THE SYNERGY OF STUDENTS' USE OF PAPER-AND-PENCIL TECHNIQUES AND DYNAMIC GEOMETRY SOFTWARE: A CASE STUDY

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This study is part of an ongoing research[1] on the interpretation of students' behaviors when solving plane geometry problems in Dynamic Geometry Software and paper-and-pencil media. Our theoretical framework is based on Rabardel's (2001) instrumental approach to tool use. We seek for synergy relationships between students' thinking and their use of techniques by exploring the influence of techniques on the resolution strategies. Our findings point to the existence of different acquisition degrees of geometrical abilities concerning the students' process of instrumentation when they work together in a computational and paper-and-pencil media. In this report we focus on the case of a student.

INTRODUCTION

We report research on the integration of computational technologies in mathematics teaching, in particular on the use of Dynamic Geometry Software (DGS) in the context of students' understanding of plane geometry through problem solving. We focus on the interpretation of students' behaviors when solving plane geometry problems by analyzing connections and synergy among techniques used in environments, DGS and paper-and-pencil, and geometrical thinking (Kieran & Drijvers, 2006). Many pedagogical environments have been created such as Cinderella, Geometer's Sketchpad, and Cabri Géomètre II. We focus on the use of GeoGebra because it is a free DGS that also provides basic features of Computer Algebra Software. As said by Hohenwarter and Preiner (2007), the software links synthetic geometric constructions (geometric window) to analytic equations, coordinate representations and graphs (algebraic window). Our aim is to analyze the relationships between secondary students' problem solving strategies in two environments: paper-and-pencil (P&P) and GeoGebra (GGB). Laborde (1992) claimed that a task solved using DGS may require different strategies to those required by the same task solved with P&P; this fact has an influence on the feedback provided to the student.

Our broadest research question aims at how the use of GGB in the resolution of plane geometry problems interacts with the students' paper-and-pencil skills and their conceptual understanding. We analyze and compare resolution processes in both environments, taking into account the interactions (student-content, student-teacher and student-GGB). In this report we focus on two research goals as being interpreted in the case of one student, Santi. We analyze this student's instrumentation process,

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and we compare his resolution strategies when using P&P and GGB within each problem. In the whole research we work with a total of fourteen individual cases from the same class group and establish some commonalities and differences among them.

THEORETICAL FRAMEWORK

We first draw on the instrumental approach (Rabardel, 2001). According to Kieran and Drijvers (2006), a theoretical framework that is fruitful for understanding the difficulties of effective use of technology, GGB in our case, is the perspective of instrumentation. The instrumental approach to tool use has been applied to the study of Computer Algebra Software into learning of mathematics and also to Dynamic Geometry Software. The instrumental approach distinguishes between and artifact and an instrument. Rabardel and Vérillon (1995) claim the importance of stressing the difference between the artifact and the instrument. A machine or a technical system does not immediately constitute a tool for the subject; it becomes an instrument when the subject has been able to appropriate it for her/himself. This process of transformation of a tool into a meaningful instrument is called instrumental genesis. This process is complex and depends on the characteristics of the artifact, its constraints and affordances, and also on the knowledge of the user. The process of instrumental genesis has two dimensions, instrumentation and instrumentalization:

- Instrumentation is a process through which "the affordances and the constraints of the tool influence the students' problem solving strategies and the corresponding emergent conceptions" (Kieran & Drijvers, 2006, p. 207). "This process goes on through the emergence and evolution of schemes while performing tasks" (Trouche, 2005, p. 148).
- Instrumentalization is a process through which "the student's knowledge guides the way the tool is used and in a sense shapes the tool" (Kieran & Drijvers, op. cit., p. 207).

In our research, we select different problems for being solved first with P&P and then with the help of GGB. In order to analyze the connectivity and synergy between the students' resolution strategies in both environments, the problems are to be somehow similar. The basic space of a problem is formed by the different paths for solving the problem. We transfer the similarity of the problems to the similarity of their basic spaces. For example, the problems considered in this article, share common strategies for reaching the solution such as equivalence of areas due to complementary dissection rules, application of formulas (area of a triangle), particularization, etc.

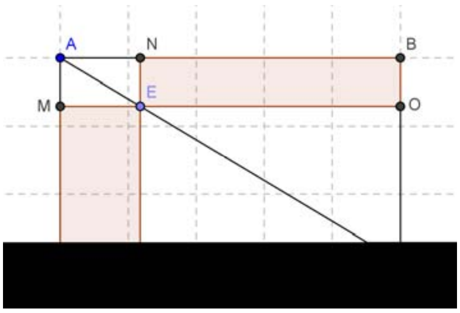
We plan to design an instructional sequence, focusing on a systematization of the interactions produced between artifacts (P&P, GGB), the mathematical actions and the didactical interactions. The theoretical framework is based on instrumental approach and activity theory (Kieran & Drijvers, 2006). We connect the activity theory as part of the "orchestration" (Trouche, 2004). The actions consist in different problem sequences to be proposed by the teacher to the students, to be solved in both media. The teacher proposes different indications or new problems. For each problem, we prepare a document with pedagogical messages that provide differing levels of information, and we group them according to the phases of the solving processes which are being carried out: familiarization, planning, execution, etc. We classify the pedagogical messages, for each phase, in three levels. Level 0 contains suggestions that do not imply mathematical contents or procedures in the solving process. The messages of level 1 only convey the name of the implied mathematical contents or procedures. Level 2 provides more specific information on these contents or procedures. For the problems to be solved in a technological environment we also prepare contextual messages. These messages are related to the use of GGB. The teacher can help the students in case they have technical difficulties with GGB.

We also specify some terms that will be used in this study of students'GGB resolutions such as figure and drawing. We use these terms with their usual meaning in the context of the Dynamic Geometry Software (Laborde & Capponi 1994). We use this distinction between Figure and drawing in order to describe the way in which students interpret the representations generated on the computer.

CONTEXT AND METHOD

The study is conducted with a group of fourteen 16-year-old students from a regular class in a public high school in Spain. These students are used to working on Euclidean geometry in problem solving contexts. They have been previously taught GGB. The main source of data for this paper comes from the experimentation with two problems:

1. Rectangle problem: *Let E be any point on the diagonal of a rectangle ABCD such as AB =8 units and AD=6 units. What relation is there between the areas of the shaded rectangles in the figure below?*



2. Triangle problem: *Let P be any point on the median [AM] of a triangle ABC. What relation is there between the areas of the triangles APB and APC?*

These problems have to do with comparing areas and distances in situations of plane geometry. They admit different solving strategies; they can be solved by mixing graphical and deductive issues, they are easily adaptable to the specific needs of each student, and they can be considered suitable for the use of GGB. For all the problems, we start by exploring the basic space of the problems in the P&P and GGB environments. After having identified the different resolution strategies and conceptual contents of the problems, the focus is on analyzing the necessary knowledge to solve them. Finally, we prepare a document with the pedagogical and technical messages that provide differing levels of information.

All the activities with students are planned to take four sessions of one hour each with an average of two problems per session. The two problems above were developed in the first two lessons in which the students worked on their own. The inquiry-based approach to the lessons leads the students to assume the responsibility for the development of the task. The teacher fosters the students' autonomy by only intervening in certain moments and giving some messages, established a priori, concerning the resolution.

For the experimentation with each problem, the whole set of data is: a) the solving strategies in the written protocols (P&P and GGB); b) the audio and video-taped interactions within the classroom (student-teacher, student-content and student-GGB); and c) the GGB files. All these data were examined in order to inform about our research goals. The integration of data concerning these goals led us to the description of the students' process of instrumentation. For the description, different variables were considered, among them: the students' heuristic strategies (related to geometric properties, to the use of algebraic and measure tools or to the use of both…); the use of GGB (visualization, geometrical concepts, overcoming difficulties…); the obstacles encountered in each environment (conceptual, algebraic, visualization, technical obstacles…); etc.

For each case, we first analyze the P&P resolution with data coming from the tapes and the protocols. We consider the student's solving strategies and the use of mathematical contents. Then we analyze the GGB resolutions with data coming from the tapes and especially from those tapes that show the screen. We consider again the student's solving strategies, the use of mathematical contents and now we also pay attention to instrumented techniques and technical difficulties. After having developed these two types of analysis, we compare GGB and P&P resolutions by looking at the use of the two environments within each problem. To analyze the problem solving process, we also consider the phases of the problem solving process (Schoenfeld, 1985) as a whole in each group of problems (GGB and P&P).

THE CASE OF SANTI: An episode of exploration/analysis

The mathematical content of the problem was dealt with in courses prior to the one Santi is currently taking. Santi has procedural knowledge relating to the application of formulas for calculating the area of the Figure, and sufficient knowledge of the concepts associated with geometric constructions. He is a high-achieving student. Santi is asked to solve the first problem with P&P and the second problem with the help of GGB. In this section we summarize his problem solving process for both problems.

- Resolution of the rectangle problem (P&P):

In the resolution of the first problem, after reading the statement of the problem, Santi observes the figure and then he states that he does not have enough numerical data. The teacher suggests the student to consider a particular case (heuristic cognitive message of level 1 in the planning /execution phase). Santi reacts to this message, considering the particular case in which E is the midpoint of the diagonal and he conjectures that both areas should be equal. Then he tries to prove the conjecture for the particular case in which the length AE is 2 units. The student reaches a solution to the particular case by using trigonometry. He obtains the angles in the triangle EAN (Figure 1) and he calculates the measures of the sides, AN and AM. Finally he obtains the numerical value of both areas and he observes that he gets different values. Santi requests a message about the solution because he expected to obtain equal values. The teacher remarks that there is an algebraic mistake in his resolution and suggest Santi to review the process he has followed because there are algebraic mistakes (metacognitive message of level 1 in the verification phase). The student finds the mistake and obtains the equal values of both areas (Figure 1). He then tries to use the same strategy for the general case using the relation: $\frac{8}{6} = \frac{AN}{AM}$.

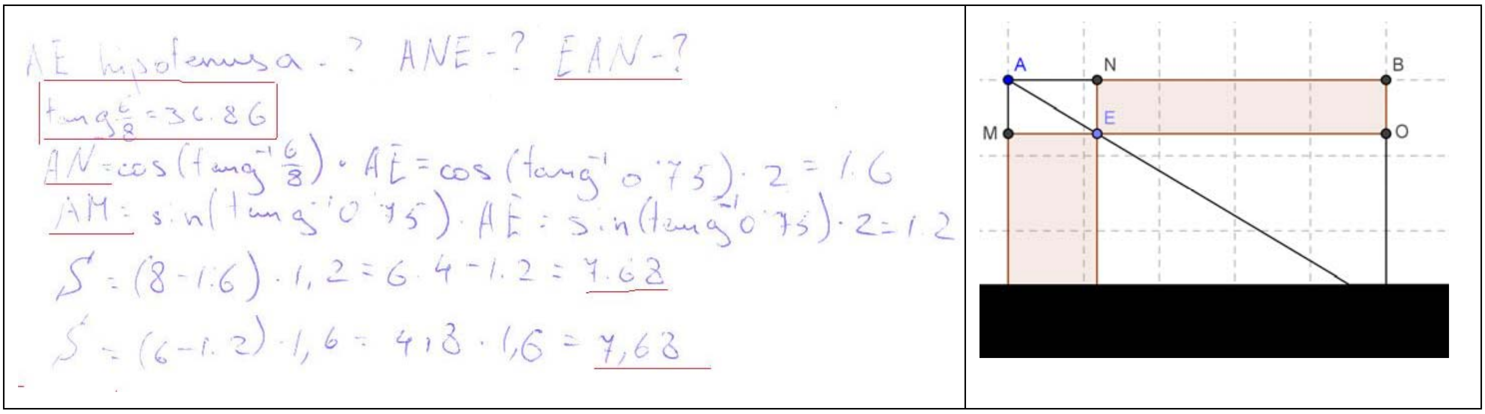


Figure 1: Resolution with paper and pencil of the first problem (Santi)

Santi bases his resolution strategy on applying trigonometry and he does not try to use the strategy based on comparing areas of congruent triangles (strategy based on equivalence of areas due to complementary dissection rules). The teacher proposes other problems to be solved with P&P and with GGB. In the following paragraph we consider one of these problems.

- Resolution of the triangle problem (GGB):

After reading the statement of the problem, Santi draws a graphic representation without coordinate axes before constructing the figure with GGB. The teacher observes that Santi has considered the point P in the side AC of the triangle instead of the median. The teacher gives Santi the following message: "*Try to understand the conditions of the problem*" (metacognitive message of level 0 in the familiarization phase). Santi constructs a new figure with GGB (Figure 2) and he observes the figure trying to find a solving path. Then he proposes a conjecture and asks the teacher for verification: "*the triangles APC and APB have a common side and the same area (he* *verifies this with the tool area of a polygon). How could I prove that these two triangles are equal* [congruent]*"? I have tried to prove that they have the same angles but I don't see it..."*

We observe that Santi does not validate his conjecture with the help of GGB (using measure tools for instance). The teacher gives him a validation message of level 1*"Are you sure that these triangles are congruentl?* Santi reacts to this message changing the triangle ABC. He drags the vertex A (Figure 3) and he observes without measure tools that the triangles are different.

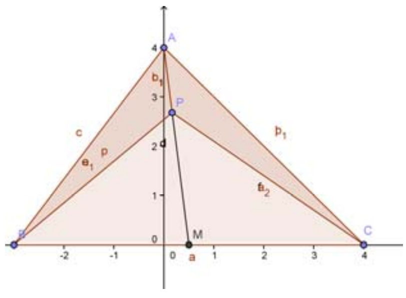


Figure 2: Construction with GGB of the triangle ABC and its median. Santi uses the tool polygon to construct the triangles.

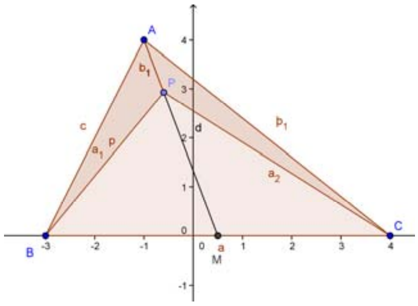


Figure 3: He moves the vertex A to obtain a general triangle. We observe that he tries to define vertices with coordinates that are integer numbers.

The last graphic deduction marks the beginning of the search for a new strategy. He observes the figure, without dragging its elements. More than five minutes have gone without doing anything in the screen. Santi requests again the help of the teacher (Table 1, line 1) for the familiarization phase of the problem.

		Interactions
1	Santi	Is P any point in the segment AM? Isn't it the midpoint? [Santi tries to consider particular cases]
2	Teacher	P is any point in the median [AM]. The triangle ABC is also a general triangle [cognitive message of level 1 for the familiarization phase]
....	Santi	[Santi reacts to this message modifying the initial triangle. He drags again the vertices to obtain the triangle in Figure 3].
3	Santi	I think that I see it!...The triangles have a common side and the same height [the segments [BM] and [MC] (wrong deduction)]
4	Teacher	Are you sure about that?
5	Santi	[Santi reacts to this message observing the triangle without doing any action on the screen. Then he states:] No. These lines are not perpendicular! [(AM) and (BC)]. But, this was a good trial...

		Have they the same base? [he refers to the common side of both triangles]
6	Teacher	Yes

Table 1: How Santi tries a new solving path

For the first time, Santi tries to drag the vertices of the triangle trying to find invariants. While he drags the vertexes he looks in the algebraic window for invariants. We observe here the simultaneous use of the algebraic window and the geometric window. He observes again that the triangles have the same area in all the cases and a common side. He tries to prove that the heights are equal but he wrongly considers that the side [BM] is the height of the triangle BAP (Figure 3). The teacher gives him a message of level 0 for the validation phase (Table 1, lines 3 to 6). Santi reacts to this message constructing with GGB the perpendicular line from the vertex B to the base of the triangle (Figure 4). He tries to follow the same strategy (proving that the heights have the same length) and he drags continuously the vertexes A, B and C, changing the orientation of the triangle, and observing the constructed lines on the geometric window.

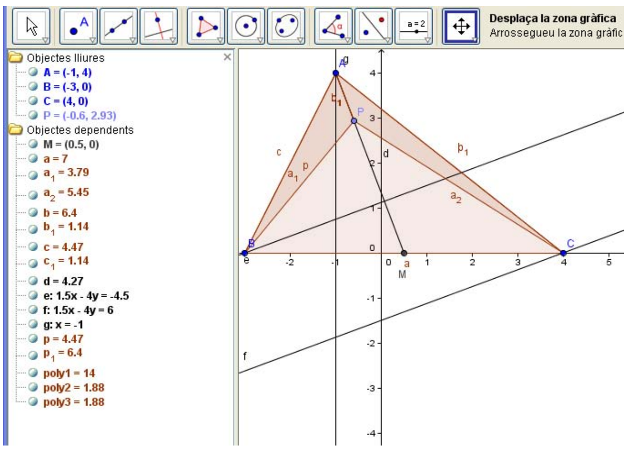


Figure 4: Construction of the height of the triangle BPA and perpendicular line through C to the median.

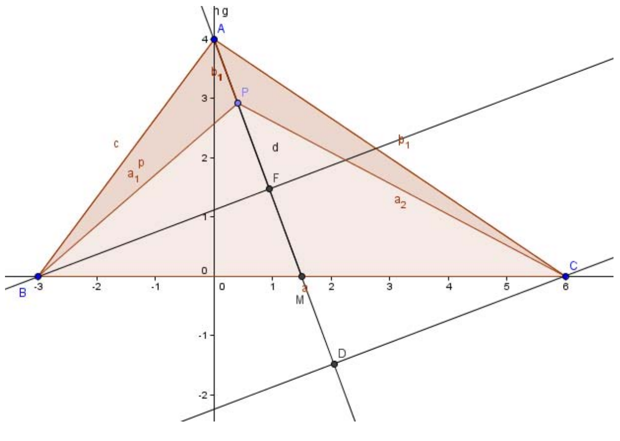


Figure 5: the heights have the same length (congruent triangles BFM and MCD)

In this time, he observes again the figure (Figure 4) without dragging. He is lost. This is the beginning of a new phase. We wonder if Santi had found a proof for his conjecture if he had constructed the heights of both triangles. Nevertheless, he does not construct the points F and D (Figure 5) and he abandons the solving strategy. Santi requests again the help of the teacher for the planning/execution phase and he states: “*Is it possible to solve the problem with trigonometry?*”.The teacher gives him a new message: “*Could you think of some way of breaking the triangle ABC into triangles and look for invariants with the help of GGB”* (cognitive message of level 2 for the planning phase). Santi reacts to the previous message of the teacher and starts a new exploration phase. He erases the perpendicular lines and drags continuously the vertexes of the triangle ABC. He observes in the algebraic window the changing values looking for invariants. He extracts the inner triangles BPM and CPM which have the same area (Table 2, line 1) from the initial configuration. This observation will suggest him a new solving path based on comparing areas. He makes a new conjecture and requests the help of the teacher for validating his deductions (Table 2).

		Interactions
1	Santi	Are the triangles BPM and PMC equal? (Figure 2)
2	Teacher	What do you mean by equal?
3	Santi	The triangles have the same area
4	Teacher	Yes. You should justify this fact.
5	Santi	If I subtract two equal areas from two equal areas, do I get the same area?
6	Teacher	Yes
6	Santi	Ok! I justify this with paper and pencil.

Table 2: Strategy based on comparing areas

Finally Santi justifies his deductions with P&P, he proves that the median of a triangle divides the triangle into two triangles of same area. We wonder if the use of GGB helps Santi to find a strategy based on comparing areas.

FINAL REMARKS

We observe in this study that Santi appropriates the software in few sessions of class and he bases his constructions on geometric properties of the figures. He also combines the simultaneous use of the algebraic window and the geometric window and he tends to reason on the figure. We consider that the affordances of the software and teacher's orchestration have influenced Santi's resolution strategies. We have identified the following instrumented schemes: 'dragging combined with perceptual approach to find a counter-example' and 'dragging combined with perceptual approach to distinguish geometric properties of the figure (perpendicularity, congruence of triangles, equality of areas). In the ongoing research (longer teaching experiment) we have also observed some common heuristic strategies in both environments such as the strategy of supposing the problem solved and the strategy of particularization. We have also observed that Santi tends to use more algebraic strategies when he works only with P&P than when he works in a technological environment. Moreover he tends to produce more generic resolutions, independent of numerical values, fostered by a proposal of problems that accept these kinds of solving strategies. Nevertheless, given that students have different relationships with the use of GGB and the detailed study of Santi gives us some insight of a future classification of typologies in the instrumental genesis. In our broader research we try to follow the instrumental genesis for a group of fourteen students to observe different students' profiles. Future research should help to better understand the process of appropriation of the software and to analyze the co-emergence, connectivity and synergy of computational and P&P techniques in order to promote argumentation abilities in secondary school geometry.

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