INTERNET AND MATHEMATICAL ACTIVITY WITHIN THE FRAME OF "SUB14"

Hélia Jacinto, *EBS Prof. Dr. Francisco Freitas Branco*

Nélia Amado, *FCT, University of Algarve and CIEFCUL*

Susana Carreira, *FCT, University of Algarve and CIEFCUL*

In this paper we analyze and discuss the use of ICT, particularly the Internet, in the context of a mathematical problem-solving competition named "Sub14", promoted by the University of Algarve, Portugal. Our purpose is to understand the participants' views regarding the mathematical activity and the role of the technology they've used along the competition. Main results revealed that the participants see the usage of Internet quite naturally and trivially. Regarding the mathematical and technological competences elicited by this competition, evidences were found that develops mathematical reasoning and communication, as well as it increases technological fluency based on the exploration of everyday ICT tools.

A GLIMPSE OVER THE MATHEMATICAL COMPETITION "SUB14"

Sub14 (www.fct.ualg.pt/matematica/5estrelas/sub14) is a mathematical problem-solving web based competition addressed to students attending 7^{th} and 8^{th} grades.

It comprises two stages. The *Qualifying* consists of twelve problems, one every two weeks, and takes place through the Internet. The Sub14 website is used to publish every new problem; it provides updated information and allows students to send their answers using a simplified text editor in which they can attach a file containing any work to present their solution. The participants may solve the problems working alone or in small teams and using their preferred methods and ways of reasoning. They have to send their solution and complete explanation through the website mailing device or using their personal e-mail account. Every answer is assessed by the organizing committee, who always replies to each participant with some constructive feedback about the given answer.

The word problems are selected according to criteria of diversity and involve several aspects of mathematical thinking not necessarily tied to school mathematics. Their aim is to foster mathematical reasoning, either on geometrical notions, numbers and patterns, and logical processes, among others. There is a concern on presenting problems that allow different strategies and also some that have multiple solutions.

In Iona's class the students had to elect a delegate and a co-delegate. Each student wrote two names in a voting sheet by order: the first for the delegate and the second for the co-delegate. There are 13 students in the class. How many ways have a student to vote if his or her own name is allowed?

Fig. 1: A problem aiming to elicit the abilities of organizing and counting

The *Final* consists of a one-day tournament where the finalists solve five problems, individually, with paper and pencil, and explain their reasoning and methods. This *Final* also provides some recreational activities addressed both to contestants and accompanying persons, namely parents and teachers.

Joanna, Josephine and Julia are all very fond of sweets. As the summer aproaches they decide to go on a diet. Their father has a large scales and they used it to weigh themsleves in pairs.
Joanna and Josephine together wheighed 132 kg
Josephine and Julia together wheighed 151 kg
Julia and Joanna together weighed 137 kg.
What is the weight of each one?

Fig. 2: A problem from the Final on identifying and relating variables and numbers

Demanding a clear description of the reasoning, methods and procedures was a strong concern of the committee. Moreover, the feedback sent to each participant had an essentially formative role (Diego & Dias, 1996), aimed at stimulating self-correction and valuing students' own ideas. Every two weeks the Sub14 committee publishes a proposal of the solution of the previous problem, stressing the diversity of strategies that students could have applied. Hence, the committee selects noteworthy excerpts from student's solutions, whether due to the originality of their reasoning, their creativity or the interesting usage of technological tools.

A THEORETICAL FRAMEWORK

In this paper we are addressing a part of a larger study and consequently we refer to a few theoretical aspects of the overall framework. There are four main focuses in the theoretical approach: (a) looking at mathematics as a human activity, (b) taking problem solving as an environment to develop mathematical thinking and reasoning, (c) exploring the concept of being mathematically and technologically competent and finally (d) considering the role of home ICT in out of school mathematics learning.

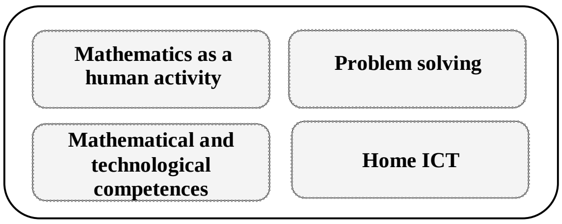

Fig. 3: Main conceptual elements of the theoretical framework

Mathematics as a human activity

Doing mathematics may be recognized as a human activity based upon a person's empirical knowledge, in search of a formalized understanding of the everyday problematic situations. From this point of view, Freudenthal (1973, 1983) states that human activity, which comprises empirical knowledge, guides oneself from the simple observation and interpretation of phenomena – horizontal mathematizing – to its abstract structuring and formalization – vertical mathematizing.

One of the criteria observed in launching a problem in Sub14 refers to the expectation that participating students will be able to activate their empirical knowledge and their experience to tackle mathematical problems. This perspective on mathematical activity is shared by many authors who emphasize the importance of exploring mathematical situations starting from common sense knowledge (Hersh, 1993, 1997; Ernest, 1993; Ness, 1993; Matos, 2005). As Schoenfeld (1994) claims, easiness in the use of mathematical tools, like abstraction, representation or symbolization, does not guarantee that a person is able to think mathematically. Rather mathematical thinking requires the development of a mathematical point of view and the competence to use tools for understanding.

This is the perspective that is present in Sub14 and which expresses the prevailing concept of mathematical activity arising from the perspective of Realistic Mathematics Education: bringing student's reality to the learning situation so that he/she is the one who does the mathematics, drawing on his/her knowledge and resources.

Mathematical knowledge and problem solving

Several authors from the field of mathematics education have proposed problem solving as a privileged activity "for students to strengthen, enlarge and deepen their mathematical knowledge" (Ponte et. al, 2007, p. 6).

This view on mathematical problem solving entails a conception of mathematical knowledge that is not reducible to proficiency on facts, rules, techniques, and computational skills, theorems or structures. It moves towards broader constructs that entails the notion of mathematical competence (Perrenoud, 1999; Abrantes, 2001) and problem solving as a source of mathematical knowledge. In solving a problem there are several cognitive processes that have to be triggered, either separately or jointly, in pursuing a particular goal: to understand, to analyze, to represent, to solve, to reflect and to communicate (PISA, 2003).

According to Schoenfeld (1992), the concept of mathematical problem can move between two edges: (i) something that needs to be done or requires an action and (ii) a question that causes perplexity or presents a challenge. The educational value of a problem increases towards the second pole where the solver has the possibility of coming across significant mathematical experiences. One of the purposes of mathematical problems should be to introduce and foster mathematical thinking or adopting a mathematical point of view, which impels the solver to mathematize: to model, to symbolize, to abstract, to represent and to use mathematical language and tools (Schoenfeld, 1992, 1994).

The formative aims of the problems proposed in Sub14 are essentially in line with the perspective of giving students the experience of mathematical thinking and also the opportunity to bring forth mathematical models and particular kinds of reasoning.

Communication, home technologies and learning

Considering that mathematics is a language that allows communicating your own ideas in an accurate and understandable way (Hoyles, 1985), Sub14 intends to develop that relevant communicational aspect, as stated in the current National Curriculum: “students must be able to communicate their own ideas and interpret someone else’s, to organize and clearly present their mathematical thinking” and “should be able to describe their mathematical understanding as well as the procedures they use” (Ponte et. al, 2007, p. 5). Conversely, the importance of developing the competence of mathematical communication draws on a strong connection between language and the processes that structures human thought, as it is referred by Hoyles (1985). Accordingly, language takes up two different roles in mathematical education: communicative, where students show the capacity to describe a situation or reasoning act; and cognitive, which may help to organize and structure thoughts and concepts. Hence, there is a multiplicity of capacities and competences, both mathematical and technological, which are triggered through the combination of facts and resources in order to solve each problem of the competition.

Technologies and particularly the Internet, which gave life to Sub14, had a somewhat “neutral” or “trivial” role since the main focus of students’ concerns was on the actual mathematical activity involved. Noss and Hoyles (1999) used the “window image” to emphasize this phenomenon: a window allows us to look beyond, and not only at the object itself. Although every new technology tends to draw attention to the tool itself, we soon need to “forget” the tool and concentrate on the potentialities it has to offer, namely on the learning and cognition field.

Using Lévy’s (1990) ideas, Borba and Villarreal (2005) claim that technology mediates the processes that are responsible for the rearrangement of human thinking. In fact, knowledge is not only produced by humans alone, but it’s an outcome of a symbiotic relationship between humans and technologies – which the authors entitled *humans-with-media*: “human beings are impregnated with technologies which transform their thinking processes and, simultaneously, these human beings are constantly changing technologies” (p. 22).

Indeed, human thought used to be defined as logical, linear and descriptive. Nowadays it is hastily changing into a *hypertextual thinking*, comprising many forms of expression that go beyond verbal or written forms, such as image, video or instant messaging. These social changes allow youngsters to develop a large number of competences, which grants them the skills and sophistication required to learn outside the school barriers.

Towards the conclusions of the ImpaCT2 project, that took place in Great Britain, Harrison (2006) asserted that the model used to measure the influence of new technologies on youngster’s school achievement was too simplistic and induced to settle on the absence of such influence. This author then proposed a new model that emphasized the importance of social contexts in which learning takes place. Harrison (2006) was thus able to conclude that learning at home must not be neglected, but be faced as a partner of the school curriculum.

Although knowledge gathered outside the school is frequently seen as worthless, it is clear that children are capable of watching a YouTube's video, talk to their friends through MSN, and also solve the Sub14 problems and express their thinking using an ordinary technological tool. These "digital natives" (Prensky, 2001, 2006) access information very fast, are able to process several tasks simultaneously, prefer working when connected to the Web and their achievement increases by frequent and immediate rewards.

METHODOLOGY

The purpose of this study was to identify and understand the participants' perceptions regarding the (i) mathematical activity, (ii) the competences involved and (iii) the role of the technological tools they've used along the competition.

A case study methodology reveals itself appropriated in cases where relevant behaviours can't be manipulated, but it is possible and appropriate to proceed to focused interviews, attempting to understand the surrounding reality (Yin, 1989). Since we intended to get diversity and interpret results, eleven participants were chosen intentionally, from the 120 finalists, hoping they would provide interesting data according to the research questions.

The field work began collecting data that would allow a complete understanding of the competition, in order to adjust the approach to the participants. Later on, we used other data collecting techniques: a *questionnaire* to the finalists, *video records* from the Final, *documental data* from participants (such as their solutions to the Sub14 problems, or their interactions with the Sub14 committee, using e-mail). That data allowed the planning of interviews to the eleven participants, as well as to their parents, aiming at collecting descriptive data, in their own language, hoping for an understanding on how they viewed certain aspects of Sub14 and of their involvement.

For the data analysis we used an interpretative perspective (Patton, 1990) and an inductive process (Merriam, 1988), based on content analysis. Thus, the objective was to understand the significance of the events from the interviewees' perspective, within the scope of the theoretical assumptions defined prior to the interviews.

THE INTERNET – THE SUB14 LIFE SUPPORT

The first evidence produced about students' perceptions on the problem solving environment was the fact that the Internet and the technologies used within Sub14 assumed, from the point of view of students, a neutral role in the development of their mathematical activity. However several aspects of their products and statements showed evidence of the importance and usefulness of different tools, behind their apparent indifference to technology if put in abstract terms. Therefore, we may state that the Internet undoubtedly is the technology that brings Sub14 to life; all the learning processes and the competences involved derive from the interaction provided and nourished by this tool.

Trivializing Technology

Resorting to the Internet and other technologies was seen as absolutely natural by some participants.

> "As I see it, reasoning comes from the mind; therefore I think no technology will help us to really solve a problem." [Bernardo]

Trivializing the role of the Internet and the technology involved in the competition can be found in the model proposed by Harrison (2006), which highlights the importance of the social context surrounding the learning process. These participants show all the characteristics of a digital native (Prensky, 2001), i.e., they start using computers at an early age, with a great variety of purposes, which can be related or not to school learning. Furthermore, these participants can also be considered as "humans-with-media", or particularly, "humans-with-Internet", according to the definitions proposed by Borba and Villarreal (2005), since their personality is being built, simultaneously, through the daily interaction with the Internet and other technologies.

The Role of Communication and Feedback

Essentially, the participants like the feedback sometimes provided immediately by the Sub14 committee, resulting from the analysis of their answers to each problem. The possibility of correcting little mistakes or even change the resolution completely, using the hints from the feedback, increase their self-esteem and motivation to remain in the competition. For the interviewed students, this is the characteristic that distinguish Sub14 from other similar competitions.

> "This year I also participated in another competition. We send an answer to a problem, but they don't reply to us, and the Sub14 committee keeps sending hints". [Isabel]

As students pointed out there is someone who receives their answer to the problem, their questions or even their complaints.

> "It's not something that we send and no one will care about, they are always there." [Lucia]

As mentioned above, the feedback is almost immediate and this is only possible due to the communicability that the Internet enables. The constant request for auto-correction forces the participants to reflect on their own reasoning and the mistakes given, stimulating them to submit a correct answer as quickly as possible. Some of them sent messages to Sub14 several times a day, until they get the confirmation that their answer was correct.

Another positive aspect of this bilateral communication is the request of a complete, coherent and clearly written explanation of the participant's reasoning. This way, the feedback provided by the organizing team respects and nourishes the reasoning of each participant, as well as the processes used. We have even noticed a development on the correctness of the answers that the participants submitted throughout the competition.

> "In the beginning it was somehow strange. I wasn't used to it. I'd put the calculations and that was it. But we had to present all our thinking. It was as if I had to write what I was thinking. Thus, I would think out loud and split it into parts. But from the 3rd or 4th problem I was already used to it." [Isabel]

This feedback originated a change of attitudes in some participants within their mathematics classroom when facing assessment situations. The students themselves observed they took more care while answering to questions posed by the teachers, presenting all the necessary justifications and showing a greater predisposition to interpret a problematic situation, find a reasoning path or procedure in order to explain the solution in a convincing way.

> "[…] I now pay more attention to little details that sometimes others don't, and it reflects on the tests and on the problems that the teacher gives us, some of them really tricky… but now I am tuned!" [Lucia]

"Home Technologies"

The dynamic nature of the bidirectional communication can be felt in other aspects revealed by the participants. First off all, we note the usage of the Sub14 website: the participants use it frequently and think that the available information is important and useful, they like the design, the way it is organized and the fact that it is permanently updated:

> "I like having an organized website (…) the 'Press Conference' page was always updated." [Ana]

The purpose of posting submitted solutions was to show the methods used by some of the participants, hoping to improve their performance by the positive reinforcement of seeing their works and their names posted online.

> "Yes! Sometimes I would go there to see if any of the posted solutions was mine! Once or twice I found my answer and I was very happy and shouted… 'Daddy, daddy, come here!'" [Bernardo]

Bernardo's enthusiasm, as well as many other participants', supports the pedagogical and motivational aspects of the methodology adopted in Sub14. Not only it promotes the diversification of reasoning strategies and points out the several problem solving phases, but it also increases self-esteem and improves innovation and creativity as "special" answers are selected to be published online.

Moreover, the fact that Sub14 is a digital competition allowed the participants the opportunity of communicating their reasoning in an inventive way, since they could resort to any type of attachments, particular the ones they felt more comfortable with or the ones they found adequate to the problem itself. Therefore, the participants used mainly the text editor, MSWord, but they also used drawing and spreadsheet programs, like MSPaint and MSExcel, all examples of home technology.

MSWord was used to compose text, organize information in tables, and insert images, automatic shapes, WordArt objects or Equation expressions. It was elected the favorite between the participants, since it is the one they better understand and constantly are asked to use for several school assignments.

> "[Word] is the simplest to use, it's the one that I have more confidence on to do school tasks, and I'm used to it. It's the one I'm good at." [Lucia]

Using images was a strategy that seven of the interviewee used. Nevertheless, some of them only inserted images that had something to do with the problem context, more like an illustration. In this case, we may consider that resorting to images had mainly an aesthetic function, as it didn't help presenting or clarifying the reasoning and strategy used to solve the problem. However, other interviewees sketched their own images using MSPaint in order to improve the intelligibility of their thoughts:

> "Anything that I thought that could help to improve the reasoning, I would draw it [in paper] and then I'd put it in the computer." [Bernardo]

> "We were playing with some straws and we reached the solution by trial and error. Then [we took some pictures] with the digital camera [and] put them in the computer so that we could send them." [Alexandra]

In this way the image usage assumed, essentially, two roles in the answers of these participants. Firstly, it was merely a visual detail, which may be influenced by the type of work done in students' school assignments. Secondly, the creation of images within the context of their interpretation of the problems is an evidence of their efforts on expressing their reasoning in the best possible way. Moreover, we can notice their awareness of the different representations that could materialize their reasoning and even some decision ability when facing the options they had at hand.

Two interviewees used Excel to present their answers. One of them used this tool to solve every Sub14 problem, showing however a narrow usage of the program as a means to organize the information and his answer. Seldom using the function "SUM", he essentially resorted to tables and images, considering that the spreadsheet was better than a text editor. The referred simplicity seems to come from the fact that he has been exposed to this tool from an early age:

> "Sometimes, when I was a kid – I got my first computer when I was six – I liked to get there [MS Excel] and do squares with the cells, paint them and that sort of things…" [Bernardo]

Another participant used the spreadsheet to solve five of the twelve problems, showing that he knew some of the advantages of this tool. Therefore, these participants were confident enough in using MSExcel, nonetheless not as a result of work within the school context, but rather of their domestic "findings".

ANOTHER LOOK AT SUB14 AS A LEARNING ENVIRONMENT

Solving the Sub14 mathematical problems requires looking at a problem situation from a mathematical perspective. This can be seen as a mathematizing process, since the participant is stimulated to express the way in which thinking was organized and progressed. In this competition, the participants found a place where they could freely communicate their ideas, had someone who listened and advised them, helping to make their mathematical thinking and expression become clearer. Moreover, when solving a problem, they faced the transition from convincing themselves to convincing the others (Mason, 2001). This led participants to develop their own understanding of the problem, promoting the usage of domestic technologies to communicate, thus adding competences that sometimes school neglects or forgets.

As a learning environment, although being external to the school context, Sub14 is aligned with school mathematics, and promotes a set of competences that fit within current mathematical education purposes and curricular targets. The fact that the competition occurs in a loose institutional context allows a greater family commitment and complicity with the participant's learning process, fostering the discussion on mathematical questions and problems outside the school environment, especially at home, maybe around dinner table.

Further work on this field shall include a future experience to investigate the possibility of allowing participants to communicate amongst them, within the website, bearing in mind the idea of a connected learning environment.

REFERENCES

Abrantes, P. (2001). Mathematical competence for all: Options, implications and obstacles. *Educational Studies in Mathematics*, **47**, p. 125-143.

Borba, M. & Villarreal, M. (2005). *Humans-with-Media and Reorganization of Mathematical Thinking: Information and Communication Technologies, Modeling, Experimentation and Visualization.* New York, NY: Springer.

Diego, S. & Dias, P. (1996). Feedback educativo: contributo para o reconhecimento da sua importância em software educativo. *Actas do Simpósio Investigação e Desenvolvimento de Software Educativo*. Costa da Caparica.

Ernest, P. (1993). Mathematical activity and rhetoric: a social constructivist account. In I. Hirabayashi et al., (Eds). *Proceedings of the 17th Annual Conference of the International Group for the Psychology of Mathematics Education*, (pp. 238-245). University of Tsukuba, Japan.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Reidel.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht: Reidel.

Harrison, C. (2006). Postmodern Research and E-Learning: Anatomy and representation. *European Educational Research Journal*, 5(2), p. 80-93.

Hersh, R. (1993). Humanistic Mathematics and the Real World. In A. White (Ed.). *Essays in Humanistic Mathematics*, (pp. 15-18). Washington D.C.: MAA.

Hersh, R. (1997). *What is Mathematics Really?*. New York: Oxford University Press.

Hoyles, C. (1985). What is the point of group discussion? *Educational Studies in Mathematics,* 16 (2), p. 205-214.

Lévy, P. (1990). *As Tecnologias da Inteligência. O Futuro do Pensamento na Era da Informática.* Lisboa: Instituto Piaget.

Mason, J. (2001). Convincing Myself & Others: Discussing with mummy and justifying to daddy. *Mathematics Teaching*, **177**, p. 31-36.

Matos, J. F. (2005). Matemática, educação e desenvolvimento social – questionando mitos que sustentam opções actuais em desenvolvimento curricular em Matemática. In L. Santos et al. (Orgs.), *Educação e Matemática: caminhos e encruzilhadas*. *Actas do encontro internacional em homenagem a Paulo Abrantes*. (pp. 69-81). Lisboa: APM.

Merriam, S. (1988). *Case Study Research in Education – a qualitative approach*. San Francisco: Jossey-Bass Publishers.

Ness, H. M. (1993). Mathematics, an Integral Part of Our Culture. In A. White (Ed.). *Essays in Humanistic Mathematics*, (pp. 49-52). Washington D.C.: MAA.

Noss, R. & Hoyles, C. (1999). *Windows on Mathematical Meanings. Learning Cultures and Computers.* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Patton, M. (1990). *How to use qualitative methods in evaluation*. Newbury Park: Sage.

Perrenoud, P. (1999). *Construir as competências desde a Escola*. Porto Alegre: Artes Médicas Sul.

PISA (2003). *Conceitos Fundamentais em Jogo na Avaliação da Resolução de Problemas*. Ministério da Educação, GAVE. Lisboa.

Ponte, J. et al. (2007). *Programa de Matemática do Ensino Básico*. M.E., Lisboa.

Prensky, M. (2001). Digital Natives, Digital Immigrants. *On the Horizon*. 9(5), October, (n/p.). NCB University Press.

Prensky, M. (2006). *Don't bother me, Mom, I'm learning! How computer and video games are preparing your kids for 21st century success and how you can help!*. St. Paul, MN: Paragon House.

Schoenfeld, A. (1992). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*, (pp. 334-370). Reston: NTCM.

Schoenfeld, A. (1994). Reflections on doing and teaching mathematics. In A. Schoenfeld (Ed). *Mathematical Thinking and Problem Solving*. (pp. 53-70). Hillsdale, NJ: Lawrence Erlbaum Associates.

Yin (1989). *Case Study Research*. Design and Methods. Thousand Oaks, CA: Sage Publications.