THE ROBOT RACE:

UNDERSTANDING PROPORTIONALITY AS A FUNCTION WITH ROBOTS IN MATHEMATICS CLASS

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This paper presents and discusses the use of robots to help 8th grade students learn mathematics. An interpretative methodology was used and data analyses were supported by Situated Learning Theories and Activity Theory. These tools allowed the accurate description and analysis of student's practices in mathematics classes. The results indicate that the use of robots to study proportionality as a function aided and supported student learning.

INTRODUCTION

Educational systems the world over are investigating new and engaging mechanisms in order to better present complex concepts and challenging domains such as mathematics. The implementation and exploration of technologies in classrooms is a promising general approach. However, we should not neglect the real world where the actual students live – a world more and more dependent on technologies. Consequently, it is essential to combine computation aids and new educational aims with a redefinition of teaching processes and teachers role's in the classroom. It is in this context that the project DROIDE was initiated in 2005.

DROIDE[2]: "Robots as mediators of Mathematics and Informatics learning" is a project with three main objectives:

(1) to create problems in Mathematics Education/Informatics areas which are suitable to be solved using robots; **(2)** to implement problem solving using robotics at three points in the educational system: mathematics classes at K-9 and K-12 levels; Informatics in K-12 levels; Artificial Intelligence, Didactics of Mathematics and Didactics of Computer Science/Informatics at the university level; **(3)** to analyze and understand students' activity during problem solving using robots.

This paper discusses research on the second issue (the implementation of problem solving using robots in mathematics class) at the K-9 level. It addresses the following research problem: to describe, analyse and understand how students learn mathematics using robots as mediators of learning. It particularly focuses on the mathematical concept of proportionality as a function.

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THEORETICAL BACKGROUND

The research approach is derived from Situated Learning Theories (Lave & Wenger 1991, Wenger, 1998, Wenger et al, 2002) and Activity Theory (especially the 3rd generation introduced by Engeström, 2001). A key element of Situated Learning theories is the notion of a community of practice and the suggestion that learning is a situated phenomenon. In this paper, this viewpoint is used to reflect upon emergent learning within students' mathematical practices.

The Concept of Practice

According to Wenger, McDermott and Snyder (2002) practice[3] is constituted of a set of "work plans, ideas, information, styles, stories and documents that are shared by community members" (p.29). Practice is the specific knowledge that the community develops, shares and maintains. Practice evolves as a collective product integrated in participants' work and the organisation of knowledge in ways that are useful and reflect the community's perspectives (Matos, 2005).

Wenger (2002) proposes three dimensions in which practice is the source of coherence in a community: mutual engagement, joint enterprise and shared repertoire. Mutual engagement is a sense of "doing things together"; the sharing of ideas and artefacts, with a common commitment to interaction between community members. Joint enterprise is having (and being mutually accountable for) a communal common goal, a procedure which rapidly becomes an integral part of practice (Matos, Mor, Noss and Santos, 2005). Shared repertoire refers to a set of agreed resources for discussions and negotiations. This includes artefacts, styles, tools, stories, actions, discourses, events and concepts.

The Concept of Mediation

Engeström (1999) conceptualizes an activity model formed by three elements – the subject, the object and the community – with mediation relations between them. In the context of this research, the mathematics classroom forms such an activity system. The subject is part of a collective; reflecting the fact that we do not act individually in the world. The subject is part of a system of social relations.

The concept of mediation has a central role in Activity Theory[4]. It is based on the presupposition that the subject does not act directly on the environment; that it has no direct access to the objects. The relation between subject and object is mediated by artefacts (Werstch, 1991); things constructed by individuals and maintaining a dialectic relation between people and activity (Werstch, 1991). To say that a tool or

[3] The term practice is sometimes used as an antonym for theory, ideas, ideals, or talk. In Situated Learning theories that is not the idea. In Wenger's sense of practice, the term does not reflect a dichotomy between the practical and the theoretical, ideals and reality, or talking and doing. The paper extension does not allow the development of the idea of practice. For discussion of practice related with mathematics education see Fernandes (2004).

[4] For a more general vision of Activity Theory see http://pparticipar-t-act.wikispaces.com/

artefact is mediator of learning means that it gives power to the process of transformation of objects; that it is a tool with which people think (Piteira, 2000).

This paper claims that robots can be artefacts, mediators of the learning of functions. The veracity of this claim is demonstrated in the following sections.

METHODOLOGY

The work reported in this paper was organised into three stages:

First stage – analysis of School Mathematics and Informatics curriculum; selection of didactical units where robotics can be used; creation of problems/tasks to be solved in Mathematics and Informatics classes.

Second stage –implementation of problems/tasks in Mathematics and Informatics classes; data collection through video recordings of students.

Third stage – analyses of student activity during learning with robots using interpretative methods introduced in Situated Learning Theories and Activity Theory. The unit of analysis was "(...) the activity of persons-acting in setting" (Lave, 1988, p.177).

LEARNING AS PARTICIPATION: ANALYSING STUDENTS MATHEMATICAL ACTIVITY WHEN USING ROBOTS

A brief description of mathematics class

In mathematics classes students worked in small groups. In the initial phase, the work involved construction of the robots and basic programming to solve simple tasks. This activity took place on a Windows® desktop environment and the students used a visual programming tool that ships with the robot kits. Subsequently, students used the robots to recognise and apply concepts in coordinate systems, to understand the meaning of function, to represent one function (proportionality) using an analytic expression and to intuitively relate a straight line slope with the proportionality constant, in functions such as $x \mapsto kx$.

General plan of work for functions unit

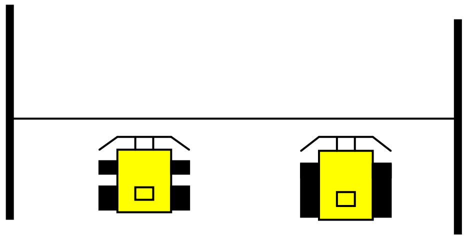
The first mathematical unit students worked on involved functions. Four sets of problems were prepared. **Problem set 1** presented examples and counterexamples of functions explaining things that take place in everyday situations. **Problem set 2** showed more complex graphs (beyond straight lines) and taught students to also recognize then as functions. In **problem set 3** it was intended that students learn proportionality as a function. The definition of proportionality emerged from the mathematical activity of students as they used robots. Finally, **problem set 4** was concerned with affin functions, such as y-intersect and slope. It also dealt with the relation between the graphical appearance of these kinds of function those of proportionality shown earlier. This paper[5] analyses students solving problem 3.

In the classroom

We will describe and analyse mathematical activity of two groups of students. One group consisted of four girls with similar mathematical levels and abilities (**C**, **La**, **Li** and **S**). When they started to work together, they had experienced considerable difficulty, even going so far as to repeatedly suggest that the problem could not be solved, at least individually. Eventually, they understood the problem could be solved if they teamed up and learned to work cooperatively. The other group featured three boys (**M**, **P** and **Ma**), in which one of them had a higher level of mathematical ability than the other two.

The class started with the teacher distributing materials to each group: one robot (either *Roverbot* or *Tank*), one laptop, one tape-measure and a worksheet including the following tasks[6]:

I. Let's compare the two robots speed: Roverbot and Tank. Probably the first idea that occurs to you is to hold a robot race, to find out which is the quickest. However, that is not the best way to determine speed values and compare them accurately.



a) Through experimentation of Roverbot (programming, test and registration of data) complete the following table:

Time(seconds)	1	3	6
Distance covered (cm)			

(i) Calculate the quotient between distance covered and time. (ii) Do the values 'distance covered' and 'time' vary in direct proportion? Justify your answer. (iii) Which is the proportionality constant? In this situation what does the proportionality constant mean? (iv)Comment upon the following affirmation: "The correspondence between the distance covered by Roverbot and the time spent to cover that distance is a function."

[5] For a more general discussion about mathematical activity of students using robots to learn functions see Fernandes, Fermé and Oliveira (2006, 2007) and Oliveira, Fernandes and Fermé (2008).

[6] After the realization of several tests we verified that the time that the robot needs to reach the standard velocity as well as the braking time are negligible. So we can assume that, to the end of this question, time and distance covered vary in proportion.

Practice as meaning

According to Engeström (1999), in the structure of an activity we can identify subjects that act over objects, in a process of reciprocal transformations that culminates with the achievement of certain results.

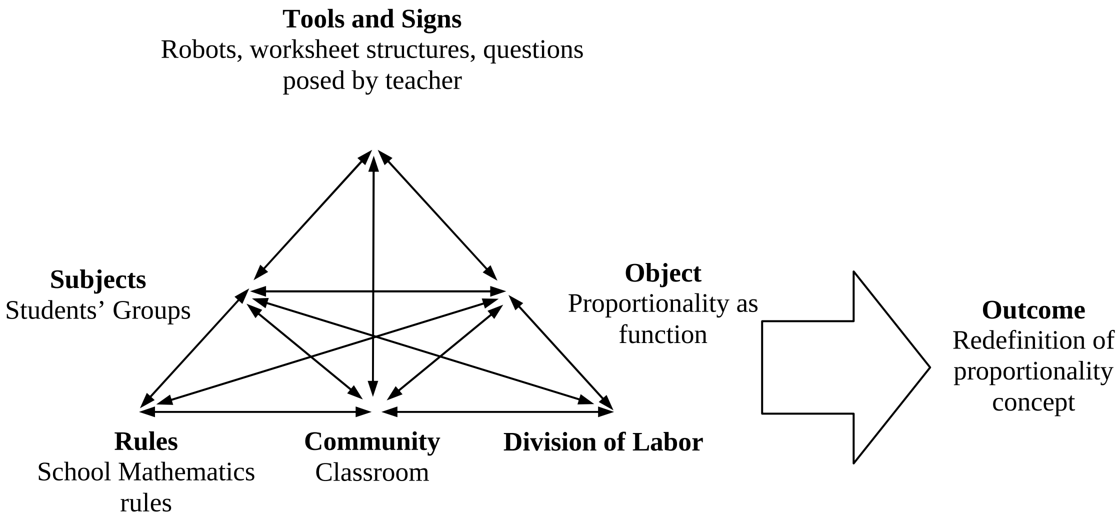


Figure 1 – School mathematics activity structure

Figure 1 shows activity during school mathematics class when robots were used to study proportionality as a function. In this case the term *subject* (figure 1) is collective and is represented by the different groups of students. The *community* is the class and its work methodology. The *object* is the 'raw material' at which the activity is directed and which is transformed (with the help of mediating instruments) as its outcome. In the situation considered here, the object is proportionality as a function and the instruments were the robots, the worksheet structure and the way the teacher posed questions to students. The episode described below shows how one of the groups solved the task described above.

Each student read the task. **C** programmed the Roverbot to move forward one second, then measured the distance covered. 33cm was recorded in the table. **S** followed the some process for 3 seconds and they registered 99cm. Then **C** programmed the robot to move forward 6 seconds. However, the desk on which they were working was too short for this last course. **Li** suggested they try out on the floor. This was done and 178 cm was recorded in the table. The students then began to discuss the results for the first time. They started to calculate the quotient between space covered and time, more or less the first times they speak. There dialogue is shown below:

1. C: 33/1 = 33 [data recorded on the worksheet].
2. C: 99/3 = 33
3. Li: 178:6 = 29.6666
4. S: It can't be. It has to be 33.
5. C: Let's programme and measure all again. Something is wrong. [They repeat all the process and the values were again 33, 99 and 178].
6. S: But it can't be. It has to be 33 (referring to the value of the quotient between the two variables)
7. La: 33 x 6 is 198. Let's put 198 on the table.

They erased 178 on the table and wrote 198. Teacher came near to the group and saw 198 (but he had previously seen 178).

8. Teacher: Wasn't the result of measuring 178?
9. C: Yes, but 33/1 is 33, 99/3 is 33
10. La: So we changed 178 by 198 because 33 times 6 is 198.
11. S: Let's programme and measure all again.

Meanwhile another group calls teacher. They programmed again the robot to forward one second and then they measured the distance covered over the desk.

12. La: Oh! I know… We measured in two different places. We have to measure always on the floor.

The results obtained of measuring the distance covered were 30, 89 and 178 for 1, 3 and 6 seconds respectively.

13. The results of the quotient were 30, 29,(6) and 29,(6) respectively. Students accepted them as good and answered that time and distant covered are in direct proportion.

Wenger (1998) states that "meaning is a way of talking about our (changing) ability - individually or collectively – to experience our life and the world as meaningful" (p. 5). He describes meaning as a learning experience.

The concept of proportionality is studied in mathematics class from 5th grade onwards. It refers to a constant relationship between two variables and is usually discussed abstractly, such as in the example below:

Verify that there is no proportionality between the following variables.

a	13	26	39	52.08
b	1	2	3	4

Many times, in school mathematics, proportionality is discussed without context; only numbers matter and the emphasis is on the mathematical concept instead of in the meaning of mathematical concept. This process makes difficult the learning experience in Wenger (1998) sense.

In the episode presented above, the students believe that the variables time and distance should be in proportion. Analysing the episode we can not determine the origin of that belief. But we can conjecture that it comes from the presence of the robot (a car) or from the way the question is written in the worksheet (question iii). Although we are guessing at its source, it is clear that the idea of proportionality is meaningful for the students, as they choose to recapture their data several times in the face of results that violate this principle. Only when an inconsistency appears, do the students begin to discuss where they made a mistake and what to do in order to solve it. But the idea that time and distance should be in proportion is really meaningful for them. This can be seen when they changed the result (from 178 to 198) to ensure that the calculations adhere to the rule and neglecting the fact of the last quotients are not equal. In spite of the evidence of the measurements, students believed that values should be in proportion. This shows that the ‘dogmatic’ knowledge of direct proportionality is more *entrenched*[7] than their confidence in their ability to successfully run experiments and, consequently, they neglected the evidence of the experiment.

The use of unusual artefacts in mathematics class (tape-measure, robots, laptops) associated to a methodology of work where students can stand up, measure, program the computer and experiment with data helped students to construct and rebuild meaning about the concept of proportionality.

From the perspective of activity theory, students groups acted on robots, which were mediators’ elements, between them and the object. The robots were a facilitator of activity that they empowered students during the process of object transformation.

In the second student group, students had a different experience. After programming the robot for 6 seconds they had the following discussion:

> M: It’s 172 cm [referring to the space covered by the robot in 6 seconds].
>
> P: 172?
>
> M: 172 or 173.

[7] The term entrenchment refers to Goodman (1954). He claims that the criterion to decide between two predicates (in our case, the rule and the evidence) is the degree of entrenchment of the predicates. The entrenchment of a predicate depends of the history of the past projections and their success or failure. In our case, the students have more history records where they must leave their proper ideas when confronted with the formal concepts (teacher knowledge, textbook).

P: But it can’t be. It’s not correct. It should be 180. And the other value should be 90 [referring to the space covered by the robot in 3 seconds].

Ma: Why?

P: I have done it in the calculator. If in one second the robot covers 30cm, I multiplied it by 3 and it’s 90. And for 6 seconds it is 180.

M: But it’s not correct. Aren’t you seeing the tape measure? It’s 173cm.

In this dialogue we can notice that one of the students of the group knows the scholarly notion of proportionality well and applies it to compare with the results of the experiment. He seems to trust more in the mathematical rules that he knows than in the evidence of the measurement experiment.

The two students groups reacted differently to the inconsistency between mathematical rules and the empirical evidence: one believed the values they obtained through measurement and considered that the values they obtained by approximation from the quotient were enough to guaranty the proportionality (as shown the episode above); the others calculated values after they knew the space covered by the robot in one second. Where does this difference in attitude (in the face of the same evidence) come from?

The division of labour (figure 1) refers both to the horizontal division, of the tasks between different members of the community, and the vertical, of power and status. The vertical division of labour is connected with the fact that, in the groups, there are students with more power than others (due to their superior performance in mathematics class, assessed through evaluation by their co-students) and these lead the search to solve the problem. Therefore, by analysing the horizontal division of labour we can say that it has emerged naturally between different students of the groups and represents the way how they organized their work in order to solve the problem proposed by teacher.

Finally the *rules* (figure 1) refer to the explicit or implicit regulation, to norms and conventions that constrain actions and interactions in the activity system. What students believe to be mathematics class, the way they see mathematical rules, the way they interpret the question put by the teacher and the worksheet structure (that is connected with the way they see mathematics class and mathematics) impose a certain form to the students’ actions. As we have said before we have two different reactions to the inconsistency between correctness of mathematical rules and the inexactness of the empirical evidence – for one group the rules won and for other the empirical evidence.

FINAL CONSIDERATIONS

Robots helped students to renegotiate the meaning of proportionality that they had previously encountered (during seven years of school mathematics) as depending uniquely and exclusively of the quotient between two variables. The negotiation of the meaning evolves through the interaction of two process – participation and reification (Wenger, 1998). When concepts are presented to students as reified objects participation (in Wenger's sense) becomes difficult. Learning through experience, essentially negotiating meaning through participation, helps students' better grasp mathematical concepts. Most of the students in the study described here redefined the concept of proportionality as a function directly because of the work done in this mathematics class and the robots had an important role in this process (Fernandes *et al.*, 2006, 2007, Oliveira *et al.* 2008). Furthermore, as this result was not explicitly intended. Instead, it was an emergent aspect of the students' mathematical practice and study of functions. In the course of their experience with robots, students transitioned from the abstract perfection of mathematics (the definition of proportionality in school mathematics) to the practical reality (proportionality in action) of everyday experience.

References

Engeström, Y. (1999). Expansive visibilization of work: An activity-theoretical perspective. Computer Supported Cooperative Work 8 pp. 63-93.

Engeström, Y. (2001). Expansive Learning at Work: toward an activity theoretical reconceptualization. Journal of Education and Work, Vol. 14, No. 1, 133-156.

Fernandes, E., Fermé, E., e Oliveira, R. (2007). Viajando com Robots na Aula de Matemática. V Conferência Internacional de Tecnologias de Informação e Comunicação na Educação – Challenges 2007: Ambientes Emergentes, O Digital e o Currículo e Avaliação Online. Universidade do Minho. Braga.

Fernandes, E., Fermé, E., and Oliveira, R. (2006) Using Robots to Learn Function in Math Class. In L. H. Son, N. Sinclair, J. B. Lagrange and C. Hoyles (Eds) Proceedings of the ICMI 17 Study Conference: background papers for the ICMI 17 Study. Hanoi University of Technology.

Fernandes, E. (2004) Aprender Matemática para Viver e Trabalhar no Nosso Mundo. Tese de Doutoramento. Faculdade de Ciências da Universidade de Lisboa.

Goodman, N. (1954) Fact, Fiction, and Forecast. University of London: Athlone Press.

Lave, J. & Wenger, E. (1991). Situated learning: Legitimate Peripherial Participation. Cambridge University Press.

Matos, J. F. (2005) Aprendizagem como participação em comunidades de prática mediada pelas TIC, Paper presented at IV Conferência Internacional sobre

Tecnologias de Informação e Comunicação na Educação - Challenges 2005 11-13 Maio, Braga, Portugal.

Matos, J.F., Mor, Y., Noss, R. and Santos, M. (2005) Sustaining interaction in a mathematical community of practice, Paper presented at CERME 4, 14-21 Fev, St Feliu de Guíxols, Spain.

Oliveira, R., Fernandes, E., and Fermé, E. (2008) Proporcionalidade directa como função: da perfeição à realidade a bordo de um robot. *Quadrante*. XVI (1). APM. Lisboa.

Piteira,G. (2000) Actividade Matemática Emergente com Ambientes Dinâmicos de Geometria Dinâmica. Tese de Mestrado. Universidade de Lisboa.

Wenger, E. (1998). *Communities of Practice: Learning, Meaning and Identit*y. Cambridge, UK: Cambridge University Press.

Wenger, E., McDermott, R. Snyder, W. M. (2002). *Cultivating communities of practice*. Boston, Massachusetts, USA: Harvard Business School Press.

Werstch, J. V. (1991). *Voices of the mind*. Cambridge, MA: Harvard University Press.