended, also through the development of a specific web-based tool, to involve all the stakeholders in the design, development and deployment of teaching and learning activities involving the use of technologies.

The outlined pathway includes researcher’s explicitation of the actual role played by theoretical frameworks in the effective use of ICT tools and the individuation of the gap between theory and practice. This can help reducing the distance with teachers. The tool for pedagogical planning developed in the ReMath project is aimed at the same goal by involving teachers, from the beginning, also in the design of teaching activities with ICT-based tools. Such activities are seen as integral part in the design process of a technology. In this way we believe it can be possible to develop communities of practice that bring together teachers and researchers so that teaching practice and research could nurture one from each other favouring a better impact of technology enhanced learning in school practice.

## NOTES

1. TELMA teams (whose acronyms are indicated in brackets) belong to the following Institutions: Consiglio Nazionale delle Ricerche, Istituto Tecnologie Didattiche, Italy (ITD); Università di Siena, Dipartimento di Scienze Matematiche ed Informatiche, Italy (UNISI); University of Paris 7 Denis Diderot, France (DIDIREM); Grenoble University and CNRS, Leibniz Laboratory, Metah, France (LIG); University of London, Institute of Education, United Kingdom (IOE); National Kapodistrian University of Athens, Educational Technology Laboratory, Greece (ETL-NKUA).

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# THE DESIGN OF NEW DIGITAL ARTEFACTS AS KEY FACTOR TO INNOVATE THE TEACHING AND LEARNING OF ALGEBRA: THE CASE OF ALNUSET

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*The integration of CAS systems into school practices of algebra is marginal. To integrate effectively digital technology in the teaching and learning of algebra, it is necessary to go beyond the experience of CAS and of their instrumented techniques and to face the design of new artefacts. In this paper we discuss design problems faced in the development of a new digital artefact for teaching and learning of algebra, the Alnuset system. We present the key ideas that have oriented its design and the choices we have worked out to instrument its incorporated algebraic techniques. We compare the quantitative, symbolic and functional instrumented techniques of Alnuset with those of CAS highlighting crucial differences in the teaching and learning of algebra.*

**Keywords**: *Alnuset, Instrumented technique, CAS, Algebraic learning*

## INTRODUCTION

In the last 15 years a scientific debate on the role of technology in supporting teaching and learning processes in the domain of algebra has been going on. This debate originates from research studies carried out in different countries with the purpose of studying the use of Computer Algebra Systems (CAS) in school contexts. In particular, near benefits (Heid, 1988, Kaput, 1996, Thomas, Monaghan and Pierce, 2004) obstacles and difficulties have been identified in using this technology by students and teachers (Mayes, 1997, Drijvers, 2000, Drijvers, 2002, Guin & Trouche, 1999). Results of these research works (Artigue, 2005) highlight that the integration of CAS systems into the school practice of algebra remains marginal due to different reasons. CAS expands the range of possible task-solving actions. As a matter of fact, techniques involved in a CAS (instrumented techniques) are in general different from those of the paper and pencil environment. Managing the complexity of CAS instrumented techniques and highlighting the potential offered by the machine to the student is hard work. As shown by some experiments (Artigue, 2005), CAS use may cause an explosion of techniques which remain in a relatively simply-crafted state. Moreover, any technique that goes beyond a simple, mechanically learnt gesture, should be accompanied by a theoretical discourse. For the paper and pencil techniques this discourse is known and can be found in textbooks. For instrumented techniques it has to be built and its elaboration raises new, specific difficulties. Even if the use of CAS seems fully legitimate in the class, in general, instrumented techniques cannot be institutionalised in the same way as paper and pencil ones (Artigue, 2005).

## THE RATIONALE

To frame the results carried out by these research studies and the complexity of the processes involved in the educational use of CAS, some French researchers (Lagrange 2000, Artigue, 2002, Lagrange, Artigue, Guin and Trouche, 2003) have elaborated a theoretical framework, named 'instrumental approach', integrating both the ergonomic theory (Rabardel, 1995) and the anthropological theory (Chevallard, 1992). The ‘instrumental approach’ provides a frame for analyzing the processes of instrumental genesis both in their personal and institutional dimensions, and the effect of instrumentation issues on the integration of CAS in the educational practice. Using this framework, Artigue observes that CAS are extremely effective from a pragmatic standpoint and for this reason professionals (mathematicians, engineers..) are willing to spend time to master them (Artigue, 2002). At pragmatic level the effectiveness often comes with the difficulty to justify, at a theoretical level, the instrumented techniques used. In particular, this is true for users who do not fully master mathematical knowledge and techniques involved in the solution of the task. As a consequence, the epistemic value of the instrumented technique can remain hidden. This can constitute a problem for the educational context where technology should help not only to yield results but also to support and promote mathematical learning and understanding. In educational practice, techniques should have an epistemic value contributing to the understanding of objects involved. “*Making technology legitimate and mathematically useful from an educational point of view, whatever be the technology at stake, requires modes of integration that provide a reasonable balance between the pragmatic and the epistemic values of instrumented techniques*" (Artigue, 2007, p. 73). These results might account for the marginalization of CAS integration into the school algebraic practices. For some researchers, to integrate digital technology effectively in the domain of algebra, it is necessary to go beyond the experience of CAS and of their instrumented techniques and to face the design of new artefacts. As underlined by Monaghan (2007) up to now CAS-in-education workers have paid little attention to design issues, preferring, in general, to work with the design supplied by CAS designers (Monaghan, 2007). Moreover, it should be noted that no comparison between the design of CAS and of technological tools for education has been developed so far. This article aims at pointing out design issues that can effectively support teaching and learning processes in algebra. This goal will be pursued considering the design of ALNUSET (ALgebra on the NUmerical SETs), a system developed to improve teaching and learning of crucial topics involved in the mathematical curricula such as algebra, functions and properties of numerical sets. In particular, in this article we compare design aspects of Alnuset and of CAS and we highlight the relevance of differences in their instrumented techniques for the teaching and learning of algebra.

## PROBLEMS OF DESIGN IN DEVELOPING NEW DIGITAL ARTEFACTS

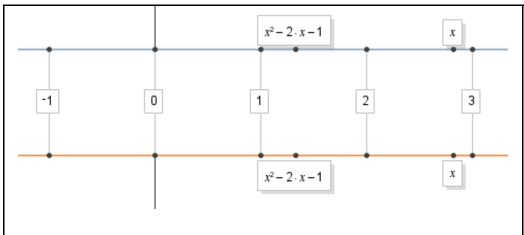
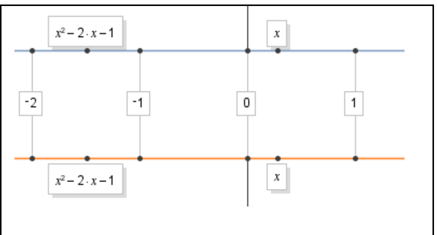
Going beyond the design of CAS requires new creative ideas to instrument techniques for mathematical activity different from those of CAS. The advent of both the dynamic geometrical artefacts and of spreadsheets has evidenced that even a single creative idea can determine a new typology of innovative artefacts. This can occur when new creative ideas allow to instrument mathematical techniques characterizing them with new operative and representative dimensions such as the drag of the variable point of a geometrical construction, as in the case of dynamic geometrical software, or the automatic re-computation of formulas of the table, as in the case of spreadsheet. Moreover, when a technique must be instrumented on the basis of an idea, various types of design problems emerge. They regard the way tasks and responsibilities have to be distributed between user and computer and the management of the interactivity, namely the operative modalities of the input by the user, the representation of the result by the computer (output), the visualisation of specific feedback to support the user action or to accompany the presentation of the result. Moreover, problems of design regard also the way in which the instrumented techniques have to be connected between each other. The way these problems are solved affects the accessibility of techniques, their usefulness for the task to be solved, the meaning that the instrumented technique evidences in the interaction, the discourse that can be developed about it. Hence, the way these problems are solved affects the balance between pragmatic and epistemic values of instrumented techniques within the didactical practice and this can affect mathematics teaching and learning. The anthropological framework is the theoretical tool used to analyse the way in which techniques are implemented and their effectiveness on the educational level. Ideas are evaluated on the base of this framework. We discuss these general assumptions in the domain of algebra referring to Alnuset System.

## ALNUSET: IDEAS AND CHOICE OF DESIGN

ALNUSET is a system designed, implemented and experimented within the ReMath (IST - 4 - 26751) EC project that can be used to improve the teaching and learning of algebra at lower and upper secondary school level. The design of ALNUSET is based on some ideas that have oriented the realisation of the three, strictly integrated components: the Algebraic Line component, the Algebraic Manipulator component, and the Function component. These three components make available respectively techniques of quantitative, symbolic and functional nature to support teachers and students in developing algebraic objects, processes and relations involved in the algebraic activity. In the following we present the main ideas that have oriented the realisation of the three components of Alnuset and illustrate the choices and decisions taken to instrument algebraic techniques so that an appropriate balance between their epistemic and pragmatic values can emerge when used in the educational practice.

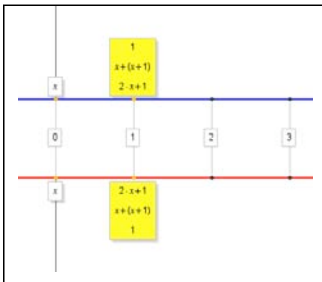
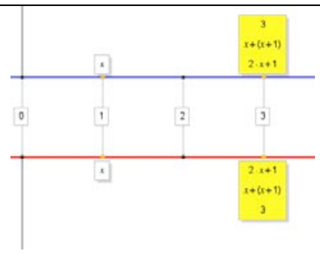
*Algebraic line component*

The main idea in the design of the Algebraic line component is the representation of algebraic variables on the number line through mobile points associated to letters, namely points that can be dragged on the line with the mouse. In this component the user can edit expressions to operate with. The computer automatically computes the value of the expression on the basis of the value of the variable on the line and it places a point associated to the expression on the algebraic line. When the user drags the mobile point of a variable, the computer refreshes the positions of the points corresponding to the expressions containing such a variable in an automatic and dynamic manner. This is possible only thanks to the digital technology that allows to transform the traditional number line into an algebraic line. The following two figures report the representation of a variable and of an algebraic expression on the lines of this component. Note that the presence of two lines is motivated by operative necessities regarding the use of the algebraic editor based on geometrical models that is available in this component. This editor is not considered in this report.

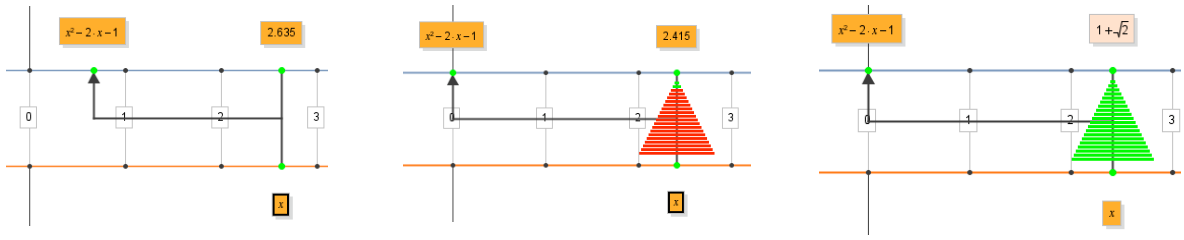
| | | |
|---|---|---|
|  | The drag of the variable x determines the dynamic movement of the expression containing it |  |

Through its visual feedback, this technique can be used either to explore what an expression indicates in an indeterminate way or to compare expressions. The design of this component is associated to every point represented on the line by a post-it. The computer automatically manages the relation among expressions, their associated points and post-it. The post-it of a point contains all the expressions constructed by the user that denote that point. By dragging a variable on the line, dynamic representative events can occur in a post-it. They might be very important for the development of a discourse concerning the notions of equality and equivalence between expressions. As a matter of the fact, the presence of two expressions in a post-it may mean:

- A relationship of equality, if taking place at least for one value of the variable during its drag along the line
- A relationship of equivalence, if taking place for all the values assumed by the variable when it is dragged along the line.
- A relationship of equivalence with restrictions, if taking place for every value of the variable when it is dragged along the line, but for one or more values, for which one of the two expressions disappears from the post-it and from the line.

| | | |
|---|---|---|
|  | The expressions x+(x+1) and 2*x+1 are equivalent, because they refer to the same point on the Algebraic line and they are contained in the same post-it whatever the value of the x variable is during the drag. |  |

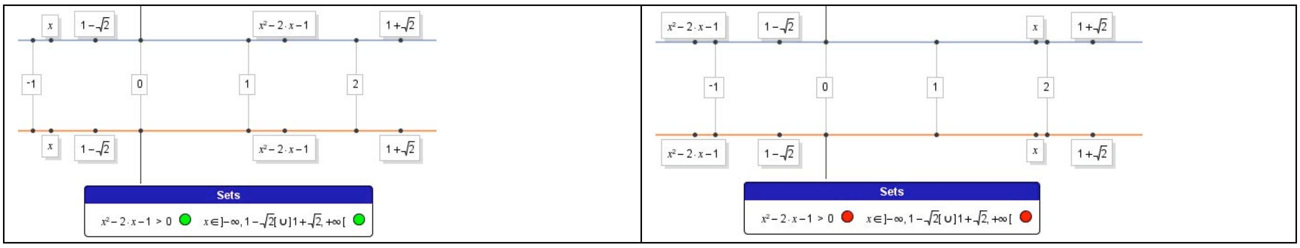
Moreover, the algebraic line component has been designed to provide two very important instrumented techniques for the algebraic activity, i.e. for finding the roots of polynomial with integer coefficients and for identifying and validating the truth set of algebraic propositions. The root of a polynomial can be found dragging the variable on the algebraic line in order to approximate the value of the polynomial to 0. When this happens, the exact root of the polynomial is determined by a specific algorithm of the program and it is represented as a point on the line.



This technique, that can be controlled by the user through his visual and spatial experience, is effective not only at a pragmatic level but also at an epistemic level, because it can concretely support the development of a discourse on the notion of root of a polynomial, as value of the variable that makes the polynomial equal to 0. The truth set of a proposition can be found through the use of a specific graphical editor. Let us consider the inequation $x^2$-2x-1>0, that once edited, is visualised in a specific window of this component named “Sets”. Once the root of the polynomial associated to the inequation has been represented on the line, a graphic editor can be used to construct its truth set (see the figure).

Two open intervals on the line, respectively on the right and on the left side of the roots of the polynomial $x^2$-2x-1, have been selected with the mouse. The system has translated the performed selection into the formal language.

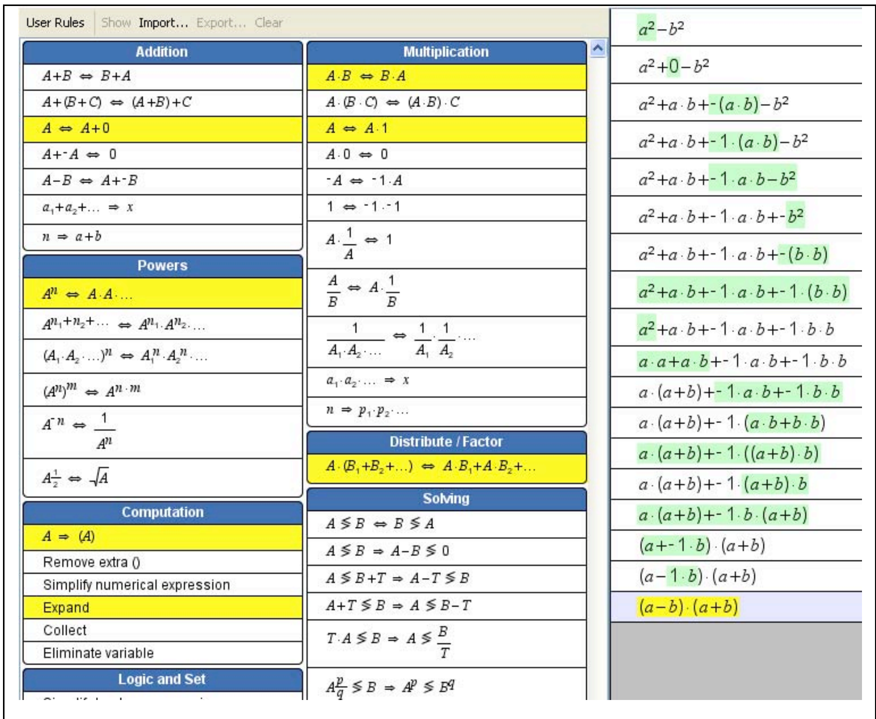
Once the truth set of a proposition has been edited, it can be validated using a specific feedback of the system. In the set window propositions and numerical sets are associated to coloured (green/red) markers that are under the control of the system. The green/(red) colour for the proposition means that it is true/(false) while the green/(red) colour for the numerical set means that the actual variable value on the line is/(is not) an element of the set. Through the drag of the variable on the line, colour accordance between proposition marker and set marker allows the user to validate the defined numerical set as truth set of the proposition (see figure below). The validation process is supported by the accordance of colour between the two markers and by the quantitative feedback provided by the position of variable and of the polynomial on the algebraic line during the drag.



This feedback offered by the system during the drag of the variable is important to introduce the notions of truth value and of truth set of an algebraic proposition and to develop a discourse on their relationships. All the described instrumented techniques that are specific of the Algebraic line component make a quantitative and dynamic algebra possible.

*Algebraic manipulator component*

The interface of this component has been divided into two distinct spaces: a space where symbolic manipulation rules are reported and a space where symbolic transformation is realised.



This figure shows a part of the commands available with the interface and an example of algebraic transformation.

The figure shows a characteristic of the interactivity of this manipulator: the selection of a part of an expression determines the activation of the commands of the interface that can be applied on it. This characteristic can help students to explore the systems of rule for the algebraic transformation and the effects they produce

The main idea characterizing the design of the Algebraic Manipulator component is the possibility to exploit pattern matching procedures of computer science to transform algebraic expressions and propositions through a structured set of basic rules that are deeply different from those of the CAS. In CAS pattern matching procedures are exploited according to a pragmatic perspective oriented to produce a result of symbolic transformation that could be also very complex, as in the case of command like factor or solve. As a consequence, the techniques of transformation can be obscure for a not expert user. In the Algebraic Manipulator component of Alnuset pattern matching procedures have been exploited according to three specific pedagogical necessities. The first necessity is to highlight the epistemic value of algebraic transformation as formal proof of the equivalence among algebraic forms. To this aim we have designed this manipulator with a set of basic rules that correspond to the basic properties of addition, multiplication and power operations, to the equality and inequality properties between algebraic expressions, to basic logic operations among propositions and among sets. Every rule produces the simple result of transformation that is reported on the icon of its corresponding command on the interface, and this makes the control of the rule and the result easy to control. Moreover a fundamental function of this component allows the student to create a new transformation rule (user rule) once this rule has been proved using the rules of transformation available on the interface. For example, once the rule of the remarkable product $a^2$-$b^2$= (a+b)*(a-b) has been proved, it can be added as new user rule in the interface $a^2$-$b^2$$\leftrightarrow$(a+b)*(a-b) and it can successively be used to transform other expressions or part of them whose form match with it. Moreover, a specific command allows to represent every transformed expression on the algebraic line automatically. Through this command it is possible to verify quantitatively the preservation of the equivalence through the transformation, observing that all the transformed expressions belong to the same post-it when their variables are dragged along the line. These characteristics of the algebraic manipulator of Alnuset can have a great epistemic importance because they can be effectively exploited to support the comprehension of the algebraic manipulation in terms of formal proof of the equivalence between two algebraic forms. The second necessity is to support the integration of practice of quantitative and manipulative nature. In this manipulator three rules allow the user to import the root of a polynomial, the truth set of a proposition and the value assumed by a variable on the algebraic line from the Algebraic line component to be used in the algebraic transformation. For example the rule “Factorize” uses the root of polynomial found in the Algebraic Line to factorize it. The way in which this rule works, makes the factorization technique of Alnuset different from that of CAS. In CAS this technique is totally under the control of the system, and the result can appear rather obscure for not expert users. In Alnuset, the factorization can be applied on the polynomial at hand only if its roots have been previously determined on the algebraic line. In Alnuset the distribution of tasks between user and computer and the way they interact, can contribute to understand the link between the factorization of a polynomial and its roots. The third necessity is to offer more powerful rules of transformation when needed for the activity and when specific meaning of algebraic manipulation have been already constructed. Two specific rules, also present in the CAS are available in this manipulator. They determine the result of a numerical expression and the result of a computation with polynomials respectively. These rules of transformation contribute to increase the pragmatic value of the instrumented technique of algebraic transformation in Alnuset and they can be used to introduce to the use of CAS

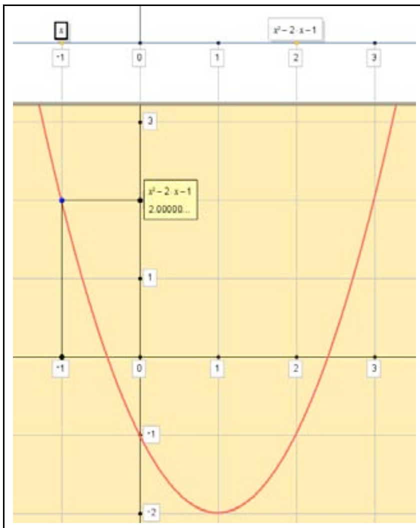
Moreover, the technique of algebraic transformation has been instrumented in this manipulator to provide non expert users with cognitive supports in the development of specific manipulative skills. A first support is the possibility to explore, through the mouse, the hierarchical structure that characterises the expression or the proposition to be manipulated. By dragging the mouse pointer over the elements of the expression or proposition at hand (operators, number, letters, brackets…), as feedback the system dynamically displays the meaningful part of the selected expression or proposition. In this way it is possible to explore all meaningful parts of an expression in the different levels of its hierarchical structure. Another feedback occurs when a part of expression has been selected. Through a pattern matching technique, the system, as feedback, activates only the rule of the interface that can be applied on the selected part of expression. This is a cognitive support that can be used to explore the connection among the transformational rules of the interface, the form on which it can be applied, and the effects provided by their applications.

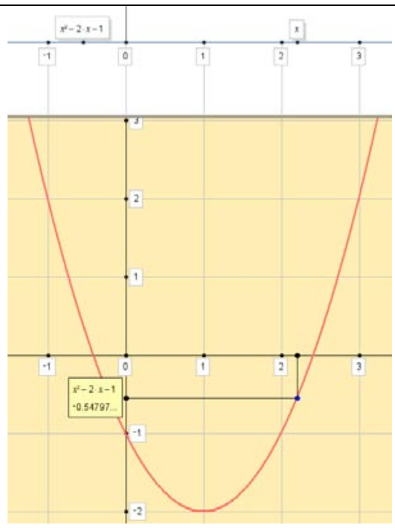
*Functions component*

The main idea characterizing the design of the Functions component is the possibility to connect a dynamic functional relationship between variable and expression on the algebraic line with the graphical representation of the function in the Cartesian plane. As a consequence, the interface of this component has been equipped with the Algebraic line and a Cartesian plane. This idea makes this component deeply different from other environment for the representation of function in the Cartesian plane. Through a specific command and the successive selection of the independent variable of the function, an expression represented on the Algebraic line is automatically represented as graphic in the Cartesian plane. Dragging the point corresponding to the variable on the algebraic line, two representative events occur:

- on the algebraic line, the expression containing the variable moves accordingly
- on the Cartesian plane, the point defined by the pair of values of the variable and of the expression moves on the graphic as shown in the following figure.

This instrumented technique supports the integrated development of a dynamic idea of function with a static idea of such a notion (Sfard 1991). The functional relationship between variable and expression is visualized dynamically on the algebraic line through drag of the variable point, and statically in the Cartesian plane through the curve. The movement of the point along the curve during the drag of the variable on the algebraic line supports the integration of these two ideas, showing that the curve reifies the infinite couples of values corresponding to the variable and to the expression on the line. This instrumented technique can be very useful to orient the interpretation of the graphics on the Cartesian plane and to develop important concepts of algebraic nature.





For example, it contributes to assign an algebraic meaning to the intersection of two curves (for the value of the variable that determines the intersection, the two expressions are contained in the same post-it on the algebraic line) or to the intersection of a curve with the x-axis (in this case the expression is contained in the post-it of 0).

Other examples are related to the construction of meaning for the sign of a function (position of the corresponding expression on the line with respect to 0), or to order among functions (positions of the expressions on the algebraic line )

## CONCLUSIONS

In this paper we have presented the main ideas that oriented the realisation of Alnuset and the choices we made to instrument specific functions of algebraic activity that can be useful for the teaching and learning of algebra. We have shown that the quantitative, symbolic and functional techniques available in the three environments of Alnuset to operate with algebraic expressions and propositions have characteristics that are deeply different from the instrumented technique of CAS. The technique of Alnuset was designed having in mind two types of users, different from the target user considered by CAS designers. The former type of user is the student who is not an expert of the knowledge domain of algebra and uses the instrumented techniques of Alnuset to learn it carrying out the algebraic activity proposed by the teacher. The latter type of user is the teacher who has difficulties to develop algebraic competencies and knowledge in students and who uses the instrumented technique of Alnuset to acquaint them with objects, procedures, relations and phenomena of school algebra. The technique of Alnuset was designed to be easily controlled during the solution of algebraic tasks, to produce results that can be easily interpreted and to mediate the interaction and the discussion on the algebraic meaning involved in the activity. The techniques of Alnuset structure a new phenomenological space where algebraic objects, relations and phenomena are reified by means of representative events that fall under the visual, spatial and motor perception of students and teachers. This contributes to provide an appropriate balance between the pragmatic and epistemic values of the techniques made available by Alnuset. In the phenomenological space determined by the use of the instrumented technique of Alnuset algebra can become a matter of investigation as evidenced by Trgalova et al. (WG4) and Pedemonte (WG2) of CERME6.

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# CASYOPÉE IN THE CLASSROOM: TWO DIFFERENT THEORY-DRIVEN PEDAGOGICAL APPROACHES

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*The ReMath project is a European project that addresses the task of integrating theoretical frames on mathematical learning with digital technologies at the European level. A specific set of six dynamic digital artefacts (DDA) has been currently developed, reflecting the diversity of representations provided by ICT tools. Here we considerer the DDA Casyopée which was experimented in two different countries: Italy (Unisi team) and France (Didirem team). The paper focuses on the influence of the theoretical frames in the design of these Teaching Experiments.*

## PROBLEMATIC OF THE REMATH PROJECT

The project focuses on the primary and secondary school level giving a balanced attention to both teachers and students and incorporating a range of innovative and technologically enhanced traditional representations. Specific attention is given to cultural diversity: seven teams from four countries are involved in this project. The work is based on evidence from experience involving a cyclical process of

a) developing six state-of-the-art dynamic digital artefacts for representing mathematics involving the domains of Algebra, Geometry and applied mathematics,

b) developing scenarios in a common format for the use of these artefacts for educational added value,

c) carrying out empirical research involving *cross-experimentation* (Cerulli et al. 2008) in realistic educational contexts, aiming at enhancing our understanding of meaning-making through representing with digital media, in particular by providing new insight into means of using technologies to support learning, and into learning processes in relation to the use of technologies.

Many recent studies highlight the existence of a multiplicity of theoretical frameworks for addressing those themes, and there is a shared increasing need of overcoming the resulting fragmentation (Artigue, 2007). This need is also felt within ReMath project, in which a variety of educational paradigms is present. The issue is addressed through the development of specific methodological tools, some of which are drawn and re-elaborated from the experience of TELMA project (Cerulli et al., 2008).

In this paper we present two different Teaching Experiments designed and carried out within ReMath project, respectively by Didirem team of the University Paris 7 (France), and by Unisi team of the University of Siena (Italy). Both the TEs were designed around the use of the software Casyopée (partly developed within the project). After describing the main features of Casyopée (exploited in the Teaching experiments) we will focus on the design of the Teaching Experiments, and we will compare them relying on the construct of Didactical Functionality (Cerulli, Pedemonte and Robotti, 2006). Though it would be interesting, a discussion on the actual implementation of the plans in classroom is out of the goals and of the possibilities of the present paper.

## THE CONSTRUCT OF DIDACTICAL FUNCTIONALITY

The construct of Didactical Functionality is meant to provide a minimal common perspective, hopefully independent from specific theoretical frameworks, to frame diverse approaches (possibly depending on theoretical references) to the use of ICT tools in mathematics education, as well as the theoretical reflections regarding the actual use of ICT tools in given contexts.

By Didactical Functionality of an ICT tool, one means the system constituted by three interrelated poles: a set of features of the tool, a set of educational goals, and the modalities of employing the specified features of the tool for achieving the envisaged educational goals.

Trivially, through the construct of Didactical Functionality one intends to acknowledge that an ICT tool (or part of it) can be used in different ways for achieving different educational goals, that is one can design or identify different Didactical Functionalities of a given tool. In particular different theoretical perspectives can lead to designing different Didactical Functionalities of a given tool.

## THE DDA CASYOPEE

The DDA Casyopée (Lagrange and Chiappini, 2007) is built as an open problem-solving environment with the aim of giving students a means to work with algebraic representation, progressively acquiring control of the sense of algebraic expressions and of their transformations. Functions are the basic objects in Casyopée. Using this tool, students can explore and prove properties of functions. Casyopée takes into account the potentialities that Computer Algebra Systems offer to teaching and learning: going beyond mere numerical experimentation and accessing the algebraic notation; focusing on the purpose of algebraic transformations rather than on manipulation and connecting the algebraic activities. It is expected that students will make sense of algebraic representations by linking these with representations in these domains. See below a screen copy on the algebraic representations provided by Casyopée, it splits into two windows: a symbolic one and a graphical one.

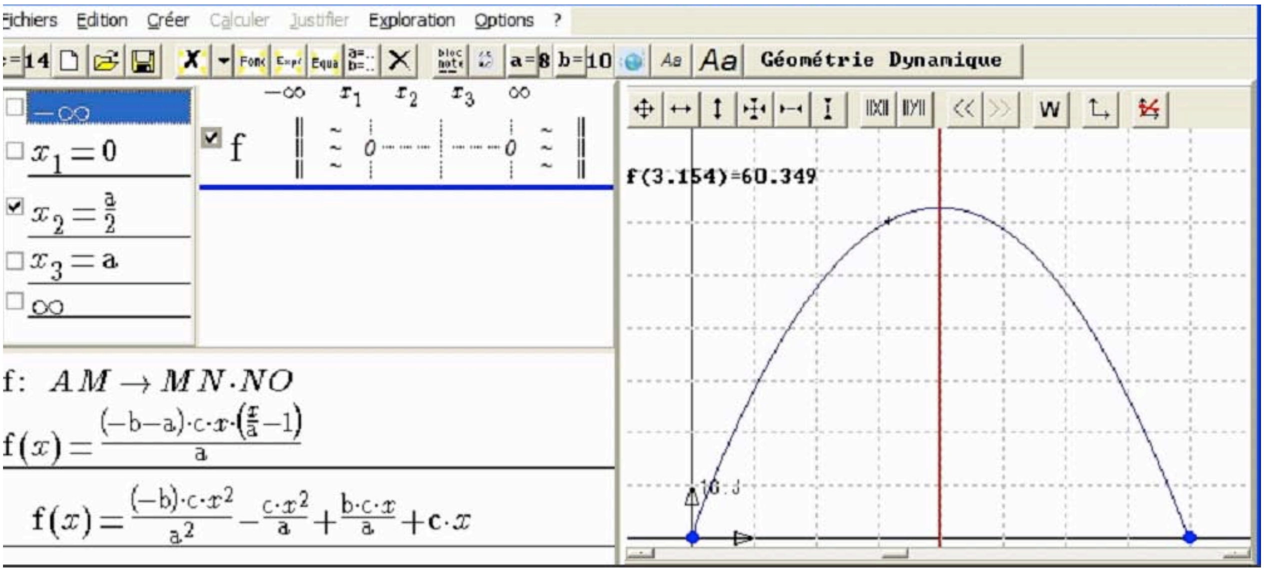


**Figure 1: the algebraic setting in Casyopée**

In the Remath project, Casyopée has been extended with a geometrical module. The aim is to explore what can be an interesting cooperation between a geometrical problem and its analytic treatment. The goal is not to develop a whole geometric dynamic environment but rather to see how geometric and analytic environments can articulate each other. For instance, a geometrical figure can be a domain to experiment with geometrical calculations. In the screenshot below, students can ask for the measure of the area of the rectangle MNOP. Then an algebraic model can be built choosing one of the measures as an independent variable and the other as a dependant variable. Properties of the dependency can be conjectured and proved: they take sense both in the algebraic and in the geometrical settings.

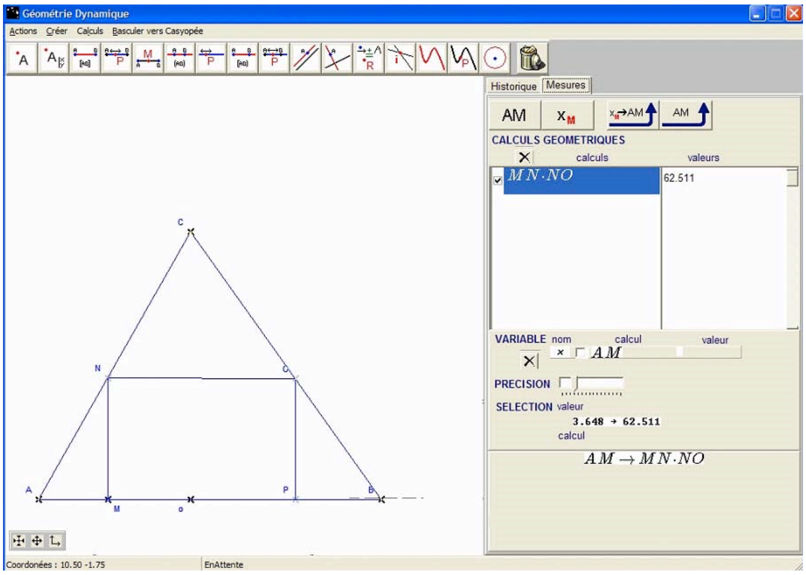


**Figure 2: the geometrical window in Casyopée**

The main specificity of Casyopée among other dynamic geometrical artefacts is to connect geometric and algebraic approaches. More precisely, the geometrical frame allows one to consider a geometric calculation and to export it in the algebraic environment. This transfer is allowed by choosing an adequate variable for the geometrical situation. At this point, Casyopée gives a feedback on the choice of this independent variable.

The representations offered by Casyopée have been thought to be close to institutional ones. Casyopée allows students to work with the usual operations on functions such as algebraic operations, analytic calculations and graphical representations. The geometric environment offers commands usually available in other dynamic geometry environments such as creating fixed and free geometrical objects (points, lines, circles, curves)

## UNISI AND DIDIREM PEDAGOGICAL PLANS

In the introduction we recalled that different specific methodological tools have been developed within ReMath for fostering the comparability of studies dealing with the use of ICT tools in mathematics education. A new conceptual model of the pedagogical scenario, called Pedagogical Plan (Bottino et al. 2007), is one of those methodological tools. A Pedagogical Plan has a recursive hierarchical structure: each pedagogical plan is conceived as a tree whose nodes and leaves are pedagogical plans themselves. Several components are attached to each pedagogical plan: including the articulation of the educational goals, of the class activities, the specification of the features of the ICT tool used and how they are used, and of the rationale underpinning the whole pedagogical plan and of the theoretical frames inspiring it. A web-based tool (Pedagogical Plan Manager, PPM) has been also developed for supporting teams in designing their pedagogical plans.

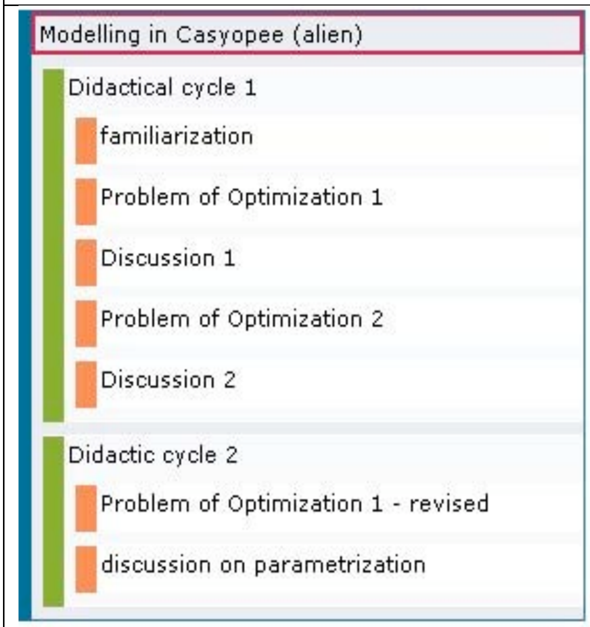
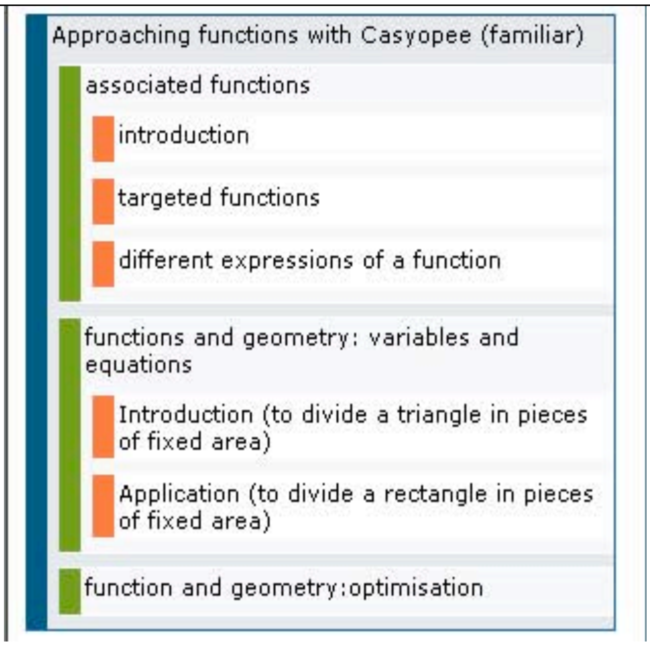
| UNISI | DIDIREM |
|---|---|
|  |  |





**Figure 3: synthetic view of Unisi and Didirem pedagogical plans in the PPM**

Figure 3 displays a screenshot from the PPM, and it is meant to provide an overview of the structures of the pedagogical plans designed by the Unisi and Didirem teams.

**Details of the Unisi pedagogical plan**

The Unisi pedagogical plan is inspired by the Theory of Semiotic Mediation (Bartolini Bussi and Mariotti, 2008) drawn from a Vygotsijan perspective. This theory guided both the specification of the educational goals (starting from an analysis of Casyopée) and the overall structure of the planned activities.

The designed educational goals are (a) to foster the evolution of students' personal meanings towards the mathematical meanings of function as co-variation. That regards also the notions of variable, domain of a variable… and (b) to foster the evolution of students' personal meanings towards mathematical meanings related to the algebraic modelling of geometrical situations.

Students are expected to have already received some formal teaching on the notions of variable, function and graph of a function, and on its graphical representation in a Cartesian plane. Moreover, a common experience of researchers and teachers is that meanings related to those notions are rarely elaborated in depth. The aim is to mediate and weave those meanings in the more global frame of modelling.

Hence, the pedagogical plan is not meant to help students become able to use Casyopée for accomplishing given tasks, but instead to foster the students' consciousness-raising of the mathematical meanings at stake.

The whole pedagogical plan is structured in cycles entailing: students' pair or small group activity with Casyopée for accomplishing a task, students' personal rethinking of the class activity (through the request to students of producing individual reports on that activity), class discussion orchestrated by the teacher.

The familiarization session is designed as a set of tasks aims at providing students with an overview of Cayopée features and guiding students to observe and reflect upon the "effects" of their interaction with the tool itself, e.g.:

> Could you choose a variable acceptable by Casyopée and click on the "validate" button? Describe how did the window "Geometric Calculation" change after clicking on the button. Which new button appeared?

Besides familiarization, the designed activities with Casyopée consist of coping with "complex" optimization problems formulated in a geometrical setting and posed in generic term, e.g.:

> Given a triangle, what is the maximum value of the area of a rectangle inscribed in the triangle? Find a rectangle whose area has the maximum possible value.

The aim is to elaborate on those problems so to reveal and unravel the complexity and put into evidence step by step the specific mathematical meanings at stake.

According to the designed pedagogical plan, the teacher plays the delicate role of guiding students to unravel such complexity and to make the targeted mathematical meanings emerge. The main tool for the teacher to achieve this objective, is the orchestration of the class discussions. The development of a class discussion cannot be completely foreseen *a priori*, it should be designed starting from the analysis of students' actual activity with Casyopée and of the reports they produce, and it would still depend on extemporary stimuli. Nevertheless in the design Unisi team tried to anticipate possible development of the pedagogical plan and to plan some kind of possible canvas for the teachers for managing class discussions.

The pedagogical plan is intended for scientific high schools or technical institutes, grade 12 or 13, and can be implemented over approximately 11 school hours.

**Details of the Didirem pedagogical plan**

The Didirem pedagogical plan aims to help students construct or enrich knowledge in the following areas: meaning of functions as algebraic objects and meaning of functions as means to model a co variation in geometric and algebraic settings. It is intended for scientific high schools grade 11 or 12 and has been implemented in ordinary classes during approximately 10 school hours. It is inspired both by the Instrumental Approach (Artigue, 2002), the Theory of Situation (Brousseau, 1997) and the Theory of Anthropologic Didactic (Chevallard, 1999).

Specific importance is given to the construction of tasks with an adidactical potential, where students can choose different variables for exploring functional dependencies, and to the connection between algebra and geometry. This connection is supported in Casyopée by geometric expressions that allow expressing magnitudes in a symbolic language mixing geometry and algebra.

The pedagogical plan is built around three main types of tasks:

- First session: finding target quadratic functions by animating parameters (five different tasks according to the semiotic forms used for these functions):

  Lesson 1: Introducing associated functions (a function $g$ is associated to a function $f$ if it is defined by a formula like $g(x)=af(x)+b$ or $f(ax+b)$ or similar)

  Lesson 2: Target Functions (functions that can be graphed but whose expression is not known; each student have to guess the function graphed by his/her partner)

  Lesson 3: Different expressions of quadratic functions

So students should consolidate: the meaning of variable, the distinction between variable and parameter, the meaning of function of one variable with several registers of semiotic representation and the fact that a same function may have several algebraic expressions. The new notion of associated function is worked-out during this session.

- Second session: creating a geometrical calculus as a model of a geometrical situation to solve a problem of relationships between areas, manipulation to experiment co variation between two geometrical variables:

Lesson 4: To divide a triangle in pieces of fixed area

Lesson 5: Application; dividing a rectangle into figures of fixed area

This way students can enhance their knowledge on co variation and develop the ability to experiment and anticipate in a dynamic geometrical situation, and the ability to model a geometric situation through geometric calculations.

- Third session: creating a function as a model of a geometrical situation to solve an optimization problem.

Lesson 6: solving a problem of optimisation in geometric settings by way of algebraic modelling.

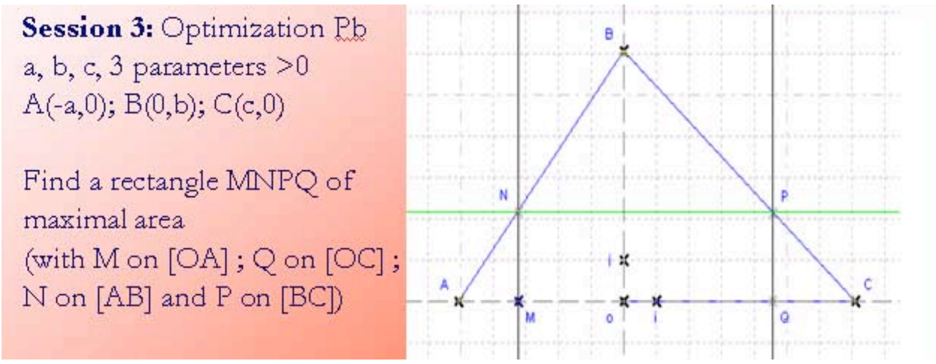



**Figure 4: statement of the session 3 in Didirem pedagogical plan**

This problem allows both to reinvest abilities to use the DDA, previous knowledge on associated functions and to introduce the notion of optimum in a geometrical situation.

## COMPARISON OF THE UNISI AND DIDIREM APPROACHES USING THE CONSTRUCT OF DIDACTICAL FUNCTIONALITY

The two pedagogical plans, described in the previous sections, evidently share some characteristics but also have apparent deep differences. In this section we use the frame provided by the construct of Didactical Functionality to develop a more systematic comparison between the two pedagogical plans.

### Tool Features

The two pedagogical plans are not generally centred on the use of the same DDA, but more specifically on the use of the same DDA features. In fact both exploit especially

(a) features of the dynamic geometry environment: the commands for creating fixed, free or constrained points, for dragging free or bonded points, for creating points with parametric coordinates, and the corresponding feedbacks of the DDA;

(b) features of the geometric calculation environment: the commands for creating "geometric calculation" associating numbers to geometrical objects, for choosing (independent) variables, for creating function between the selected variable and calculation, and the corresponding feedbacks;

(c) features of the algebraic environment, including the commands for displaying and exploring graphs of functions, for creating and manipulating parameters, for manipulating the algebraic expressions of functions, and the corresponding feedbacks.

**Educational Goals**

Different educational goals are associated to the use of those features. More precisely, one can recognize that both pedagogical plans share a common focus on some mathematical notions: function (in particular, conceived as co-variation), variables (independent and dependent) and parameters. Moreover the two pedagogical plans present, among other tasks, two optimization problems sharing the same mathematical core (see sections…). But, besides those surface similarities, there are profound differences.

Other Unisi educational goals are to mediate and weave meanings, related to the notions of function, variable and parameter. With that respect the Unisi team assumes, on the one hand, that those notions are familiar for students, and, on the other hand, that those notions are not elaborated in depth. Hence the Unisi pedagogical plan aims at helping students gain a deeper consciousness of the mathematical meanings at stake and re-appropriate them in the more global frame of modelling. In addition the Unisi objective includes the shared and decontextualized formulation of the different mathematical notions in focus.

The Didirem objectives are mainly to use potentialities of representations offered by Casyopée to introduce some new mathematical knowledge. This knowledge has been chosen for two main reasons: its affordance to the French curriculum and the importance to be studied in several frames of representations.

**Modalities of employment**

In accordance with the different objectives and the different pedagogical culture, the modalities of use are different as well.

The Unisi pedagogical plan has an iterative structure. Students' activity with Casyopée alternates with class discussions, after each session students are required to produce individual reports on the performed activities. This structure is meant to foster students' generation of personal meanings linked to the use of the DDA and their evolution towards the targeted mathematical meanings together with the students' consciousness-raising of the mathematical meanings at stake. That process is constantly fuelled by the teacher, whose role is crucial. Accordingly the teacher's role is explicitly taken into account in the design of the pedagogical plan, which provides with hints for the possible actions. The tasks used are optimization problems set in a geometrical frame. Their solution and the reflection on these solutions are fundamental steps towards the achievement of the designed educational goals. Also the familiarization with the DDA has to be considered within that perspective: as already mentioned, it aims at making students observe and reflect upon the "effects" of their interaction with the DDA itself. *Ad hoc* tasks are designed for that purpose.

Instead, the Didirem team pays specific attention to a progressive use of the DDA combining artefact and mathematical knowledge. Indeed, students work only in the algebraic window during section 1, then only in the geometrical windows in section2; finally section 3 gives an opportunity to reinvests the knowledge in the two environments. Moreover, all the tasks proposed are mathematical ones and are elaborated in order to allow students make progress alone working on the problem and to construct their new knowledge thanks the feedbacks.

## CONCLUSION

Those differences can be strongly related with the different theoretical perspectives adopted by the two teams.

The Unisi team has mainly structured its pedagogical plan according to the Theory of Semiotic Mediation which inspired both the specification of the educational goals and the organization of the activities in iterative cycles. In particular the Theory of Semiotic Mediation led the Unisi team to devote attention towards the design of the teacher’s action in the pedagogical plan. In fact, the teacher plays a crucial role throughout the whole pedagogical plan, especially for fostering the evolution of students’ personal meanings towards the targeted mathematical meanings and facilitating the students’ consciousness-raising of those mathematical meanings.

Instead, the Didirem team splits its theoretical approach into several theoretical frames which shape their pedagogical plan: the Instrumental Approach (Artigue, 2002), the theory of Situation (Brousseau, 1997) and at last the theory of anthropologic didactic (Chevallard, 1999). The first frame aims to go further than a simple familiarization with the DDA and to help the students constructing a mathematical instrument. This process goes hand in hand with the learning process. The last optimization problem is used to evaluate the progress of this process. The process is accurately designed through a careful choice of mathematical tasks, with an adidactical potential, whereas the definition of the teacher's actions and role escapes the design of the PP. Finally, the TAD is called upon to manage instrumental distance between institutional and instrumental knowledge.

No doubt that these approaches are complementary. Each team might benefit from this collective work to improve its pedagogical plan in the future. For instance, the Didirem team plans to pay more attention to the teacher’s role during the pedagogical plan conception. Nevertheless, the objective is not to elaborate a wide common consensual theoretical frame, but rather to go in depth in the clarification of didactical functionalities, in a shared language.

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# NAVIGATION IN GEOGRAPHICAL SPACE

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*This study is part of the ReMath project (Remath' – Representing Mathematics with Digital Media FP6, IST-4, STREP 026751 (2005 – 2008), http://remath.cti.gr. Twenty four 10th Grade students participated in a constructivist teaching experiment, the aim of which was to investigate children's constructions of mathematical meanings concerning the concept of function while navigating within 3d large scale spaces. The results showed that the utilization of the new representations provided by the dynamic digital media such as Cruislet could reform the way that mathematical concepts are presented in the curricula and possibly approach these mathematical notions through meaningful situations. The new representations provide the opportunity to introduce and study mathematical notions not as isolated entities but rather as interconnected functionalities of meaningful real – life situations.*

Functions are a central feature of mathematics curricula, both past and present. Many research studies indicate students' difficulty in understanding the concept of functions. This difficulty comes from a) the static media used to represent the concept, b) the introduction of function mainly as a mapping between sets in conventional curricula, c) the use of formalisation and function graphs as the only representations. With digital media, students can dynamically manipulate informal representations of function defined as co-variation and rate of change, which is an interesting and powerful mathematical concept. Tall(1996) points out a fundamental fault-line in "calculus" courses which attempt to build on formal definitions and theorems from the beginning. Moreover, he suggests that enactive sensations of moving objects may give a sense that "continuous" change implies the existence of a "rate of change", in the sense of relating the theoretically different formal definitions of continuity and differentiability. The enactive experiences provide an intuitive basis for elementary calculus built with numeric, symbolic and visual representations.

The 'Cruislet' environment is a state-of-the-art dynamic digital artefact that has been designed and developed within the Eu ReMath project. It is designed for mathematically driven navigations in virtual 3d geographical spaces and is comprised of two interdependent representational systems for defining a displacement in 3d space, a spherical coordinate and a geographical coordinate system. We consider that the new representations enabled by digital media such as Cruislet can place mathematical concepts in a central role for both controlling and measuring the behaviours of objects and entities in virtual 3d environments. The notion of *navigational mathematics* is used to describe the mathematical concepts that are embedded and the mathematical abilities the development of which is supported within the Cruislet microworld. In this study we focus on how students using spherical and geographical systems of reference in Cruislet construct meanings about the concept of function.

## THEORETICAL FRAMEWORK

A number of research studies suggest that students of all grades, even undergraduate students, have difficulties modelling functional relationships of situations involving the rate of change of one variable as it continuously varies in a dependent relationship with another variable (Carlson et all, 2002; Carlson, 1998, Monk & Nemirovsky, 1994). This ability is essential for interpreting models of dynamic events and foundational for understanding major concepts of calculus and differential equations. On the other hand, the VisualMath curriculum (Yerushalmy & Shternberg, 2001) is an a example of a function based curriculum that involves the moving across multiple views of symbols, graphs, and functions. VisualMath uses specially designed software environments such as simulations' software, or other modelling tools that include dynamic forms of representations of computational processes. Yerushalmy (2004) suggests that such emphasis on modeling offers students means and tools to reason about differences and variations (rate of change). Moreover, .Kaput and Roschelle (1998) using computer simulations study aspects of calculus at an earlier stage. These simulations (MBL tools), permit the study of change and the ways it relates to the qualities of the situation. In their study Nemirovsky, Kaput and Roschelle (1998) show that young children can use the rate of change as a way to explore functional understanding. In studying the process of the understanding of dynamic functional relationships, Thompson (1994) has suggested that the concept of rate is foundational.

Confrey and Smith (1994) choose the concept of rate of change as an entry to thinking about functions. They introduce introduce two general approaches to creating and conceptualizing functional relationships, a correspondence and a covariation approach. They suggest that “a covariational approach to functions makes the rate of change concept more visible and at the same time, more critical (p. 138). They explicate a notion of covariation that entails moving between successive values of one variable and coordinating this with moving between corresponding successive values of another variable.

Moreover, Carlson, Larsen and Jacobs (2001) stress the importance of covariational reasoning as an important ability for interpreting, describing and representing the behavior of dynamic function event. They consider covariational reasoning to be the cognitive ability involved in coordinating images of two varying quantities and attending to the ways in which they change in relation to each other. On the same line, Saldanha and Thompson (1998) introduced a theory of developmental images of covariation. In particular, they considered possible imagistic foundations for someone’s ability to see covariation. Carlson et all (2001) in their study exploring the role of covariational reasoning in the development of the concepts of limit and accumulation, suggest a framework including five categories of mental actions of covariational reasoning:

1. An image of two variables changing simultaneously
2. A loosely coordinated image of how the variables are changing with respect to each other
3. An image of an amount of change of one variable while considering changes in discrete amounts of the other variable
4. An image of the average rate-of-change of the function with uniform increments of change in the input variable
5. An image of the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function

The proposed covariation framework contains five distinct developmental levels of mental actions. Using this particular framework we will try to classify students' covariational reasoning while studying navigation within the context of Cruislet microworld. We consider navigation as a dynamic function event. The function's independent variable is the geographical coordinates of the position of the first aeroplane, which students are asked to navigate, while the dependent variable is the geographical coordinates of the position of the second aeroplane.

Our approach to learning promotes investigation through the design of activities that offer a research framework to investigate purposeful ways that allow children to appreciate the utility of mathematical ideas (Ainley & Pratt, 2002). In this context, our approach is to design tasks for either exclusively mathematical activities or multi-domain projects containing a mathematical element within the theme which can be considered as marginalized or obscure within the official mathematics curriculum (Kynigos & Yiannoutsou, 2002, Yiannoutsou & Kynigos, 2004).

## TASKS

In the tasks that are included in this teaching experiment, students actually engage with the study of the existence of a rate of change of the displacements of the airplanes which are defined in the geographical coordinate system. In particular the dispacements of two airplanes are relative according to a linear function. This function will be hidden and the students will have to guess it in the first phase of the activity based on repeated moves of aeroplane A and observations of the relative positions and moves of planes A and B. The second phase, the students will be able to change the function of relative motion and play games with objectives they may define for themselves such as move plane A from Athens to Thessaloniki and plane B from Athens to Rhodes and then to Thessaloniki in the same time period.

This scenario is based on the idea of half – baked games, an idea taken from microworld design (Kynigos, 2007). These are games that incorporate an interesting game idea, but they are incomplete *by design* in order to encourage students to change their rules. Students play *and* change them and thus adopt the roles of both player and designer of the game (Kafai, 2006).

Initially, students are asked to study the relation between the two aeroplanes, the rate of change of their displacements and consequently find the linear function (decode the rule of the game). In order to decode "the rule of the game", they should give various values to coordinates (Lat, Long, Height) that define the position of the first plane. They will be encouraged to communicate their observations about the position of the second plane to each other and form conjectures about the relationship between the positions of the two aeroplanes.

In the second phase students are encouraged to build their own rules of the game by changing the function of the relative displacements of the two aeroplanes.

## METHODOLOGY

The research methodology is a constructivist teaching experiment along the same lines as described by Cobb, Yackel and Wood (1992).The researcher acts as a teacher interacting with the children aiming to investigate their thinking. The researcher, reflecting on these interactions, tries to interpret children's actions and finally forms models-assumptions concerning their conceptions. These assumptions are evaluated and consequently either verified or revised.

Twenty four (24) students of the 1st grade of upper high school, (aged 15-16 years old) participated in this experiment. Students worked in pairs in the PC lab. Each pair of students worked on the tasks using Cruislet software.

The data consists of audio and screen recordings as well as students' activity sheets and notes. The data was analyzed verbatim in relation to students' interaction with the environment. We have focused particularly on the process by which implicit mathematical knowledge is constructed during shared student activity. As a result, in our analysis we use students' verbal transcriptions as well as their interaction with the provided representations displayed on the computer screen.

## ANALYSIS

While students were interacting with the Cruislet environment according to the tasks, several meanings emerged regarding the concept of function. We categorise these meanings in clusters that rely upon the concept of function. In particular, there are two major categories:

### Domain of numbers

Students navigating an aeroplane in the 3d map of Greece realized that the domain of the geographical coordinates is actually a closed group. The 3d map of Greece is a geographical coordinate system with specific borders. The investigation of the range of the geographical borders as the domain of the function became the subject of study and exploration through the use of the Cruislet functionalities. In particular, students exploited the two different systems of reference and, experimenting with the values of the geographical coordinates, they define the range of the latitude – longitude values. This specific range of values has been considered as the domain of the functions according to which the displacements of the aeroplanes are relative. Although students didn't refer to the values as the domain of the function, we interpret their involvement in finding them, as a mathematical activity regarding the domain of the function.

Students experimented by giving several values to the geographical coordinates of the airplane's position defining at the same time the range of the coordinates' values. In the following episode students are trying to find out the reason for not placing the airplane in a given position.

S1: Why?? It doesn't accept any value. (they gave values in procedure fly1 and the airplane couldn't go).

R: Do you remember what values the lat long coordinates have?

Isn't lat equals 58 isn't correct? (she also speaks to the next team)

S1:It doesn't accept 32 20 100 either.

S2: Greece hasn't got value 20 (student from another team speak ironically to him)

S1: Why? Was the 58 you used correct?

An interesting issue related to the domain of the function, is that the provided representations, i.e. the result of the aeroplane's displacement displayed on the screen, helped students realize that the domain of numbers of the two aeroplanes displaced in relative positions, are strongly dependent. For instance when the first moved to a given position, the second one couldn't go anywhere, but the domain of values was restricted by the first position. In the following episode students realized that the 2nd aeroplane didn't follow them when they flew at a low height. The episode is interesting as it indicates the way students realize the domain of geographical coordinate values that the first aeroplane can take in relation to the other one.

S1: There are some times that it (meaning the other aeroplane) can't follow us.

R: Where? When?

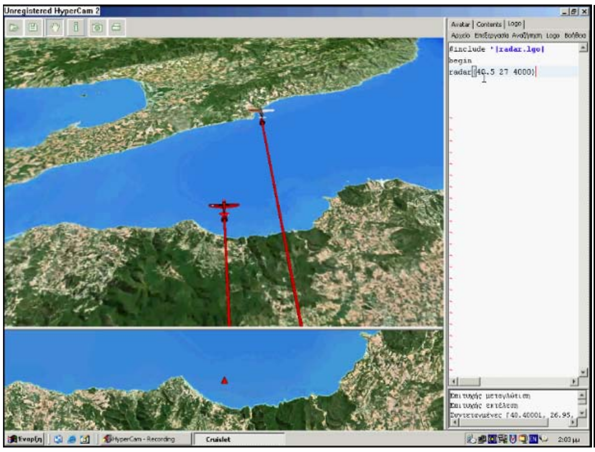
S1: When I'm getting into the sea.

We could say that the characteristics of Cruislet software, such as the visualization of the results of the objects' displacements on the map, acted as a mediator in students' engagement with the domain of function. We have to mention that although the modalities of use of Cruislet software and the communication within the groups didn't reveal that students realized or mentioned anything regarding the concept of function, they did focus on finding ways to move the aeroplanes. In other words, students didn't conceive the values of the coordinates as the domain of the function, although they used it in this way. The interpretation of students' actions relies upon our educational goals, which conceive this as a mathematical activity that was related to the notion of function and particularly, its domain.

**Function as covariation**

During the implementation of the tasks, students engaged with the notion of function, through their experimentation with the dependent relationship between two aeroplanes' positions, which was defined by a black – box Logo procedure. Trying to find out the hidden function, students' actions and meanings created, suggested they were able to coordinate changes in the direction and the amount of change of the dependent variable in tandem with an imagined change of the independent variable. Our results indicate that students developed covariational reasoning abilities, resulting in viewing the function as covariation.

Initially most of the students expressed the covariation of the aeroplanes' positions using verbal descriptions, such as behind, front, left, etc. as they were visualizing the result of the airplanes' displacements. In the following episode students express the dependent relationship while looking at the result displayed on the screen.



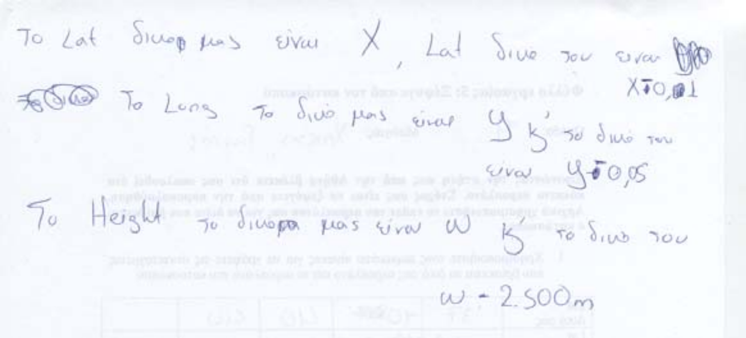
S1: So, he always wants to be close to us on our left.

R: Yes.

S1: And he is beneath, further down to us. Beneath.

S2: And behind.

Students experimented by giving several values to geographical coordinates in Logo and formed conjectures about the correlation between the aeroplanes' positions. Through their interaction with the available representations, they successfully found the dependent relation of the function in each coordinate, resulting in their coming into contact with the concept of function as a local dependency. In fact, one of the teams conceived the relationship among each coordinate as a function, as is obvious in their notes on the activity sheet.



Translation

| |
|---|
| Our Lat is x, his Lat is x – 0.1 |
| Our Long is y and his is y – 0.05 |
| Our Height is ω and his is ω – 2500m. |

It is interesting to mention that students separated latitude and longitude coordinates on the one hand and that of height on the other as they were trying to decode the hidden functional relationship between the airplanes' height coordinates. In particular, they didn't encounter difficulties in decoding latitude and longitude relationship in contrast to their attempts to find the height dependency. Although all three functions regarding coordinates were linear, students conceived the functional relationship between height mainly as proportional, in contrast to latitude and longitude that were comprehended as linear, from the beginning. In the following episode, students endeavor to apply the rate of change of the function to decode the height relationship. As they thought the height coordinates had a proportional relationship, they suggested carrying out a division to find it.

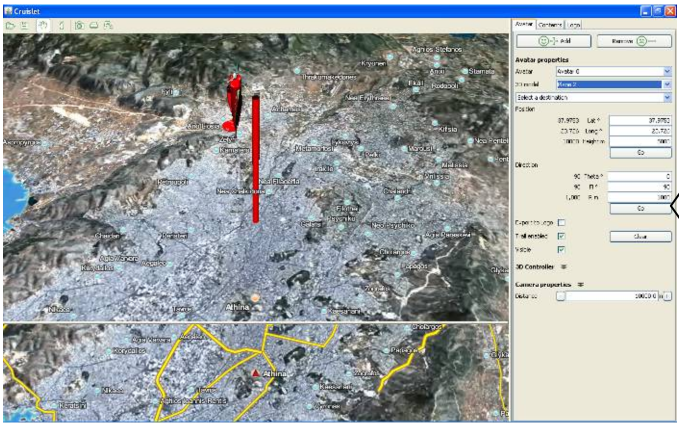
S2: When we go up 1000, he goes up 1000.

R: Do you mean that if we go from 7000 to 8000 he goes from… let's say 2500 to 3500.

S2: He is at… 3000. No. Give me a moment. At 8000 he was at 5500. At 7000 he was at 4500. At 5000 he is as 2500. And then….

S1: We could do the division to see the rate.

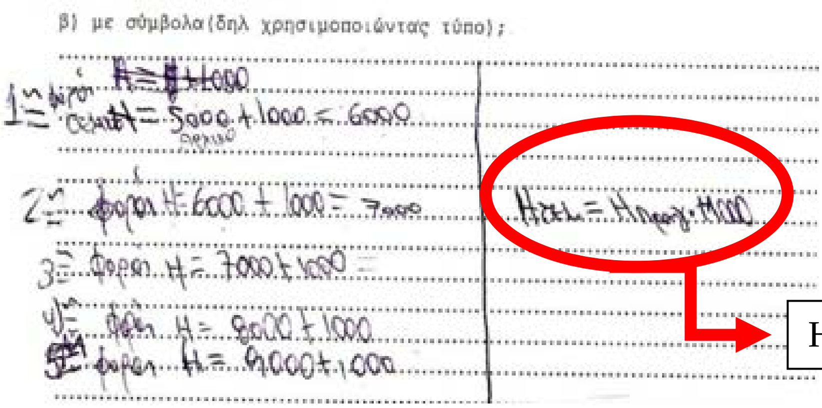
An interesting example was the cases of the variation of the height of the aeroplane every time they pushed the button 'go' in spherical coordinates, when they wanted to make a vertical displacement. In particular, by defining the vector of a vertical upward displacement, students observed that height was the only element that changed in the position of the displacement. Through a number of identical displacements students identified and expressed verbally, symbolically and graphically the interdependency between direction functionality and the height of the aeroplane. Students' reasoning: "the more times we push the button GO the higher the aeroplane goes", suggests that students developed a covariational reasoning ability similar to the second level proposed by Carlson et al (2001) of how the variables change with respect to each other. Moreover, the retrospective symbolic type developed by students (h2= h1+1000) indicates that they realized that the rate of change of the height is constant. In the following figures we can see the result displayed on the screen (figure 1) as well as students' writings on the activity sheet (figure 2).



Students' actions:

1. Define the spherical coordinates (theta = 0 fi = 90 R = 1000).
2. Push the "Go" button in "Avatar properties" tab resulting in the vertical displacement of the aeroplane.
3. Watch the displacement of the aeroplane on the GUI.
4. Focus on the changes of the height coordinate.

Figure 1




$H_{final} = H_{before} + 1000$

Figure 2

The provided representations of Cruislet software became a vehicle to engage students with concepts related to the concept of function and their expression in a mathematical way. The result of airplanes' displacements on the screen, gave them the chance to realize the dependent relation in 'visual terms' and then express it in mathematical terms. We believe that the results are mainly based on the way that these characteristics were used in the task activity. In particular, the activity was based on the idea of the 'Guess my function' game and the dependent relationship, (built in Logo programming language), was hidden at first. Due to this choice, students focused primarily on the observation of the relative displacements and not on the Logo code underneath it. At the same time perceiving the activity as a game encourages the engagement of students with the activity.

## CONCLUSIONS

The study indicated that students exploiting Cruislet functionalities can construct meanings concerning the concept of functions. The provided linked representations (spherical and geographical coordinates), as well as the functionalities of navigating in real 3d large scale spaces actually enable students to explore and build mathematical meanings of the concept of function within a meaningful context. They explore the domain of numbers of a function within a real world situation distanced from the "traditional" formal definitions. On the other hand, they built the concept of function as covariation exploring the variation of the spherical and geographical coordinates. The provided context gave students the opportunity to cope with and explore mathematical concepts at different levels. They navigate within 3d large scale spaces controlling the displacement of an avatar and develop their visualization abilities building mathematical meanings of the concept of function while at the same time they explore the mathematical concepts of spherical and geographical coordinates.

The functionalities of the new digital media such as Cruislet provide a challenging learning context where the different mathematical concepts and mathematical abilities are embedded and interconnected. The role of the teacher becomes crucial in designing mathematical tasks where students' enactive explorations will reveal these mathematical notions and put them under negotiation. In the case of Cruislet, navigational mathematics becomes the core of the mathematical concepts that involves the geographical and spherical coordinate system interconnected with the concept of function and the visualization ability.

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# MAKING SENSE OF STRUCTURAL ASPECTS OF EQUATIONS BY USING ALGEBRAIC-LIKE FORMALISM

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*This paper reports on a design experiment conducted to explore the construction of meanings by 17-year-old students, emerging from their interpretations and uses of algebraic-like formalism. The students worked collaboratively in groups of two or three, using MoPiX, a constructionist computational environment with which they could create concrete entities in the form of Newtonian models by using equations and animate them to link the equations' formalism to its visual representation. Some illustrative examples of two groups of students' work indicate the potential of the activities and tools for expressing and reflecting on the mathematical nature of the available formalism. We particularly focused on the students' engagement in reification processes, i.e. making sense of structural aspects of equations, involved in conceptualising them as objects that underlie the behaviour of the respective models.*

## INTRODUCTION

In this paper we report on a classroom research [1] aiming to explore 17-year-old students' construction of meanings, emerging from the use of algebraic-like formalism in equations used as means to create and animate concrete entities in the form of Newtonian models. The students worked collaboratively in groups of two or three using a constructionist computational environment called "MoPiX" [2], developed at the London Knowledge Lab (http://www.lkl.ac.uk/mopix/) (Winters et al., 2006). MoPiX allows students to construct virtual models consisting of objects whose properties and behaviours are defined and controlled by the equations assigned to them. We primarily focused on how students interpreted and used the available formalism while engaged in *reification* processes (Sfard, 1991), i.e. making sense of structural aspects of equations, involved in conceptualising them as objects that underlie the behaviour of the respective models.

## THEORETICAL BACKGROUND

Recognising the meaning of symbols in equations, the ways in which they are related to generalisations integrated within specific equations and also the ways in which a particular arrangement of symbols in an equation expresses a particular meaning, are all fundamental elements to the mathematical and scientific thinking. Research has been showing rather conclusively that the use of symbolic formalisms constitutes an obstacle for many students beginning to study more advanced mathematics (Dubinsky, 2000). Traditional approaches to teaching equations as part of the mathematics of motion or mechanics seem to fail to challenge the students' intuitions since they usually encompass static representations such as tables and graphs which are subsequently converted into equations. Lacking any chance of interacting with the respective representations, students fail to identify meaningful links between the components and relationships in such systems and the extensive use of mathematical expressions (diSessa, 1993). Indeed, students tend to use and manipulate physics equations in a rote manner, without understanding the concepts they covey (Larkin et al., 1980). Sherin (2001) argued that, in order to overcome this obstacle, students need to acquire knowledge elements that he termed *symbolic forms*. The acquisition of *symbolic forms* would help students make connections between an algebraic expression's conceptual content and its structure, which is considered to be crucial for the understanding, meaningful use and construction of physics equations.

In the mathematics education field, the relevant research is mainly based on the distinction between the two major stances that students adopt towards equations: the process stance and the object stance (Kieran, 1992; Sfard, 1991). The process stance is mainly related with a surface "reading" of an equation, concentrated into the performance of computational actions following a sequence of operations (i.e. computing values). In contrast, according to the object stance, an equation can be treated as an object on its own right, which is crucial to the students' development of the so-called *algebraic structure sense* (Hoch and Dreyfus, 2004), i.e. the act of being able to see an algebraic expression as an entity, recognise structures, sub-structures and connections between them, as well as to recognise possible manipulations and choose which of them are useful to perform. This development, linking procedural and structural aspects of equations, has been termed *reification* (Sfard, 1991) and has been considered to underlie the learning of algebra in general.

Recently, students' uses and interpretations of symbolic formalism in understanding mathematical and scientific ideas have been studied in relation to the representational infrastructure of new computational environments designed to make the symbolic aspect of equations more accessible and meaningful to children, especially through the use of multiple linked representations (Kaput and Rochelle, 1997). Adopting a broadly constructionist framework (Harel and Papert, 1991), we used a computer environment that is designed to enhance the link between formalism and concrete models, allowing us to study the ways in which the use of formalism, when put in the role of an expression of an action or a construct (a model), can operate as a mathematical representation for constructionist meaning-making. Our central research aim was to study students' construction of meanings emerging from the use of mathematical formalism when engaged in reification processes. We mainly focused on the development of their understanding on the structure of an equation based primarily on the conception of it as a system of connections and relationships between its component parts.

## THE COMPUTATIONAL ENVIRONMENT

MoPiX (Winters et al. 2006) constitutes a programmable environment that provides the user the opportunity to construct and animate in a 2d space, models representing phenomena such as collisions and motions. In order to attribute behaviours and properties to the objects taking part in the animations generated, the user assigns to the objects equations that may already exist in the computational environment's Equations Library or equations that she constructs by herself.

Figure 1 shows a red ball performing in the MoPiX environment a combined motion both in the vertical and horizontal axis, leaving a green trace behind. As one may observe, the equations attributed to the object incorporate formal notation symbols (Vx, x, t) as well as programming–natural language utterances (ME, appearance, Circle). However, their main characteristic is that they constitute functions of time, as it is stated by the second argument on the parentheses on their left side. For example, the horizontal motion equations attributed to the ball define the object's: horizontal position at the 0 time instance (1), horizontal position at any time instance (2), the horizontal velocity at the 0 time instance (3), the horizontal velocity at any time instance (4) and the horizontal acceleration at any time instance (5). The MoPiX environment constantly computes the attributes given to the objects in the form of equations and updates the display, generating on the screen the visual effect of an animation.

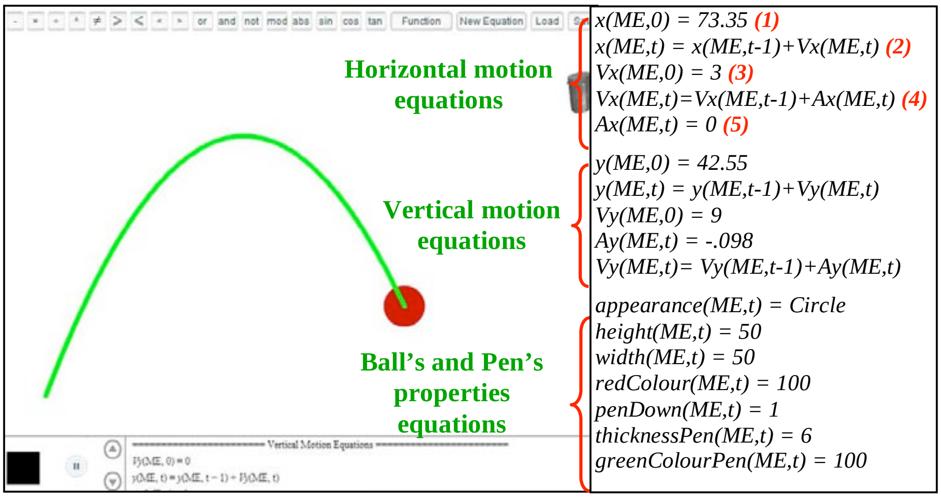


**Figure 0. The MoPiX environment**

Some specific features of MoPiX, underlying the novel character of the representations provided, may offer students opportunities to further appreciate utilities of the algebraic activity around the use of equations. The first of these features is that MoPiX offers a strong visual image of equations as containers into which numbers, variables and relations can be placed. The meaningful use of the environment may allow students to easily make connections between the structure of an equation and the quantities represented in it. The second feature of MoPiX is that it allows the user to have deep structure access (diSessa, 2000) to the models animated. The equations attributed to the objects and underpin the models' behaviour do not constitute "black boxes", unavailable for inspection or modifications by the user (for a discussion on black and white box approaches see Kynigos 2004). The third feature of MoPiX is that the manipulations performed to a model's symbolic facet (e.g. changing a value or removing an equation from the model) produce a visual result on the Stage, from which students can get meaningful feedback. "Debugging" a flawed animation demands students' engagement in a back and forth process of constructing a model predicting its behaviour, observing the animation generated, identifying the equations that are responsible for the "buggy" behaviour and specifying which and how particular parts need to be fixed.

## TASKS

For the first phase of the activities we developed, using exclusively “Library” equations, the “One Red Ball” microworld which consisted of a single red ball performing a combined motion in the vertical and the horizontal axis. The students were asked to execute the model, observe the animation generated, discuss with their teammates and other workgroups the behaviours animated and write down their remarks and observations on a worksheet. In order to provoke discussions regarding the equations’ role and stimulate students to start using the equations themselves, we asked them to try to reproduce the red ball’s motion. In this process, we encouraged them to interpret and use equations from the “Library”, add and remove equations from their objects so as to observe any changes of behaviour and link the equations they used to the behaviours they had previously identified. As we deliberately made the original red ball move rather slowly, near the end of this phase, we expected students to start expressing their personal ideas about their own object’s motion (e.g. make it move faster) and thus start editing the model’s equations, using the “Equations Editor”, so as to describe the new behaviours they might have in mind.

For the second phase of the activities we designed a half–baked microworld (Kynigos 2007), i.e. a microworld that incorporates an interesting idea but it is incomplete by design so as to invite students to deconstruct it, build on its parts, customize and change it. In this case we built a game–like microworld –called “Juggler” (Kynigos 2007)– consisting of three interrelated objects: a red ball and two rackets with which the ball interacted. The ball’s behaviour was partially the same as the “One Red Ball’s”. However, certain equations underpinning its behaviour, did not derive from the environment’s “Library” but were created by us. Using the mouse the rackets could be moved around and make the ball bounce on them, forcing it to move away in specific ways.

We asked the students to execute the Juggler’s model, observe the animation generated and identify the conditions under which each object interacted with each other. The students were encouraged to discuss with their teammates on how they would change the “Juggler” microworld and embed in it their own ideas regarding its behaviour. In the process of changing the half–baked microworld, students were expected to deconstruct the existing model so as to link the behaviours generated on the screen to its equations’ formalism and reconstruct the microworld, employing strategies that would depict their ideas about the new model’s animated behaviours.

## METHOD

The experiment took place in a Secondary Vocational Education school in Athens with one class of eight 12$^{th}$ grade students (17 years old) studying mechanical engineering and two researchers -the one acting also as a teacher- for 25 school hours. Students were divided in groups of two or three. The groups had at their disposal a PC connected to the Internet, the MoPiX manual, translations in Greek of selected equations' symbols and a notebook for expressing their ideas. The adopted methodological approach was based on participant observation of human activities, taking place in real time. The researchers circulated among the teams posing questions, encouraging students to explain their ideas and strategies, asking for refinements and revisions when appropriate and challenging them to express and implement their own ideas. A screen capture software was used so as to record the students' voices and at the same time capture their interactions with the MoPiX environment. Apart from the audio/video recordings, the data corpus involved also the students' MoPiX models as well as the researchers' field notes. For the analysis we transcribed verbatim the audio recordings of two groups of students for which we had collected detailed data throughout the teaching sequence and also several significant learning incidents from other workgroups. The unit of analysis was the episode, defined as an extract of actions and interactions performed in a continuous period of time around a particular issue. The episodes which are the main means of presenting and discussing the data were selected (a) to involve interactions with the available tool during which the MoPiX equations were used to construct mathematical meaning and (b) to represent clearly aspects of the reification processes emerging from this use.

## ANALYSIS AND INTERPRETATIONS

### Interpreting existing equations' symbols

In the first phase of the experimentation, the students in their attempt to reproduce the red ball' motion, started interpreting and using equations that already existed in the environment's "Equation Library". The natural language aspect incorporated in the MoPiX formalism was the element that guided their actions. The equations that they chose to assign first to their object were those whose symbols (at least some of them) were close to everyday language utterances and provided them some indication on the kind of the behaviour they described (e.g. the "amIHittingtheGround" symbol). Equations that contained symbols that didn't satisfy the "natural language" criterion (e.g. the "Ax") were simply disregarded.

As they continued their experimentations with MoPiX, the students seemed to gradually abandon the "natural language" criterion and shifted their attention into identifying the meaning of the symbols. The students of Group B for instance came across two "Library" equations that seemed to describe the velocity in the x axis, the "Vx(ME,0)=3" and the "Vx(ME,t)=Vx(ME,t-1) + Ax(ME,t)". Their decision to attribute the second one to their object, so as to define its velocity at any time instance, came as a result of a comparison between the two equations' left parts. Yet again, the students seemed to interpret specific symbols of the equations and completely disregard others (e.g. the "Ax" on the second equation's right part).

In a number of subsequent episodes, the same students seem to articulate their understanding not just about particular symbols but also about the whole string of the equation's symbols and the relations among them. In the following excerpt the students of Group B talk about the "x(ME,t)=x(ME,t-1)+Vx(ME,t)" equation.

| | |
|---|---|
| S1 | It *[i.e. x(ME,t)]* is the object *[i.e. "ME"]* in function with time *[i.e. "t"]*. |
| R2 | What does this mean? |
| S1 | *[goes on disregarding the question and points at the x(ME,t-1)]* It's your object *[i.e. "ME"]* in function with time minus 1 *[i.e."t-1"]*. |
| R2 | What does "in function with time" mean? Can you explain it to me? |
| S1 | How much... In every second, for example, how much it moves. |
| R2 | Meaning? |
| S2 | Wait a minute! *[Showing both parts of the equation]* The equation is this one. All of this. It's not just these two *[i.e. the x(ME,t) and the x (ME,t-1)]*. |
| S1 | Minus 1, which means that in every second of your time it subtracts always 1, resulting to something less than the current time. Plus your velocity. |

Drawing on his previous experience with the MoPiX equations, S1 starts to independently interpret the equation's symbols moving from left to right. Having interpreted the first two of them, he attempts to also interpret the relationship between them and defines it as the distance that the object has covered in a second of time. S2, who understands the kind of correlation S1 has made, intervenes and stresses the fact that he hasn't taken into account all the symbols in the equation. S1, who up to that point disregarded the "Vx(ME,t)" on the right part, takes an overall view of the equation and interprets it not by merely referring to the comprising symbols but by also referring to the connection between them. It is noticeable that at this point the students' actions demonstrate an emerging awareness of the equation's structure as a system of connections and relationships between the component parts.

## Variables and numerical values to control motion animations

As students gained familiarity with the MoPiX formalism, they started expressing their own personal ideas about the ways their objects should move. In order to put into effect those ideas, the students initially modified the existing equations' symbols and left the structure intact. One of the main elements that they often altered was the equations' arithmetic values. The students of Group B, for instance, attributed to their object the "Vy(ME,0)=0" equation which prescribed the object's y axis initial velocity to be 0. The observation of the animation triggered the implementation of a series of changes to the equation's arithmetic values starting with the conversion of the "0" on the right part into "3". The successive changes of the arithmetic value on the equation's right part didn't cause the object to constantly move since the equation referred just to the initial velocity. To make the velocity for "all the next time instances to come" to be "3", the students replaced the "0" on the left part (i.e. an arithmetic value) with "t" (i.e. a variable).

| | |
|---|---|
| S2 | Do we need a symbol for this? |

R2 Do we need a symbol? It's a good question. How do you plan to express it?

S2 With symbols we usually express something that we can't describe accurately.

S1 Plus… t. *[He writes down Vy(ME,t)=3]. [Showing the "t"]* So, when I look at this symbol

S2 I'll know it represents the infinity.

We suggest that the students relocated their focus from just attributing arithmetic values, which indicates a process stance to equations, into forming functional relationships. The fact that they were involved in a process of recognizing which manipulations were possible and at the same time useful to perform so as to express their idea, indicates a implicit focus on the structure of the equations. Furthermore, the statements concerning the use of symbols to express "something that we can't describe accurately" seems to constitute an indication of a progressive acquisition of algebraic structure sense through "mixed cues" (Arcavi, 1994) (i.e. interpreting symbols as invitations for some kind of action while working with them).

**Relating different objects' behaviours by constructing new equations**

The next episode describes how the Group A students, in the course of changing the "Juggler" microworld, didn't just use or edit existing equations but constructed from scratch two new ones. The idea they wanted to bring into effect was to "make a ball on the Stage change its colour according to an ellipse's position". Knowing that there was no such equation in the "Library", they started talking about how they would correlate those two objects using the Y coordinates.

S1 When it *[i.e. the ball]* is situated in a Y below the Y of this one *[i.e. the ellipse]* for example.

R1 I'm thinking… Will the ball know when it is below or above the ellipse?

S2 That's what we will define. We will define the Ys.

S1 This. The: "I am below now". How will we write this?

S2 Using the Ys. Using the Ys. The Ys. That is: when its Y is 401, it is red. When the Y is something less than 400, it's green!

Having conceptualized the effect they would like their new equation to have, the students in the above excerpt decide about two distinct elements regarding the equation under construction: its content (i.e. the symbols) and its structure (i.e. the signs between the symbols). Subsequently, encountering the fact that there was no in-built MoPiX symbol to express the idea of an object becoming green under certain conditions, the students came to invent one. The "gineprasino" (i.e. "become green" in Greek) symbol was decided to represent a varying quantity taking two distinct values (1 and 0, according to if the ball was below the ellipse or not). To represent the ball's position they chose to use its Y coordinate in terms of a quantity varying over time (i.e. "y(ME,t)") while for the ellipse's position they chose to use its Y coordinate in terms of the constant arithmetic value corresponding to the object's position on the Stage at that time (i.e. "274"). Adding a "less than" sign in between, the equation eventually developed was the "gineprasino(ME,t)=y(ME,t)≤274".

Unexpectedly, this equation didn't cause the ball to become green since it described solely the event to which the ball would respond (being below the ellipse) and not the ball's exact behaviour after the event would have occurred (change its colour). To overcome this obstacle, the students decided to construct another equation in which they tried to find out ways to integrate the "gineprasino" variable. A "Library" equation which explains what happens to a ball's velocity when it hits on one of the Stage's sides and the way in which a variable similar to the "gineprasino" was incorporated in it, led students to duplicate this equation's structure, eliminate any content and use it as a template to designate what happens to the ball's colour when it is below the ellipse. The second equation encompassed in-built MoPiX symbols (the "greenColour"), the "gineprasino" variable in two different forms (not(gineprasino) and gineprasino) and numerical values (0 and 100) to express the percentage of the green colour the ball would contain in each case (i.e. the ball being above and below the ellipse). Thus, the second equation developed was the: "greenColour(ME,t) = not(gineprasino(ME,t))×0 + gineprasino(ME,t)×100".

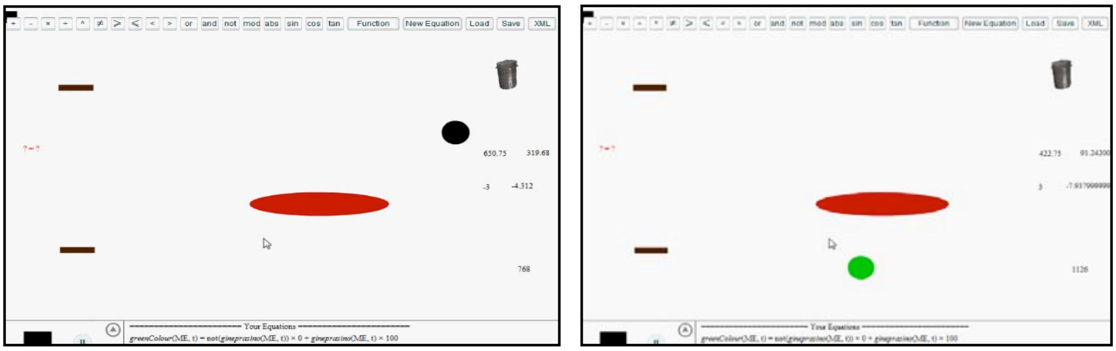


**Figure 2: The ball's different percentage of green colour according to its Y position**

The above episode contains many interesting events that indicate the existence of a qualitative transformation of the students' mathematical experience in reifying equations that emerged through their interaction with the available tools.

While building the first equation the students got engaged in processes such as inventing and naming variables, relating symbols with mathematical systems (i.e. the XY coordinate system) and manipulating inequality symbols to relate arithmetic values to variables. However, in building the second equation, the meaning generation evolved to include the students' view of equations as objects. The students extracted mathematical meaning from an equation that seemed to describe a behaviour similar to the one they intended to attribute to their ball. Conceptualizing a mapping between the ideas behind the two equations, the students duplicated the similar equation's structure and inserted new terms so as to define a completely novel behaviour for their object. This is a clear indication that they recognised the existence of structures external to the symbols themselves and used them as landmarks to navigate the second equation's construction process.

The manipulation of the second equation's new terms reveals further their developing structural approach to equations. By inserting in the second equation the the "gineprasino" variable which was introduced in the first one and providing it new forms (i.e. not(gineprasino)), the students seem to have conceptualised the first equation as a mathematical object which it could be used means to encode structure and meaning in the second equation. We think that this reflects a kind of mathematical thinking that has a great deal to do with developing a good algebraic structural sense accompanied with the acquisition of a functional outlook to equations as objects which is a warranty of relational understanding.

## CONCLUDING REMARKS

Our purpose in this paper was to illustrate a particular approach to studying the student's construction of meanings for structural aspects of equations, emerging from the use of novel algebraic-like formalism. In the first part of the results, an initial icon-driven conceptualisation of the MoPiX equations seemed to have been leading students towards the development of criteria for an isolated interpretation of the MoPiX equations' symbols. As soon as the students became familiar with testing their models and observing the animations generated on the "Stage", their interactions with the computer environment became strongly associated with the editing of the existing equations' content. As expressed in the second part of the results, the editing of equations revealed a subtle shift from a process-oriented view to equations into an object- oriented one as well as a progressive development of algebraic structure sense. In the last part of the results, students' previous experience with the MoPiX tools seemed to become part of their repertoire, allowing them to construct new equations following specific structural rules, invent variables and specify their values, and use the equations as objects to represent variables in other equations. Concluding, we suggest that in the present study reifying an equation was not a one–way process of understanding hierarchically–structured mathematical concepts but a dynamic process of meaning–making, webbed by the available representational infrastructure (Noss and Hoyles, 1996) and the ways by which students drew upon and reconstructed it to make mathematical sense.

## NOTES

1. The research took place in the frame of the project "ReMath" (Representing Mathematics with Digital Media), European Community, 6th Framework Programme, Information Society Technologies, IST-4-26751-STP, 2005-2008 (http://remath.cti.gr)

2. "MoPiX" was developed at London Knowledge Lab (LKL) by K. Kahn, N. Winters, D. Nikolic, C. Morgan and J. Alshwaikh.

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# RELATIONSHIP BETWEEN DESIGN AND USAGE OF EDUCATIONAL SOFTWARE: THE CASE OF APLUSIX

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*In this contribution, we are interested in the design process of Aplusix, a microworld for the learning of algebra and in the impact of usages on this process. In the first part, we present general principles that seem to be guiding the overall design process of the system and the development of tree representation of algebraic expressions, which has been added recently. The second part is devoted to a design and implementation of a learning scenario involving Aplusix. Examples of impact of this empirical study on the software design choices are discussed.*

***Key words**: Aplusix, algebra, tree representation, pedagogical scenario*

## INTRODUCTION

The research reported in this paper is carried out in the framework of the ReMath project (http://remath.cti.gr) addressing the issue of using technologies in mathematics classes "*taking a 'learning through representing' approach and focusing on the didactical functionality of digital media*". The digital media at the core of this research is Aplusix, software designed to help students learn algebra. The work has been developed in three phases:

(1) *Design and implementation of a new representation of algebraic expressions*. During this phase, fundamental choices for a representation of expressions in a form of a tree were made collaboratively through interactions between computer scientists and didacticians of mathematics: on the one hand, computer scientists make sure that the new developments comply with general principles of the software, on the other hand, didacticians ensure that these choices are based on didactical and epistemological hypotheses. The choice of theoretical frameworks in both domains has an impact on functionalities of the tree representation. This design phase is presented in the following section.

(2) *Design of a pedagogical scenario*. Based on the choices made in the design phase, didacticians designed a pedagogical scenario to explore possible contributions of this new representation to the learning of algebra. The scenario has to take account of institutional constraints in order to implement it in ordinary classes. The design of scenario may lead to reconsidering certain choices concerning the new representation, or suggesting other. Such cases will be presented further in the paper.

(3) *Experimentation*. The scenario has been experimented in three different classes, which allowed validating underlying didactical hypotheses, as well as assessing the way students manipulate this new representation. This phase is discussed in the last part of the paper.

## DESIGN AND DEVELOPMENT OF APLUSIX

When developing computer-based learning environments, designers need to make choices at the interface level and thus at the level of the internal universe of the environment. Thus pieces of knowledge implemented in such an environment will live not only under constraints of the didactical transposition (Chevallard 1985), but also under other constraints proper to the environment resulting from what Balacheff (1994) calls *computational transposition*. Thus, designers of computer-based learning environments have to respond to at least two types of requirements. First, they need to respect basic principles that are characteristic of the environment. The second type is related to the practice of the piece of knowledge in the institution in which it will be used.

Principles governing a design of software are not always made explicit and choices made are rarely explicitly linked to these principles. In what follows, we present a study carried out in an attempt to make explicit principles and choices that were guiding designers of Aplusix (aplusix.imag.fr), software for learning algebra, when they were developing tree representation of algebraic expressions.

### General design principles of Aplusix

Aplusix software (Nicaud et al., 2003, 2004) has been developed since 1980s. A new mode of representation of algebraic expressions, a tree representation, is being added to this software. As was already mentioned above, the new developments must not affect the coherence of the whole software and thus have to comply with fundamental principles that guide the design and development of Aplusix. Three main design principles have been identified:

(1) *The student is free to write algebraic expressions*. This principle, influenced by research in the domain of interactive learning environments, considering mainly microworlds, resulted in the development of an editor of algebraic expressions and in the necessity to consider and deal with students' errors.

However, freedom in manipulating algebraic expressions is limited by constraining the selection of sub-expressions, based on the syntactic and semantic dimensions of expressions, which seems to be another important design principle and that can be formulated as follows:

(2) *In manipulating algebraic expressions, their syntactic and semantic dimensions are taken into account*. For example, given the expression 2+3x, it is not possible to select 2+3 as a sub-expression. This principle brings the idea of scaffolding since this choice aims at helping understand algebraic expressions and make their manipulation easier.

As regards the interaction between a student and a system, there are two modes of interaction: (1) a test mode in which the student does not get any feedback from the system, and (2) a training mode, in which a feedback is provided both in terms of equivalence of a student's expression and the given one, and in terms of the correct end of the exercise. Thus the third principle is:

(3) *In a training mode, scaffolding should be provided by the system*. Scaffolding in the training mode requires taking decisions about validation of student's answers. It is important to clarify at this point that Aplusix recognizes 4 basic types of exercises: calculate, expand and simplify, factor and solve (equation, inequality or system of equations or inequalities). For these types of exercises, these decisions have been implemented. For example, for the "solve equation" exercise, it has been decided that the expression x = 2/4 will not be accepted as it is written in a non-simplified form, but will not be rejected either as it is not incorrect. Therefore a feedback message is sent to the student saying that the equation is almost solved.

## Design and development of tree representation in Aplusix

The decision to implement a new representation system into the existing Aplusix software was taken in relation with the ReMath project focusing on representations of mathematical concepts in educational software. Two possibilities were considered: tree and graphical representations. The reasons for choosing the development of tree representation system are numerous (Bouhineau et al. 2007): (1) from an epistemological point of view, trees are natural representations of algebraic expressions; (2) from a didactical point of view, the introduction of a new register of representation would allow creating activities requiring an interplay between registers, which would enhance learning of algebraic expressions (Duval 1993); (3) from a point of view of computer science, trees are fundamental objects used to define data structures. Indeed, internal objects used in Aplusix to represent algebraic expressions and their visual properties are trees; (4) graphical representation of algebraic expressions is available in a few educational systems, while tree representation is scarcer.

Let us note first that the fundamental choices related to the tree representation were discussed during several meetings among developers (computer scientists and engineers) and didacticians.

### *Different modes of tree representation*

The first idea was to develop the tree representation in a way that the student can see the articulation between the usual representation of an expression and a tree representing it: given an expression in a usual representation, a tree representation is provided progressively by the system, according to the student's command. A "mixed representation" mode has thus been designed where each leaf of a tree is a usual representation of an expression that can be expanded in a tree by clicking at the "+" button that appears when the mouse cursor is near a node; a tree, or a part of a tree, can be collapsed into a usual representation by clicking at the "-" button that appears when the mouse cursor is near a node. The developers considered this idea interesting from the learning point of view. However, it was in contradiction with the principle 1, according to which it was necessary to let the student edit freely a tree. The development of a "free tree representation" mode, where the student can freely built trees, brought new difficulties the developers had to face: notion of erroneous operator, representation of parentheses, difficulties related to the "minus" sign, to the square root… These difficulties and the ways the developers have coped with them are described elsewhere (Trgalova and Chaachoua 2008).

Based on the principle 3, the developers wished to implement an editing mode providing scaffolding to the student. Design and implementation of scaffolding requires to define new kinds of exercises that would be recognized by the system and the means of validation of these exercises. We will discuss some of these choices below. It led also to the implementation of a "controlled tree representation" mode with constraints and scaffolding when a tree is edited: internal nodes must be operators and leaves must be numbers or variables. The arity of operators must be correct. In the current prototype of Aplusix, 3 modes of editing trees are thus available: free, controlled and mixed representations.

*Choices of criteria for validating a student's answer*

According to the principle 3, when the student builds a tree in the free tree representation mode, the system should provide her/him with a feedback. Decisions about the conditions for a tree to be accepted as correct had to be taken and implemented. The student's tree is compared with the expected one: (1) when, after normalisation of the minus signs (transformation of all minus signs in opposite), the trees are identical, then the student tree is accepted; (2) when the two trees differ only by commutation, the student's tree is not accepted, but a specific message indicates that there is a problem with order; (3) when there is neither identity between the trees (case 1) nor commutation (case 2) but the two trees represent equivalent expressions, a message is generated indicating that the student's tree is equivalent but not the expected one; (4) when there is no equivalence between expressions represented by the trees, another message is generated indicating that the answer is not correct.

These choices were made by one of the developers based on *fundamental issues* present in Aplusix such as *the notion of equivalence, the notion of commutation* and *of associativity*. They are considered as *a first stage choices* that can be discussed and analysed from the didactical point of view, both in terms of messages to be generated and of considering different cases of behaviour.

## PEDAGOGICAL SCENARIO

Before presenting a pedagogical scenario we designed in order to validate design choices for the tree representation of expressions in Aplusix, we discuss some theoretical considerations that underpin the scenario.

According to Sfard (1991), mathematical notions can be conceived in two different ways: structurally as objects, and operationally as processes. An object conception of a notion focuses on its form while a process conception focuses on the dynamics of the notion. Algebraic expression, when conceived operationally, refers to a computational process. For example, the expression 5x-2 denotes a computational process “multiply a number by 5, and then subtract 2”, which can be applied to numerical values. When an expression is conceived structurally, it refers to a set of objects on which operations can be performed. For example, 5x-2 denotes the result of the computational process applied to a number x. It also denotes a function that assigns the value 5x-2 to a variable x. Yet, in the French high school, the operational conception of algebraic expressions prevails in the teaching of algebra. Specific activities are needed to favour the distinction between these two conceptions of an algebraic expression. Examples of such activities are describing the expression in natural language, which requires considering the structure of the expression, or using tree representation of an expression, which highlights its form.

Semiotic representation is of major importance in any mathematical activity since mathematical concepts are accessible only by means of their representations. Duval (1995) calls “*register of representation*” any semiotic system allowing to perform three cognitive activities inherent to any representation: formation, treatment and conversion. These activities correspond to different cognitive processes and cause numerous difficulties in learning mathematics. Duval (2006) claims that while treatment tasks are more important from the mathematical point of view, conversion tasks are critical for the learning. Consequently, conceptualisation of mathematical notions requires manipulating of several registers for the same notion allowing to distinguish between a notion and its representations. As Duval (1993) says, the conceptualisation relies upon the articulation of at least two registers of representation, and this articulation manifests itself by rapidity and spontaneity of the cognitive activity of conversion between registers. Yet, school mathematics gives priority to teaching rules concerning both formation of semiotic representations and their treatment. The amount of activities of conversion between registers is negligible, although they represent cognitive activities that are the most difficult to grasp by students.

Motivated by these considerations, in the design of our pedagogical scenario, we decided to take into account three semiotic registers of representation of algebraic expressions: natural language register (NLR), usual register (UR) and tree register (TR) and to design activities of formation, treatment and conversion between these registers. The pedagogical scenario thus aims at helping the students grasp the structure of algebraic expressions by means of introducing TR and articulating it with UR and NLR. The following hypothesis underpins the scenario: the introduction of TR and its articulation with NLR and UR will have a positive impact on students’ mastering of the usual register of representation of algebraic expressions, which is the one taught in school algebra. The scenario is composed from 4 units: *pre-test*, *learning*, *assessing*, and *post-test* (cf. Table 1). The *pre-test* aimed at diagnosing students’ difficulties in algebra, especially those related to the structural aspect of expressions. On the other hand, the results of the pre-test compared to those of the post-test should provide us with evidence about the efficiency of the pedagogical scenario. Two kinds of activities are proposed in the pre-test: (1) classical school algebra exercises (calculate, expand and simplify, factor), which are, in Duval's terms, treatment tasks in the register of usual representation, and (2) communication games between students proposing, in Duval's terms, activities of conversion between UR and NLR. The aim of the *learning* unit is to introduce the students to TR, a new register of representation of expressions, as well as to articulate it with the already familiar registers, namely NLR and UR. Then, conversion activities between TR and NLR and UR respectively are proposed. Most of the activities are to be done in a computer lab with Aplusix in the training mode. Eventually, simple tasks of treatment in TR are proposed to assess the mastery of the new register of representation by students. The unit called *assessing* aims at evaluating to what extent TR and conversion tasks between the registers are mastered by the students after having done activities of the *learning* unit. The evaluation is organized in the form of communication games between students similar to those from the pre-test, but this time, TR is involved in the tasks. In the *post-test*, tasks similar to those from the pre-test are proposed in order to enable a comparison of results. Confronting results obtained at the two tests should provide us with evidence confirming or not the underlying hypothesis.

| | *Activities* | *Description* | *Environment* | *Duration* |
|---|---|---|---|---|
| Pre-test | Treatment in UR | Calculate, Factor<br>Expand and simplify | Aplusix | 50 min |
| | Conversion<br>NLR ↔ UR | Communication games | Paper & pencil | 30 min |
| Learning | Introduction to TR | Scenario TR<br>introduction | Aplusix in video<br>projection | 55 min |
| | Conversion<br>NLR ↔ TR | Conversion NLR → TR<br>Conversion TR → NLR | Aplusix: controlled then<br>free mode<br>Paper & pencil | 90 min |
| | Conversion<br>UR ↔ TR | Conversion UR → TR<br>Conversion TR → UR | Aplusix: controlled then<br>free mode | 80 min |
| | Treatment in TR | Calculate in TR<br>Simplify in TR | Aplusix with second<br>view | 20 min |
| Ass. | Formation TR<br>Conversion<br>TR ↔ NLR (UR) | Communication games | Aplusix: free mode<br>Paper & pencil | 55 min |
| Post-test | Treatment in RU | Calculate, Factor<br>Expand and simplify | Aplusix | 30 min |
| | Conversion<br>NLR ↔ UR | Communication games | Paper & pencil | 20 min |

**Table 1. Structure of the pedagogical scenario.**

## EXPERIMENTATION

The scenario was proposed to 3 teachers with a possibility to adapt it to the constraints of their class. In this section, we present one of the experiments that took place in a Grade 10 class (15 years old students) in November 2007.

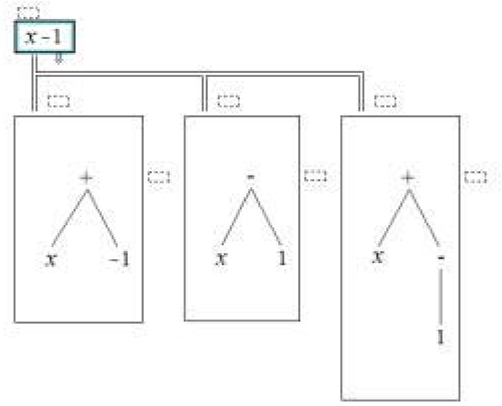
The pre-test revealed expected errors in treatment tasks within UR, in particular errors showing difficulties to take account of the structure of algebraic expressions, e.g., transforming 2+3x in 5x, and errors with handling powers and minus sign, e.g., transforming $3(-5)^2$ in $-3\times5^2$ or in $\pm3^2\times5^2$. On the other hand, we were surprised by the results obtained in communication games. Algebraic expressions given in UR were described in NLR by the students, but with characteristics of an oral register, i.e., the students described actions allowing to obtain the initial expression (cf. Table 2). This register is based on language structure used to "read" an expression in UR. It presents two specificities: left-to-right reading and presence of implicit elements.

| ***Expression given in UR*** | ***Student emitting a message*** | | ***Student receiving a message*** | |
|---|---|---|---|---|
| | *Register* | *Examples of messages* | *Correct in UR* | *Wrong in RU* |
| $2x - y$ | Oral (left-to-right) | "2 x minus y" | 14 | 0 |
| $2x - y^2$ | Oral with ambiguity | "2 x minus y squared" | 22 | 4 |
| $\frac{(3x+2)(3x-1)}{a-(x+2)}$ | Oral with brackets explicitly stated | "open a bracket, 3 x plus 2, close the bracket, open a bracket, 3 x minus 1, close the bracket, all this over a minus, open a bracket, x plus 2, close the bracket" | 7 | 1 |
| | Oral with brackets explicitly stated and with ambiguity | "open a bracket, 3 x plus 2, close the bracket, open a bracket, 3 x minus 1, close the bracket, over a minus, open a bracket, x plus 2, close the bracket" | 19 | 1 |
| ***Total*** | | | 62 | 6 |

**Table 2. Conversion from UR into NLR.**

All messages result from the oral register and they accentuate operational aspect of the expressions rather than structural one. Moreover, more than 66% of messages are ambiguous. Despite of the ambiguities, most of pairs succeeded the game thanks to implicit codes of the oral register the students share and understand and which result from didactical contract (Brousseau 1997). Thus, the goal we assigned to the communication games, namely to lead students to become aware of the limits of the oral register they use in algebra, which does not take into account the structural aspect of expressions, was not achieved.

The learning unit started by an introductory session aiming at introducing tree representation to the students. The teacher asked one of the designers of the pedagogical scenario to manage this session since he did not feel comfortable enough with the new representation implemented in the software although he uses Aplusix on a regular basis with his students. This introductory session allowed discussing with the students specificities of the tree representation of expressions and introducing vocabulary related to this new register (branch, leave, operator, argument…). Particular attention was paid to reading the expressions. Thus for example, the expression x+2y was read as "the sum of x and of the product of 2 by y", which accentuates the structure of the expression, instead of "x plus 2 y" highlighting its operational aspect. A particularity of the tree register residing in the fact that several different trees can represent a same algebraic expression was also discussed with the students based on the following example showing different meanings of "minus" sign (Fig. 1):




In the expression x-1, the minus sign can be conceived in three different ways leading to three different trees (this difference is hardly visible in UR):

- Sign of a negative number (tree on the left);
- Binary operator "difference" (tree in the middle);
- Unary operator "opposite number" (tree on the right).

**Figure 1. Three different meanings of minus sign.**

The rest of the scenario was shortened in order for the teacher to be in line with the global pedagogical program shared by all Grade 10 classes in the school. The teacher decided to individualize the implementation of the scenario according to the students in the following way: conversion NLR→TR and UR→TR in controlled mode only (only one group, denoted G1); conversion TR→NLR assigned as homework (whole class); treatment in TR optional (a few students with severe difficulties in algebra).

The G1 group was formed from rather low attaining students. The results obtained in the conversion tasks TR→NLR showed a significant difference between the two groups (cf. Table 3). These results can be considered as evidence proving efficiency of the work on conversion tasks NLR/UR→TR.

| | Answer in NLR with structural aspect | Answer in NLR with operational aspect |
|---|---|---|
| G1<br>15 students having worked on conversion tasks with Aplusix in controlled mode | 10 | 5 |
| G2<br>15 students who have not benefited from the work on conversion tasks | 3 | 12 |

**Table 3. Students' answers to the conversion tasks TR→NLR.**

As we mentioned above, the scenario, and thus the new prototype of Aplusix, had been tested in three classes. Feedbacks from students and teachers led the developers to re-examine some choices, which allowed some adaptations and improvements at the interface of Aplusix. Let us take the example of the "second view" functionality that enables visualizing a given algebraic expression represented in two registers at the same time. Initially, the second view displayed only a current step of the transformation. Observing the students using this functionality, we realized that when a student performs the next transformation step, the representation in the second view is updated and the student cannot observe the effects of the transformation in the second register. For this reason, the developers were asked to redesign this functionality in a way for the student to be able to observe the transformation s/he has performed in both registers. At present, the second view displays both current and previous steps.

## CONCLUSION

The example of the design and implementation of tree representation of algebraic expressions presented in this contribution shows that the decision to introduce a new register of representation has been motivated by the didactical considerations about the necessity of being able to represent mathematical notions in at least two different registers. Considerations of different nature had an impact on the development of the new register: (1) taking account of a didactical dimension led to make choices allowing the implementation of tasks of conversion between registers, which seem to be essential for conceptual understanding of mathematical notions (Duval 1993); (2) taking account of users' feedback allowed to make some improvements at the interface level. An example was presented in the previous section; (3) respecting the general principles of the development of Aplusix guarantees the coherence of the system after the introduction of the new register of representation of algebraic expressions. As regards the choices made in the design of the Aplusix tree module, it seems that most of them were made internally, i.e., by the developers themselves, and sometimes even individually, i.e., by one of the developers. Decisions are driven by the fundamental design principles in a way that a coherence of the whole system is preserved. Although it seems that the decisions are taken regardless the school context, both teachers and students are taken into account in the system design. The principles 1 and 3 concern especially students and their interactions with the system. Moreover, the developers are respectful towards the students' ways of editing expressions, which is shown by the decision to make it possible to recover an expression in exactly the same way as the student has edited it, even if the implementation of such a decision was difficult (Trgalova and Chaachoua 2008).

The example of the development of Aplusix illustrates a way the synergy between computer scientists, researchers in math education and users can serve a project of development of educational software.

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