SYSTEMIC INNOVATIONS OF MATHEMATICS EDUCATION WITH DYNAMIC WORKSHEETS AS CATALYSTS

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With reference to theories of cybernetics the paper proposes a general theoretical framework for initiatives aiming at systemic innovations of educational systems. It shows that it is essential to initiate incremental-evolutionary changes on the meta-level of beliefs and attitudes of the agents involved. For the theoretical foundation of concrete activities in mathematics education the didactic concept of learning environments is developed on the basis of constructivist notions of teaching and learning. Such learning environments may integrate dynamic mathematics for educational processes. So technology and especially dynamic worksheets can be considered as means and catalysts for improvements of mathematics education on system level.

Keywords: *systemic innovation, learning environment, dynamic mathematics*

INNOVATIONS IN COMPLEX SYSTEMS

There are many efforts to innovate educational systems – on regional, national and international levels – aiming at changes of teaching and learning. For understanding the structure of such initiatives a short glance at theories of cybernetics is useful.

Innovations

The OECD defines an *innovation* as the implementation of a new or significantly improved product, process or method (OECD, Eurostat, 2005, p. 46). Thus an innovation requires both an *invention* and the *implementation* of the new idea.

In the educational system we are in a situation where lots of concepts, methods and tools have been developed for substantial improvements of teaching and learning. Three examples:

(1) There is a wide range of current pedagogical theories that emphasize self-organised, individual and cooperative inquiry-based learning.

(2) There exists a huge amount of material for teaching and learning in a constructivist manner – available e.g. in electronic data bases or by print media.

(3) A large variety of software and other tools for the integration of ICT in educational processes has been developed.

But for real innovations these promising theories and products have to be implemented in the educational system. Here implementation means a good deal more than diffusion or dissemination of material (papers, guidelines, software tools etc.). And implementation should reach the real agents in the school system, i.e. the teachers and students, their thinking and their working. Let's remember the three examples from above:

(1) Teachers should teach according to current pedagogical concepts.

(2) The proposed new task culture should become standard in everyday lessons.

(3) ICT should be used as a common tool for exploring mathematics.

So for substantial innovations we do not need further material. We need changes in teachers' and students' notions of educational processes, in their attitudes towards mathematics and in their beliefs concerning teaching and learning at school. Hence the crucial question is: How can substantial innovations in the complex system of mathematics education be initiated and maintained successfully?

Complex Systems

In theories of cybernetics a system is called "complex", if it can potentially be in so many states that nobody can cognitively grasp all possible states of the system and all possible transitions between the states (Malik, 1992; Vester, 1999). Examples are the biosphere, a national park, the economic system, mathematics education in Europe and even mathematics education at a concrete school.

Complex systems usually are networks of multiply connected components. One cannot change a component without influencing the character of the whole system. Furthermore real complex systems are in permanent exchange with their environment.

Maybe this characterization of complex systems seems a bit fuzzy. But, nevertheless, it is of considerable meaning. Let us regard the opposite: If a system is not complex, someone can overview all possible states of the system and all transitions between the states. So this person should be able to steer the system as an omnipotent monarch leading it to "good" states. In contrast, complex systems do not allow this way of steering.

Steering of Complex Systems

The fundamental problem of mankind dealing with complex systems is how to manage the complexity, how to steer complex systems successfully and how to find ways to sound states.

With reference to theories of cybernetics two dimensions of steering complex systems can be distinguished (Malik, 1992). The first one concerns the manner, the second one the target level of steering activities (see figure 1).

The method of *analytic-constructive* steering needs a controlling and governing authority that defines objectives for the system and determines ways for reaching the aims. Hierarchical-authoritarian systems are founded on this principle. However, fundamental problems are caused just by the complexity of the system. In complex systems no one has the chance to grasp all possible states of the system cognitively.

So the analytic-constructive approach postulates the availability of information about the system that cannot be reached in reality.

In contrast *incremental-evolutionary* steering is based on the assumption that changes in complex systems result from natural growing and developing processes. The steering activities try to influence these systemic processes. They accept the fact that complex systems cannot be steered entirely in all details and they aim at incremental changes in promising directions. The focus on little steps is essential, since revolutionary changes can have unpredictable consequences which may endanger the soundness or even the existence of the whole system.

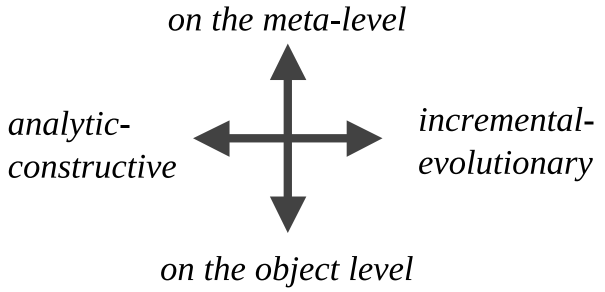

Figure 1: Steering of complex systems

The second dimension distinguishes between the object and the meta-level. The *object level* consists of all concrete objects of the system. In the school system such objects are e.g. teachers, students, books, computers, buildings etc. Changes on the object level take place if new books are bought or if a new computer lab is fitted out. Of course such changes are superficial without reaching the substantial structures of the system.

The *meta-level* comprehends e.g. organizational structures, social relationships, notions of the functions of the system etc. In the school system e.g. notions of the nature of the different subjects and beliefs concerning teaching and learning (e.g. Pehkonen, Törner 1996, Leder, Pehkonen, Törner 2002) are included.

Innovations in Complex Systems

How can substantial innovations in the complex system “mathematics education” be initiated successfully? The theory of cybernetics gives useful hints: Attempts of analytic-constructive steering will fail in the long term, since they ignore the complexity immanent in the system. Changes on the object level do not necessarily cause structural changes of the system. According to the theory of cybernetics it is much more promising to initiate *incremental-evolutionary changes on the meta-level* (see figure 2). They are in accord with the complexity of the system and do not endanger its existence. Nevertheless, they can cause substantial changes within the system by having effects on the meta-level, especially when they work cumulatively.

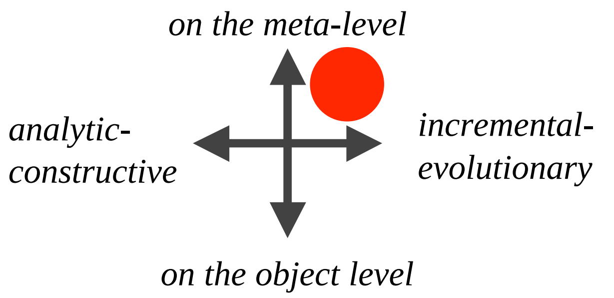

Figure 2: Innovations in complex systems

LEARNING ENVIRONMENTS WITH DYNAMIC WORKSHEETS

Aspects of Learning

Learning is a very complex phenomenon. Initiatives aiming at the development of mathematics education have to take in account the nature of learning. Let us have a very short glance at some fundamental aspects of learning (e.g. Reinmann-Rothmeier & Mandl, 1998; Haberlandt, 1997) which form a background for the latter:

- Learning is a *constructive* process. Knowledge and understanding cannot be simply transported from teachers to students. Cognitive psychology describes learning as a process of construction and modification of cognitive structures. From the view of neurobiology learning is the construction of neuronal networks. Connections between neurons develop and change.
- Learning is an *individual* process. Learning takes place inside the head of each learner. He creates his own knowledge and understanding by interpreting his personal perceptions on the basis of his individual prior knowledge and prior understanding.
- Learning is an *active* process. Cognitive activity means working with the content in mind, viewing it from different perspectives and relating it to the existing network of knowledge.
- Learning is a *self-organized* process. The learner is at least partially responsible for the organization of his individual learning processes. The degree of responsibility may vary in the phases of planning, realizing or reflecting learning processes.
- Learning is a *situative* process. It is influenced by the learning situation. A meaningful context or a pleasant atmosphere can foster learning processes, fear can hamper them.
- Learning is a *social* process. On the one hand the socio-cultural environment has great impact on educational processes. On the other hand learning in school is based on interpersonal cooperation and communication between students and teachers.

Concept of Learning Environments

Considering the aspects of learning noted above the following model seem adequate for teaching and learning processes in school:

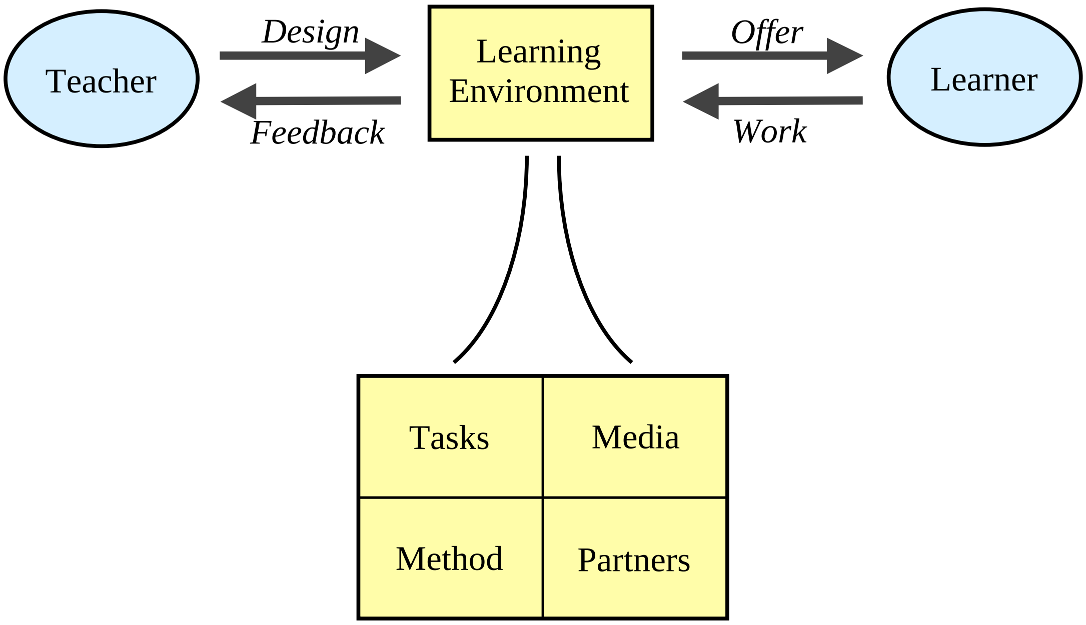

Figure 3: Working with learning environments, four components of learning environments

The *learning environment* is the essential link between the teacher and the learner. This notion includes the *tasks* for the learner's activities, the arrangement of *media* and the *method* for teaching and learning as well as the social situation with the teacher and other learners as *partners* for learning. It belongs to the teacher's field of responsibility to design the learning environment. So he offers a basis for the learner's work. This allows the teacher to get feedback about the learner as well as about the learning environment. This model is based on and extends the didactical concepts of "substantial learning environments" by Wittmann (1995, 2001) or "strong learning environments" by Dubs (1995).

The aspects of learning noted above imply fundamental consequences for the design of learning environments: Tasks should be problem-based with necessary openness for learning by discovery. They should offer meaningful contexts and view situations from multiple perspectives. The teaching methods should make the learners work actively, individually and self-organized. But not less important are the learners' communication and cooperation as well as discussions and presentations of ideas and results. Media can have several supporting functions for these processes.

Before we will discuss the relevance of this model for innovations in educational systems, we look at a specific kind of media which may carry general ideas to practice in school and serve as a catalyst for processes of change.

Dynamic Worksheets

The notion "dynamic mathematics" is currently used for software for dynamic geometry with an integrated computer algebra system, so that geometry, algebra and calculus are connected. When designing learning environments with dynamic mathematics, one faces the necessity to relate dynamic constructions to texts, e.g. for explanations or exercises for the students. For this purpose software for dynamic mathematics – like e.g. GEONExT or Geogebra – can be embedded in HTML-files. So dynamic constructions can be varied on the screen and are combined by the internet browser with texts, pictures, links and other web-elements. This kind of new media for mathematics education is called "dynamic worksheets" (Baptist, 2004; Ehmann, Miller, 2006).

With respect to the model in figure 3 dynamic worksheets are strongly related to all four components of learning environments: Of course they serve as teaching and learning *media*. Since they include text, they may provide *tasks* and instructions for the students. So implicitly they influence the teaching *method* and the cooperation between the learning *partners* (see next section). Hence, when designing learning environments with dynamic worksheets one should carefully take account of all these components and their impact on students' learning.

Figure 4 shows an example: The students are given a mathematical situation leading to an optimization problem. The text is combined with a dynamic construction which helps to understand the context. The rectangular can be moved while fitting exactly in the area between the parabola and the x-axis. The tasks help to structure the lesson according to the methodical concept described in the following section.

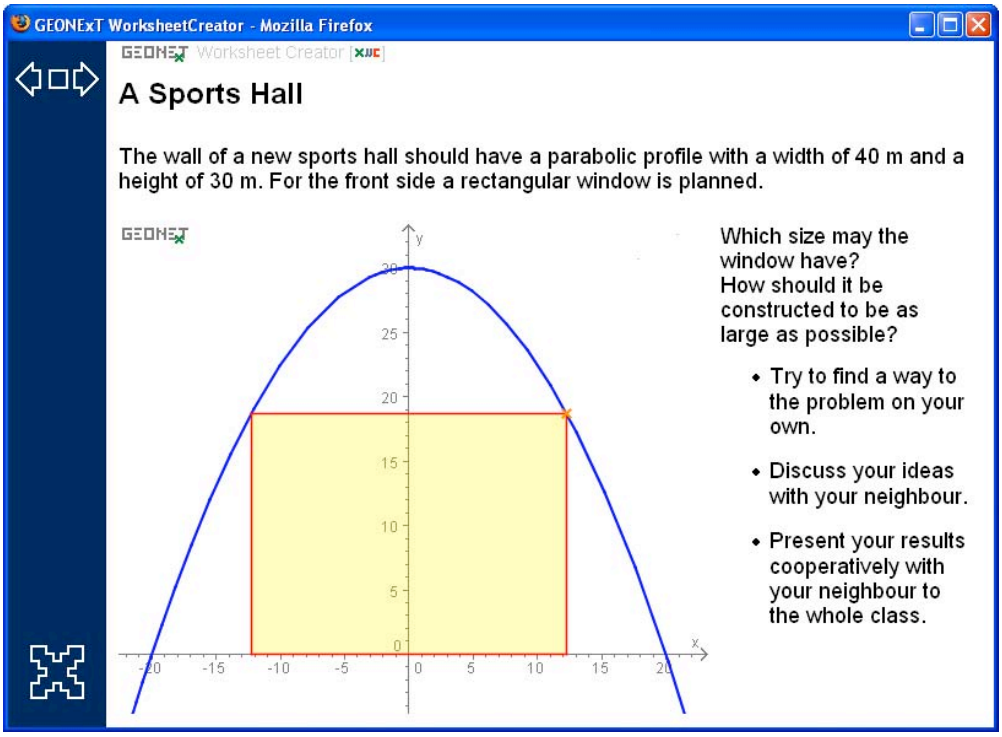

Figure 4: Screenshot of a dynamic worksheet

A Methodical Concept for Learning Environments with Dynamic Worksheets

The use of dynamic worksheets does not automatically improve mathematics education. It is crucial how these media are integrated in teaching and learning processes. If we want to initiate substantial changes on the meta-level of attitudes and beliefs concerning mathematics and mathematics education we have to organize lessons in a way that students work actively, individually, self-organized and cooperatively. They should experience that mathematics is a field for explorations and discoveries. And they should present and discuss their ideas and results cooperatively. Considering the aspects of learning noted above the following four phases structuring lessons with dynamic worksheets methodically are very natural:

1. Individual working: Learning is an individual, active and self-organized process. So at first the students work on their own. They are faced with the necessity to explore the content, to activate their prior knowledge, to develop ideas and to make discoveries. Learning environments with dynamic worksheets offer a framework for such activities and may support them.
2. Cooperation with partners: Learning is a social process. It is very natural that the students discuss their ideas with partners in small groups and that they work on the problems cooperatively. This communication helps to order thoughts and to get further ideas. Meanwhile the teacher may remain in the background or turn his attention to individuals.
3. Presentation of ideas: After having worked individually and in groups the students present their ideas and discuss them in the plenum. The different contributions reveal multiple aspects of the topic and help to view it from varying perspectives. Moreover the students train debating and presentation techniques.
4. Summary of results: Finally the students' results are summarized and possibly extended by the teacher. It is his task to introduce mathematical conventions and to consider curricular regulations. Since the students have already discovered the new content on their own paths, they can more likely integrate the teacher's explanations into their individual cognitive structures.

Table 1: Methodical concept

This methodical concept combines individual learning with working in small groups as well as in the plenum of the class in a very natural way. It is in close relationship to the methodical concepts "Think – Pair – Share" by Lyman (1981) or "I – You – We" by Gallin and Ruf (1998).

Learning by Writing: The Study Journal

The call for papers for working group 7 at CERME 6 emphasizes that technology in school should be considered within a wider range of resources for teaching and learning. Students should draw on ICT in combination with more traditional tools.

Accordingly, dynamic worksheets are only one element of rich learning environments. Especially pencil and paper do not lose relevance when student work with the computer. Noting down thoughts helps to order and arrange thoughts. Writing helps to develop understanding for new subject matters. Hence, when using dynamic worksheets students should regularly be asked to draw figures in their exercise book and to write down observations, conjectures, argumentations and personal statements. The exercise book gets the character of a personal "study journal" that accompanies students on their individual learning paths (Gallin, Ruf, 1998).

When designing dynamic worksheets for students' self-responsible learning, one should be aware of the risk that students play with the media as with a computer game quite superficially and do not get to the deeper mathematical content. The regular request of working in the exercise book decelerates the process of clicking through the learning environment. So the students are forced to take their time which is indispensable for individual learning.

Finally, the notes in the study journal ensure that ideas and results are still available when the computer is switched off. They are a basis for further presentations, discussions and summaries in the plenum of class (Baptist, 2004).

INCREMENTAL-EVOLUTIONARY SYSTEMIC INNOVATIONS WITH DYNAMIC WORKSHEETS AS PARTS OF LEARNING ENVIRONMENTS

In their plenary talks at CERME 5 Ruthven and Artigue observed that current results of activities integrating ICT in school are rather disappointing on system level.

> "Advocacy for new technology is part of a wider reform pattern which has had limited success in changing well established structures of schooling." (Ruthven, 2007) "From the very beginning, digital technologies have been considered as a tool for educational change […]. Unfortunately, the results are far from being those expected" (Artigue, 2007).

For substantial innovations in the educational system there is no lack of general ideas, pedagogical concepts or didactic tools – as discussed above. But there is a wide gap between theoretical knowledge and practice in school. So we have to develop strategies to bridge this gap.

Conclusion: A Pattern for Innovation Projects

Combining the theory of cybernetics and the concept of learning environments using dynamic worksheets we get a pragmatic, but also theory-based way of initiating innovations in school. Activities are most promising, if they focus on incremental-evolutionary changes on the meta-level of beliefs and attitudes of all agents involved. Learning environments with dynamic worksheets may serve as framework for learning processes of teachers *and* students. How can this be done concretely?

As a conclusion from all reflections above we sketch and propose a pattern for innovation projects for mathematics education. (It is realized e.g. by the current project “InnoMathEd – Innovations in Mathematics Education on European Level”, see http://innomathed.eu).

(1) The key persons for innovations in school are the teachers. Their beliefs, motivation and abilities are crucial for everyday teaching and learning in school. So regional networks of schools are established which form frameworks for teachers’ cooperative learning, exchange of experience and professional development.

(2) Universities are innovation centres for teacher education. They lead the school networks and provide regular and systematic in-service teacher education offers. This teaching and learning is designed according to the aspects of learning and the concept of learning environments described above. So the teachers get acquainted with these theories and concepts by making personal experiences in learning environments designed for them.

(3) Participating schools concentrate on one or a few areas of innovation, e.g. autonomous learning with dynamic worksheets, promoting student cooperation with dynamic worksheets or fostering key competences with dynamic worksheets. It is not promising to aim at total changes of mathematics education – because of the complexity of the system. However, such bounded fields of activity allow teachers to begin with substantial changes without the risk of losing their professional competence in class.

(4) The teachers get acquainted with general ideas and theories of teaching and learning as well as with techniques for constructing learning environments. To bridge the gap between theory and practice the teachers’ project activities are strongly related to their regular work at school. They develop learning environments for their students, they use, test and evaluate them in their classes and finally optimize them on the basis of all experiences. In this process they get guidance and coaching by the University leading the network.

(5) All learning environments which are tested, evaluated and optimized are collected in a data base and made available for public use.

(6) Teachers are given possibilities to exchange experiences with colleagues and to participate in teacher education offers on national and international level. Thus they understand that problems and necessities for development have systemic character and concern the fundaments of mathematics education far beyond their own professional sphere. Moreover, they get ideas for innovation activities from a large community.

(7) Finally, further networks of teachers and schools are essential means for dissemination processes in the long term. Experienced teachers coach colleagues from schools starting with innovation activities.

This approach may be called "theory based and material driven". On the basis of the theory of cybernetics and the theories of learning the teachers involved make incremental-evolutionary steps on the meta-level of beliefs and attitudes by designing and working with concrete learning environments for their classes.

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