CASYOPÉE IN THE CLASSROOM: TWO DIFFERENT THEORY-DRIVEN PEDAGOGICAL APPROACHES

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The ReMath project is a European project that addresses the task of integrating theoretical frames on mathematical learning with digital technologies at the European level. A specific set of six dynamic digital artefacts (DDA) has been currently developed, reflecting the diversity of representations provided by ICT tools. Here we considerer the DDA Casyopée which was experimented in two different countries: Italy (Unisi team) and France (Didirem team). The paper focuses on the influence of the theoretical frames in the design of these Teaching Experiments.

PROBLEMATIC OF THE REMATH PROJECT

The project focuses on the primary and secondary school level giving a balanced attention to both teachers and students and incorporating a range of innovative and technologically enhanced traditional representations. Specific attention is given to cultural diversity: seven teams from four countries are involved in this project. The work is based on evidence from experience involving a cyclical process of

a) developing six state-of-the-art dynamic digital artefacts for representing mathematics involving the domains of Algebra, Geometry and applied mathematics,

b) developing scenarios in a common format for the use of these artefacts for educational added value,

c) carrying out empirical research involving *cross-experimentation* (Cerulli et al. 2008) in realistic educational contexts, aiming at enhancing our understanding of meaning-making through representing with digital media, in particular by providing new insight into means of using technologies to support learning, and into learning processes in relation to the use of technologies.

Many recent studies highlight the existence of a multiplicity of theoretical frameworks for addressing those themes, and there is a shared increasing need of overcoming the resulting fragmentation (Artigue, 2007). This need is also felt within ReMath project, in which a variety of educational paradigms is present. The issue is addressed through the development of specific methodological tools, some of which are drawn and re-elaborated from the experience of TELMA project (Cerulli et al., 2008).

In this paper we present two different Teaching Experiments designed and carried out within ReMath project, respectively by Didirem team of the University Paris 7 (France), and by Unisi team of the University of Siena (Italy). Both the TEs were designed around the use of the software Casyopée (partly developed within the project). After describing the main features of Casyopée (exploited in the Teaching experiments) we will focus on the design of the Teaching Experiments, and we will compare them relying on the construct of Didactical Functionality (Cerulli, Pedemonte and Robotti, 2006). Though it would be interesting, a discussion on the actual implementation of the plans in classroom is out of the goals and of the possibilities of the present paper.

THE CONSTRUCT OF DIDACTICAL FUNCTIONALITY

The construct of Didactical Functionality is meant to provide a minimal common perspective, hopefully independent from specific theoretical frameworks, to frame diverse approaches (possibly depending on theoretical references) to the use of ICT tools in mathematics education, as well as the theoretical reflections regarding the actual use of ICT tools in given contexts.

By Didactical Functionality of an ICT tool, one means the system constituted by three interrelated poles: a set of features of the tool, a set of educational goals, and the modalities of employing the specified features of the tool for achieving the envisaged educational goals.

Trivially, through the construct of Didactical Functionality one intends to acknowledge that an ICT tool (or part of it) can be used in different ways for achieving different educational goals, that is one can design or identify different Didactical Functionalities of a given tool. In particular different theoretical perspectives can lead to designing different Didactical Functionalities of a given tool.

THE DDA CASYOPEE

The DDA Casyopée (Lagrange and Chiappini, 2007) is built as an open problem-solving environment with the aim of giving students a means to work with algebraic representation, progressively acquiring control of the sense of algebraic expressions and of their transformations. Functions are the basic objects in Casyopée. Using this tool, students can explore and prove properties of functions. Casyopée takes into account the potentialities that Computer Algebra Systems offer to teaching and learning: going beyond mere numerical experimentation and accessing the algebraic notation; focusing on the purpose of algebraic transformations rather than on manipulation and connecting the algebraic activities. It is expected that students will make sense of algebraic representations by linking these with representations in these domains. See below a screen copy on the algebraic representations provided by Casyopée, it splits into two windows: a symbolic one and a graphical one.

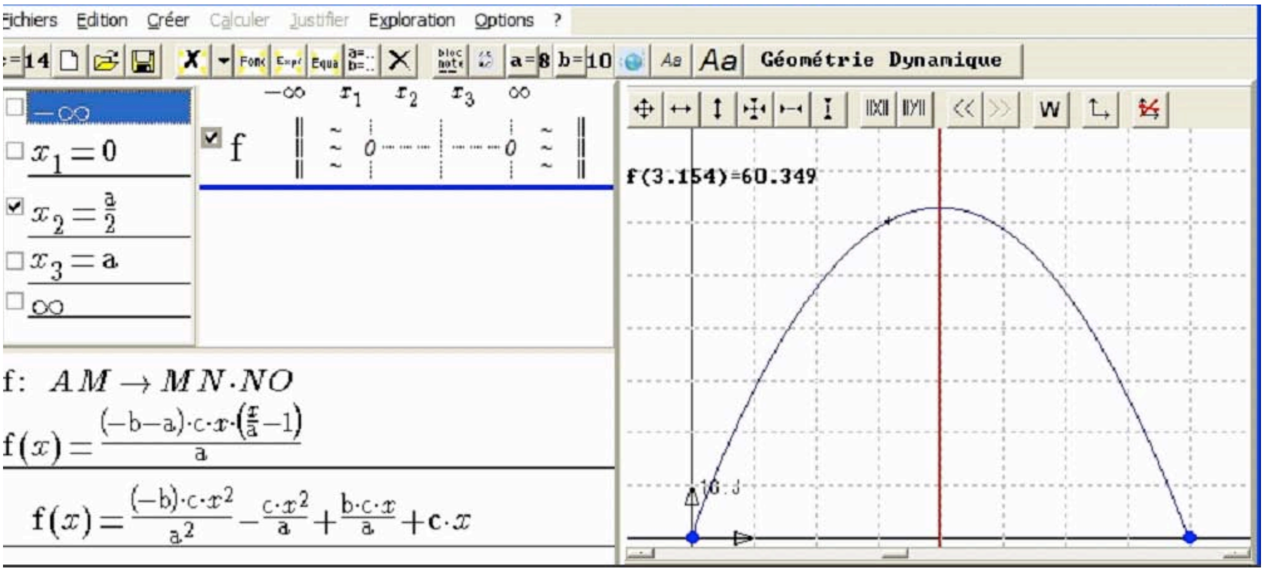


Figure 1: the algebraic setting in Casyopée

In the Remath project, Casyopée has been extended with a geometrical module. The aim is to explore what can be an interesting cooperation between a geometrical problem and its analytic treatment. The goal is not to develop a whole geometric dynamic environment but rather to see how geometric and analytic environments can articulate each other. For instance, a geometrical figure can be a domain to experiment with geometrical calculations. In the screenshot below, students can ask for the measure of the area of the rectangle MNOP. Then an algebraic model can be built choosing one of the measures as an independent variable and the other as a dependant variable. Properties of the dependency can be conjectured and proved: they take sense both in the algebraic and in the geometrical settings.

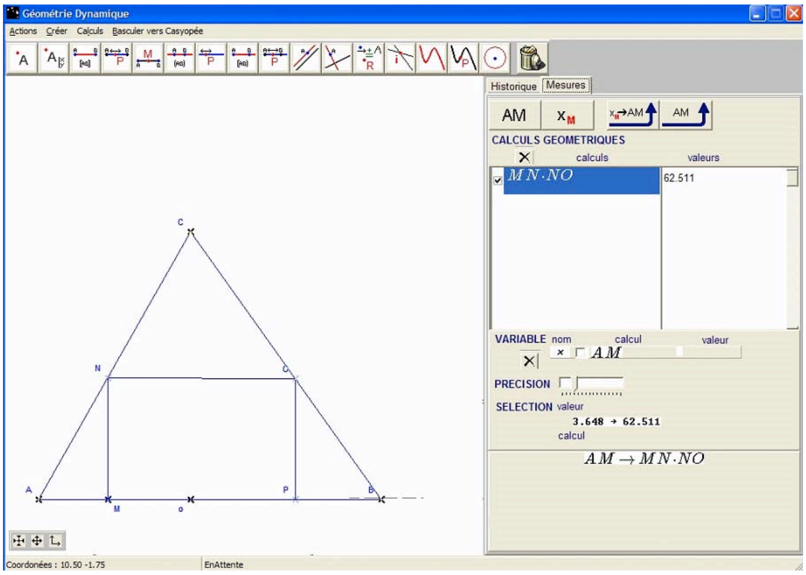


Figure 2: the geometrical window in Casyopée

The main specificity of Casyopée among other dynamic geometrical artefacts is to connect geometric and algebraic approaches. More precisely, the geometrical frame allows one to consider a geometric calculation and to export it in the algebraic environment. This transfer is allowed by choosing an adequate variable for the geometrical situation. At this point, Casyopée gives a feedback on the choice of this independent variable.

The representations offered by Casyopée have been thought to be close to institutional ones. Casyopée allows students to work with the usual operations on functions such as algebraic operations, analytic calculations and graphical representations. The geometric environment offers commands usually available in other dynamic geometry environments such as creating fixed and free geometrical objects (points, lines, circles, curves)

UNISI AND DIDIREM PEDAGOGICAL PLANS

In the introduction we recalled that different specific methodological tools have been developed within ReMath for fostering the comparability of studies dealing with the use of ICT tools in mathematics education. A new conceptual model of the pedagogical scenario, called Pedagogical Plan (Bottino et al. 2007), is one of those methodological tools. A Pedagogical Plan has a recursive hierarchical structure: each pedagogical plan is conceived as a tree whose nodes and leaves are pedagogical plans themselves. Several components are attached to each pedagogical plan: including the articulation of the educational goals, of the class activities, the specification of the features of the ICT tool used and how they are used, and of the rationale underpinning the whole pedagogical plan and of the theoretical frames inspiring it. A web-based tool (Pedagogical Plan Manager, PPM) has been also developed for supporting teams in designing their pedagogical plans.

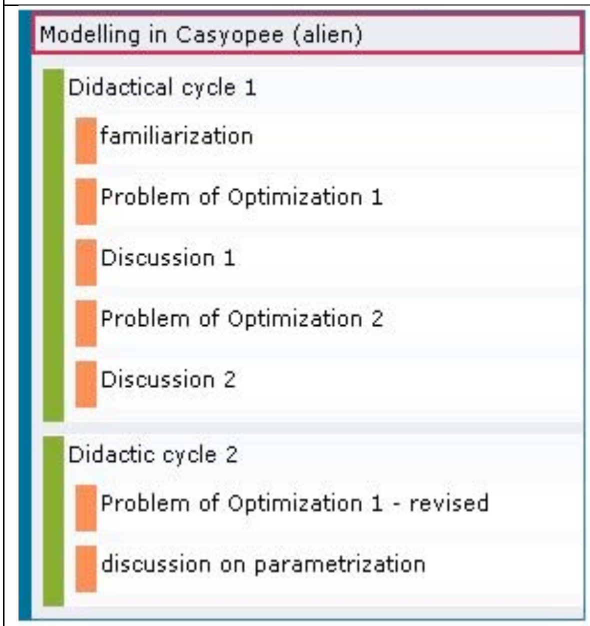
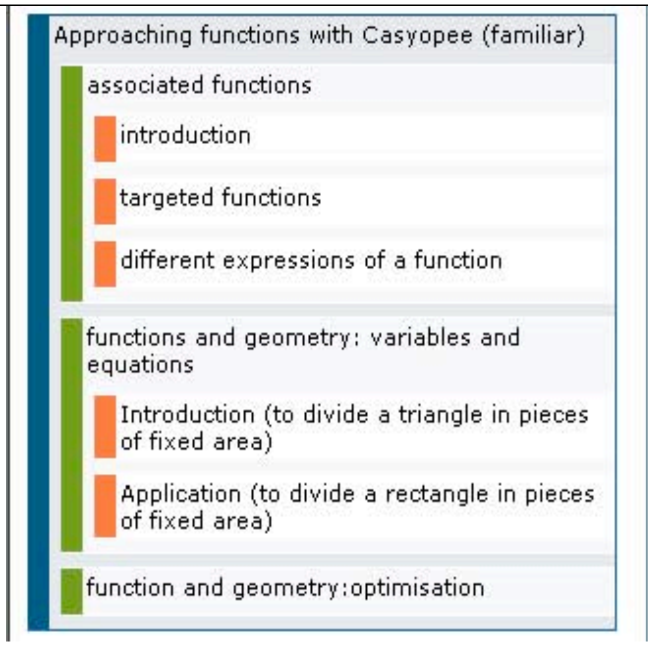
UNISI	DIDIREM
	

Figure 3: synthetic view of Unisi and Didirem pedagogical plans in the PPM

Figure 3 displays a screenshot from the PPM, and it is meant to provide an overview of the structures of the pedagogical plans designed by the Unisi and Didirem teams.

Details of the Unisi pedagogical plan

The Unisi pedagogical plan is inspired by the Theory of Semiotic Mediation (Bartolini Bussi and Mariotti, 2008) drawn from a Vygotsijan perspective. This theory guided both the specification of the educational goals (starting from an analysis of Casyopée) and the overall structure of the planned activities.

The designed educational goals are (a) to foster the evolution of students' personal meanings towards the mathematical meanings of function as co-variation. That regards also the notions of variable, domain of a variable… and (b) to foster the evolution of students' personal meanings towards mathematical meanings related to the algebraic modelling of geometrical situations.

Students are expected to have already received some formal teaching on the notions of variable, function and graph of a function, and on its graphical representation in a Cartesian plane. Moreover, a common experience of researchers and teachers is that meanings related to those notions are rarely elaborated in depth. The aim is to mediate and weave those meanings in the more global frame of modelling.

Hence, the pedagogical plan is not meant to help students become able to use Casyopée for accomplishing given tasks, but instead to foster the students' consciousness-raising of the mathematical meanings at stake.

The whole pedagogical plan is structured in cycles entailing: students' pair or small group activity with Casyopée for accomplishing a task, students' personal rethinking of the class activity (through the request to students of producing individual reports on that activity), class discussion orchestrated by the teacher.

The familiarization session is designed as a set of tasks aims at providing students with an overview of Cayopée features and guiding students to observe and reflect upon the "effects" of their interaction with the tool itself, e.g.:

> Could you choose a variable acceptable by Casyopée and click on the "validate" button? Describe how did the window "Geometric Calculation" change after clicking on the button. Which new button appeared?

Besides familiarization, the designed activities with Casyopée consist of coping with "complex" optimization problems formulated in a geometrical setting and posed in generic term, e.g.:

> Given a triangle, what is the maximum value of the area of a rectangle inscribed in the triangle? Find a rectangle whose area has the maximum possible value.

The aim is to elaborate on those problems so to reveal and unravel the complexity and put into evidence step by step the specific mathematical meanings at stake.

According to the designed pedagogical plan, the teacher plays the delicate role of guiding students to unravel such complexity and to make the targeted mathematical meanings emerge. The main tool for the teacher to achieve this objective, is the orchestration of the class discussions. The development of a class discussion cannot be completely foreseen *a priori*, it should be designed starting from the analysis of students' actual activity with Casyopée and of the reports they produce, and it would still depend on extemporary stimuli. Nevertheless in the design Unisi team tried to anticipate possible development of the pedagogical plan and to plan some kind of possible canvas for the teachers for managing class discussions.

The pedagogical plan is intended for scientific high schools or technical institutes, grade 12 or 13, and can be implemented over approximately 11 school hours.

Details of the Didirem pedagogical plan

The Didirem pedagogical plan aims to help students construct or enrich knowledge in the following areas: meaning of functions as algebraic objects and meaning of functions as means to model a co variation in geometric and algebraic settings. It is intended for scientific high schools grade 11 or 12 and has been implemented in ordinary classes during approximately 10 school hours. It is inspired both by the Instrumental Approach (Artigue, 2002), the Theory of Situation (Brousseau, 1997) and the Theory of Anthropologic Didactic (Chevallard, 1999).

Specific importance is given to the construction of tasks with an adidactical potential, where students can choose different variables for exploring functional dependencies, and to the connection between algebra and geometry. This connection is supported in Casyopée by geometric expressions that allow expressing magnitudes in a symbolic language mixing geometry and algebra.

The pedagogical plan is built around three main types of tasks:

- First session: finding target quadratic functions by animating parameters (five different tasks according to the semiotic forms used for these functions):

  Lesson 1: Introducing associated functions (a function g is associated to a function f if it is defined by a formula like $g(x)=af(x)+b$ or $f(ax+b)$ or similar)

  Lesson 2: Target Functions (functions that can be graphed but whose expression is not known; each student have to guess the function graphed by his/her partner)

  Lesson 3: Different expressions of quadratic functions

So students should consolidate: the meaning of variable, the distinction between variable and parameter, the meaning of function of one variable with several registers of semiotic representation and the fact that a same function may have several algebraic expressions. The new notion of associated function is worked-out during this session.

- Second session: creating a geometrical calculus as a model of a geometrical situation to solve a problem of relationships between areas, manipulation to experiment co variation between two geometrical variables:

Lesson 4: To divide a triangle in pieces of fixed area

Lesson 5: Application; dividing a rectangle into figures of fixed area

This way students can enhance their knowledge on co variation and develop the ability to experiment and anticipate in a dynamic geometrical situation, and the ability to model a geometric situation through geometric calculations.

- Third session: creating a function as a model of a geometrical situation to solve an optimization problem.

Lesson 6: solving a problem of optimisation in geometric settings by way of algebraic modelling.

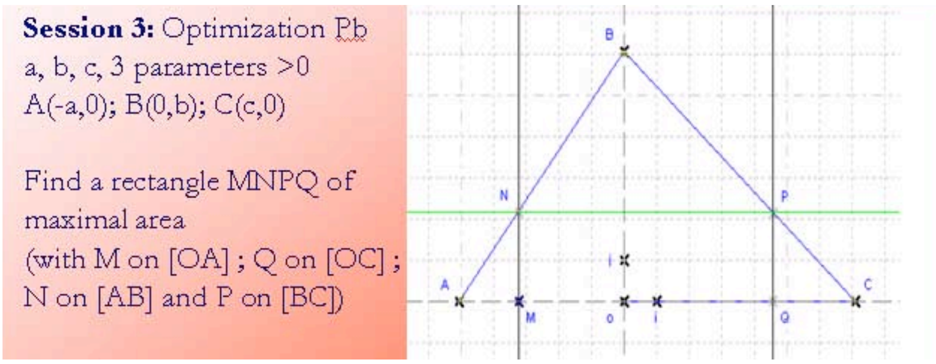


Figure 4: statement of the session 3 in Didirem pedagogical plan

This problem allows both to reinvest abilities to use the DDA, previous knowledge on associated functions and to introduce the notion of optimum in a geometrical situation.

COMPARISON OF THE UNISI AND DIDIREM APPROACHES USING THE CONSTRUCT OF DIDACTICAL FUNCTIONALITY

The two pedagogical plans, described in the previous sections, evidently share some characteristics but also have apparent deep differences. In this section we use the frame provided by the construct of Didactical Functionality to develop a more systematic comparison between the two pedagogical plans.

Tool Features

The two pedagogical plans are not generally centred on the use of the same DDA, but more specifically on the use of the same DDA features. In fact both exploit especially

(a) features of the dynamic geometry environment: the commands for creating fixed, free or constrained points, for dragging free or bonded points, for creating points with parametric coordinates, and the corresponding feedbacks of the DDA;

(b) features of the geometric calculation environment: the commands for creating "geometric calculation" associating numbers to geometrical objects, for choosing (independent) variables, for creating function between the selected variable and calculation, and the corresponding feedbacks;

(c) features of the algebraic environment, including the commands for displaying and exploring graphs of functions, for creating and manipulating parameters, for manipulating the algebraic expressions of functions, and the corresponding feedbacks.

Educational Goals

Different educational goals are associated to the use of those features. More precisely, one can recognize that both pedagogical plans share a common focus on some mathematical notions: function (in particular, conceived as co-variation), variables (independent and dependent) and parameters. Moreover the two pedagogical plans present, among other tasks, two optimization problems sharing the same mathematical core (see sections…). But, besides those surface similarities, there are profound differences.

Other Unisi educational goals are to mediate and weave meanings, related to the notions of function, variable and parameter. With that respect the Unisi team assumes, on the one hand, that those notions are familiar for students, and, on the other hand, that those notions are not elaborated in depth. Hence the Unisi pedagogical plan aims at helping students gain a deeper consciousness of the mathematical meanings at stake and re-appropriate them in the more global frame of modelling. In addition the Unisi objective includes the shared and decontextualized formulation of the different mathematical notions in focus.

The Didirem objectives are mainly to use potentialities of representations offered by Casyopée to introduce some new mathematical knowledge. This knowledge has been chosen for two main reasons: its affordance to the French curriculum and the importance to be studied in several frames of representations.

Modalities of employment

In accordance with the different objectives and the different pedagogical culture, the modalities of use are different as well.

The Unisi pedagogical plan has an iterative structure. Students' activity with Casyopée alternates with class discussions, after each session students are required to produce individual reports on the performed activities. This structure is meant to foster students' generation of personal meanings linked to the use of the DDA and their evolution towards the targeted mathematical meanings together with the students' consciousness-raising of the mathematical meanings at stake. That process is constantly fuelled by the teacher, whose role is crucial. Accordingly the teacher's role is explicitly taken into account in the design of the pedagogical plan, which provides with hints for the possible actions. The tasks used are optimization problems set in a geometrical frame. Their solution and the reflection on these solutions are fundamental steps towards the achievement of the designed educational goals. Also the familiarization with the DDA has to be considered within that perspective: as already mentioned, it aims at making students observe and reflect upon the "effects" of their interaction with the DDA itself. *Ad hoc* tasks are designed for that purpose.

Instead, the Didirem team pays specific attention to a progressive use of the DDA combining artefact and mathematical knowledge. Indeed, students work only in the algebraic window during section 1, then only in the geometrical windows in section2; finally section 3 gives an opportunity to reinvests the knowledge in the two environments. Moreover, all the tasks proposed are mathematical ones and are elaborated in order to allow students make progress alone working on the problem and to construct their new knowledge thanks the feedbacks.

CONCLUSION

Those differences can be strongly related with the different theoretical perspectives adopted by the two teams.

The Unisi team has mainly structured its pedagogical plan according to the Theory of Semiotic Mediation which inspired both the specification of the educational goals and the organization of the activities in iterative cycles. In particular the Theory of Semiotic Mediation led the Unisi team to devote attention towards the design of the teacher’s action in the pedagogical plan. In fact, the teacher plays a crucial role throughout the whole pedagogical plan, especially for fostering the evolution of students’ personal meanings towards the targeted mathematical meanings and facilitating the students’ consciousness-raising of those mathematical meanings.

Instead, the Didirem team splits its theoretical approach into several theoretical frames which shape their pedagogical plan: the Instrumental Approach (Artigue, 2002), the theory of Situation (Brousseau, 1997) and at last the theory of anthropologic didactic (Chevallard, 1999). The first frame aims to go further than a simple familiarization with the DDA and to help the students constructing a mathematical instrument. This process goes hand in hand with the learning process. The last optimization problem is used to evaluate the progress of this process. The process is accurately designed through a careful choice of mathematical tasks, with an adidactical potential, whereas the definition of the teacher's actions and role escapes the design of the PP. Finally, the TAD is called upon to manage instrumental distance between institutional and instrumental knowledge.

No doubt that these approaches are complementary. Each team might benefit from this collective work to improve its pedagogical plan in the future. For instance, the Didirem team plans to pay more attention to the teacher’s role during the pedagogical plan conception. Nevertheless, the objective is not to elaborate a wide common consensual theoretical frame, but rather to go in depth in the clarification of didactical functionalities, in a shared language.

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