NEW DIDACTICAL PHENOMENA PROMPTED BY TI-NSPIRE SPECIFICITIES – THE MATHEMATICAL COMPONENT OF THE INSTRUMENTATION PROCESS

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Relying on the collective work carried out in the e-CoLab project concerning the experimentation of the new calculator TI-nspire, we address the issue of the relationships between the development of mathematical knowledge and instrumental genesis. By analyzing the design of some resources, we first show the importance given to these relationships by the teachers involved in the project. We then approach the same issue from the student's perspective, using some illustrative examples of the intertwining of these two developments framed by the teachers' didactical choices.

INTRODUCTION

Educational research focusing on the way digital technologies impact, could or should impact on learning and teaching processes in mathematics has accumulated over the last two decades as attested for instance by the on-going ICMI Study on this theme. Questions and approaches have moved as far as research understood better the ways in which the computer transposition of knowledge (Balacheff, 1994) affects mathematical objects and the possible interaction with these, the changes introduced by digital technologies in the semiotic systems involved in mathematical activities and their functioning, and the influence of such characteristics on learning processes (Arzarello, 2007). They have also moved due to the technological evolution itself, such as the increased potential offered by technology to access mathematical objects through a network of inter-connected and interactive representations, or to develop collaborative work (Borba & Villareal, 2004). Increased technological power, nevertheless, generally goes along with increased complexity and distance from usual teaching and learning environments, and researchers have become more and more sensitive to the processes of instrumentalization and instrumentation that drive the transformation of a given digital artefact into an instrument of the mathematical work (Guin, Ruthven, & Trouche, 2004). They have revealed their underestimated complexity, and the diversity of the facets of such instrumental genesis both on the student and teacher side (Vandebrouck, 2008).

This contribution situates within this global perspective. It emerges from a national project of experimentation of the new TI-*n*spire in which we are involved. This artefact is quite innovative but also rather complex and distant from standard calculators, even from the symbolic ones. This makes the didactical phenomena and issues associated with its instrumentalization and instrumentation especially problematic and visible. In this contribution, we pay particular attention to the interaction between the development of mathematical knowledge and of instrumental genesis, analyzing how the teachers involved in the project manage it and how students experience it. Through a few illustrative examples, we point out some phenomena which seem insightful from this point of view, before concluding with more general considerations.

PRELIMINARY CONSIDERATIONS

Let us first briefly present the TI-*n*spire and its main innovative characteristics, then the French project e-CoLab and also the theoretical frame and methodology of the study.

A new tool

TI-*n*spire CAS (Computer Algebra System) is the latest symbolic 'calculator' from Texas Instruments. At first sight it undoubtedly looks like a highly refined calculator, but also just a calculator. However, it is a very novel machine for several reasons:

- Its nature: the calculator exists as a "nomad" unit of the TI-*n*spire CAS software which can be installed on any computer station;
- Its directory, file organiser activities and page structure, each file consisting of one or more activities containing one or more pages. Each page is linked to a workspace corresponding to an application: Calculator, Graphs & Geometry, Lists & Spreadsheet, Mathematics Editor, Data and Statistics;
- The selection and navigation system allowing a directory to be reorganised, pages to be copied and/or removed and to be transferred from one activity to another, moving between pages during the work on a given problem;
- Connection between the graphical and geometrical environments via the Graphs & Geometry application, the ability to animate points on geometrical objects and graphical representations, to move lines and parabolae and deform parabolae;
- The dynamic connection between the Graphs & Geometry and Lists & Spreadsheet applications through the creation of variables and data capture and the ability to use the variables created in any of the pages and applications of an activity.

When presented with the TI-*n*spire, we assumed that these developments could offer new possibilities for students' learning as well as teachers' actions. They could foster increased interactions between mathematical areas and/or semiotic representations. They could also enrich the experimentation and simulation methods, and enable storage of far more usable records of pupils' mathematics activity. However, we also hypothesized that the profoundly new nature of this calculator and its complexity would raise significant and partially new instrumentation problems both for students and teachers and that making use of the new potentials on offer would require specific constructions, and not simply an adaptation of the strategies which have been successful with other calculators.

Excerpts both from students' interviews and teachers' questionnaires carried out/ handed out at the end of the first year of experiment support our hypotheses:

> "At first it was difficult, honestly, I couldn't use it… now it's OK, but at first it was hard to understand… the teacher, other students helped us and the sheet we got helped us out… how to save, use the spreadsheet, things like that…" Student's interview

> "In my opinion the richness of mathematical activities thanks to the connection between the several registers is the key benefit […] The difficulty will be the teacher's workload to prepare such activities so to render students autonomous." Teacher's questionnaire

> "There are still a few students for whom mathematics poses a big problem and for whom the apprenticeship of the calculator still remains arduous. These students find it hard to dissociate things and tend to think that the obstacles they face are inherent to the tool rather than to the mathematics themselves." Teacher's questionnaire

Context of the research

This study took place in the frame of a two-year French project: e-CoLab (Collaborative mathematics Laboratory experiment) [1]. It was based on a partnership between the INRP and three IREM: Lyon, Montpellier and Paris. It involved six 10th grade classes, all of the pupils of which were provided with the TI-*n*spire CAS calculator. The students kept their calculators throughout the whole school year and were allowed to take them home. The groups on the 3 sites were composed of the pilot class teachers, IREM facilitators and university researchers. They met regularly on site although the exchange also continued distantly through a common workspace on the *EducMath* site, which allowed work memories to be shared and common tools (questionnaires, resources, etc.) to be designed.

All pilot teachers had a strong mathematical background but the expertise in using ICT varied from one to another. In the 1st year of the project, teachers and students were equipped with a prototype of the TI-*n*spire they had never worked with before. However, the willingness to articulate mathematical with instrumental knowledge was shared by all teachers, despite the work they later on admitted it required:

> "We have to devote an important amount of time to the instrumentation. This requires teachers to invest quite some time in order to design the activities, especially if they want to associate the teaching of mathematical concepts." Teacher's questionnaire

Theoretical framework

Two theoretical streams guide our analyses. The first one is related to the *instrumental approach* introduced by Rabardel (1997). For Rabardel, the human being plays a key role in the process of conceiving, creating, modifying and using instruments. Throughout this process, he also personally evolves as he acclimatises to the instruments, both with regard to his behaviour as well as to his knowledge. In this sense, an instrument does not emerge spontaneously; it is rather the outcome of a twofold process involved when one "meets" an instrument: the instrumentation and the instrumentalization. Rabardel's ideas have been widely used in mathematics education in the last decade, first in the context of CAS (cf. Guin, Ruthven & Trouche, 2004 for a first synthesis) then extended to other technologies as spreadsheets and dynamic geometry software, and more recently on-line resources. Recent works such as the French GUPTEn project have also used the concept of *instrumental genesis* for making sense of the teachers' uses of ICT (Bueno-Ravel & Gueudet (2008).

We are also sensitive to the semiotic aspects of students' activities. Not only are we taking into account Duval's theory of semiotic representation (Duval, 1995) and the notions attached to it (semiotic registers of representation and conversion between registers), but more globally the diversity of highly intertwined semiotic systems involved in mathematical activity including gestures, glances, speech and signs, *i.e*. the "semiotic bundle" (Arzarello, 2007). In particular, when examining students' activity, we pay specific attention to the embodied and kinesthetic dimension of it (Nemirovsky & Borba, 2004) via the pointer movement or students' gestures.

Methodology

We are interested in the students' instrumental genesis of the TI-*n*spire and in particular in considering the role mathematical knowledge plays in this genesis. Such analysis cannot be done without taking into account the characteristics of the tasks proposed to students and the underlying didactical intentions. Our methodology thus combines the analysis of task design as it appears in the resources produced by the e-CoLab group, and the unfolding of students' activity.

The analysis of students' activity relies on screen captures of students' activities made with the software Hypercam. HyperCam, already used in other research involving the study of students' use of computer technology (see for e.g. Casyopée, Gélis & Lagrange (2007)), enables us to capture the action from a Windows screen (e.g. 10 frames/sec) and saves it to an AVI movie file. Sound from a system microphone has also been recorded and some of the activities have been video-taped.

When relevant, we also back up our analysis by relying on students' or teachers' interviews/questionnaires carried out independently from the activities.

TEACHERS' INSTRUMENTATION – DIDACTICAL INTENTIONS

Didactical intentions

The pilot teachers involved in the experiment cannot be said to be "ordinary teachers". All of them have been involved, in one way or another, in the IREM's network, thus they were all somehow sensitive to didactical considerations and shared a fairly common pedagogical background. The relative success of the project was in part due to this familiarity, as one teacher acknowledged: "It is easier to communalize if we share the same pedagogical principles."

In particular, the willingness of intertwining mathematical content with instrumental knowledge was commonly held and despite the hard work that it meant, the joint work was perceived as a true added value as teachers seemed to work in harmony:

"We have to carry the instrumentalization and the mathematical learning in parallel. Activities are not evident to think of and take time to design. The help from others make us gain time and provide us with new ideas." Teacher's questionnaire

Imprint on resources

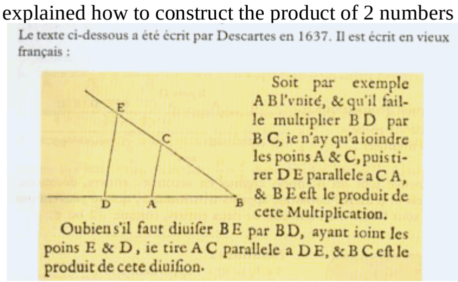
Around 25 resources were designed during the two years of the project. There are two kinds of resources: those created essentially to familiarize pupils with the new technological instrument (presentation of the artifact and introduction of some of its potentials), and those constructed around (and we should add "for") the mathematics activity itself [2]. In what follows, we mainly focus on the resources that support the teaching/learning of mathematical concepts and examine how teachers managed to articulate mathematical concepts with instrumental constituents.

The didactical intentions previously mentioned are clearly visible when examining the resources teachers designed, showing that these were built *from* the mathematical component yet at the same time *planning* a progressive instrumentation.

The *Descartes* resource is very enlightening in this sense. Teachers who have designed it acknowledged it appeared to be useful as an introduction into the dynamic geometry of the calculator, articulated with an application of the main geometrical notions and theorems introduced in Junior High School. It also offered the advantage of linking the work which had just been performed on numbers and geometry.

In this resource, several geometrical constructions are involved, enabling products and quotients of lengths to be produced and also the square root of a given length to be constructed. For the first construction proposed, the geometrical figure is given to the pupils together with displays of the measurements required to confirm experimentally that it does provide the stated product (fig. 1). The pupils simply had to use the pointer to move the mobile points and test the validity of the construction. Secondly, for the quotient, the figure provided only contained the support for the rays [BD) and [BE). The pupils were required to complete the construction and were guided stepwise in the successive use of basic tools as "point on", "segment", "intersection point", "measurement" and "calculation". Thirdly, they were asked to adapt the construction to calculate the inverse of a length. Finally for the square root they had the Descartes figure and were required to organise the construction themselves. Instructions were simply given for the two new tools: "midpoint" and "circle".

In his treatise on Geometry, Descartes explained how to construct the product of 2 numbers


Le texte ci-dessous a été écrit par Descartes en 1637. Il est écrit en vieux français :

Soit par exemple A B l'vnité, & qu'il faille multiplier B D par B C, ie n'ay qu'a ioindre les poins A & C, puis tirer D E parallele a C A, & B E eſt le produit de cete Multiplication.

Oubien s'il faut diuiſer B E par B D, ayant ioint les poins E & D, ie tire A C parallele a D E, & B C eſt le produit de cete diuiſion.

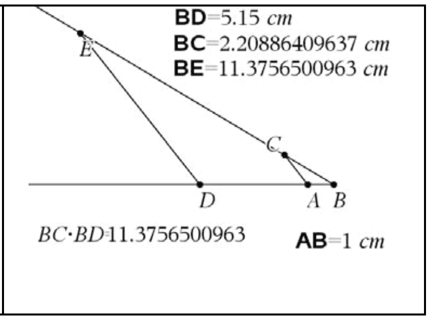


Figure 1. *First part of the Descartes resource (extracted from the pupil sheet and the associated tns file)*

In what concerns the resource *Equal areas*, the mathematical support is an algebraic problem with geometrical roots; it consists in finding a length OM such that two given areas are equal (fig. 2). The expressions of the two areas as functions of OM are 1st and 2nd degree polynomials and the problem has a single solution with an irrational value. This therefore falls outside the scope of the equations which the observed students are able to solve independently. In the first version of the resource, their work was guided by a sheet with the following stages: geometrical exploration and 1st estimate of this solution, refining the exploration with a spreadsheet to give the required value within a tolerance of 0.005, the use of CAS to obtain an exact solution, and finally the production of the corresponding algebraic proof by paper/pencil.

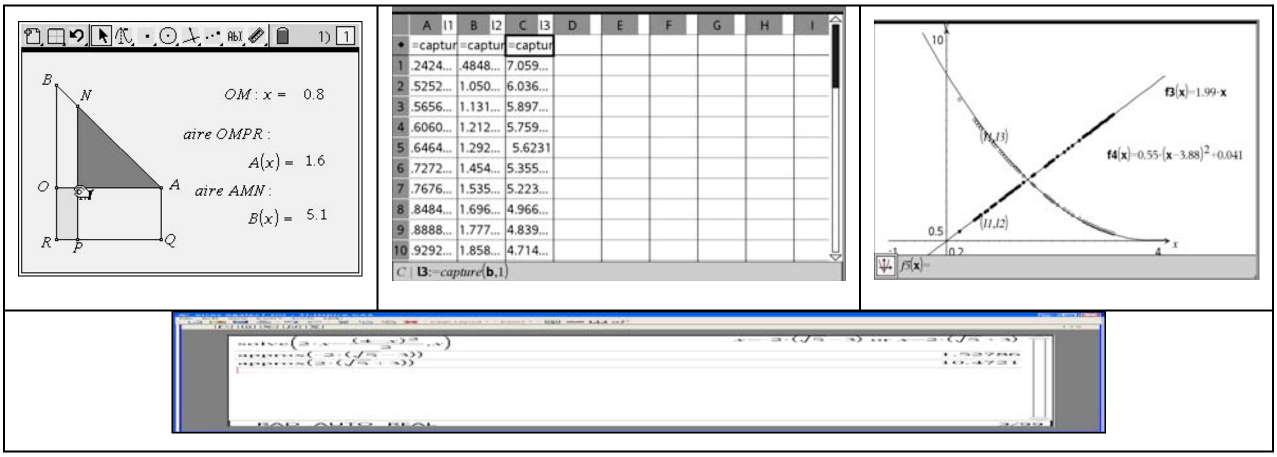


Figure 2. *Exploring progressively the problem of Equal Areas using different applications*

Experimentations led to the development of successive scenarios where more and more autonomy was given to the students in the solving of this problem, yet still requiring the use of several applications, discussing the exact or approximate nature of the solutions obtained, and the global coherence of the work.

MERGING MATHEMATICS AND INSTRUMENT – STUDENTS' VIEWPOINT

Our analysis will rely on the experimentation of two particular resources already mentioned (*Descartes* and *Equal areas*) for the following reasons: they have been designed with an evident attention to both mathematical and instrumental concerns, but take place at different moments of students' learning trajectory and have different mathematical and instrumental aims. *Descartes* has been proposed early in the school year; it aims at introducing the dynamic geometry of TI-*n*spire while revisiting some main geometrical notions of junior high school, and connecting these with numbers and operations. *Equal areas* was given to students several months later, at the end of the teaching of generalities about functions. It aims at the solving of a functional problem from diverse perspectives, and at discussing the coherence and complementarities of the results that these perspectives provide. It also aims at informing us about the state of students' instrumental genesis after 6 months of use of the TI-*n*spire.

Students and the *Descartes* resource

Two sessions and some homework were associated with this resource in the experimentation, and an interesting contrast was observed between the two sessions. The smooth running of the first session evidenced that a first level of instrumentalization of the dynamic geometry of the TI-*n*spire was easily achieved in this precise context. The successive difficulties met in the second session illustrated both the limits of this first instrumentalization and the tight interaction existing between mathematics and instrumentation. In what concerns the instrumentalization, we could mention students who inadvertently created a point that could superimpose on the points of the construction and invalidate measurements; the fact that they could not handle short segments on the calculator, or that they had not understood how to “seize” length variables in the geometry window for computing with them…

Regarding the interaction between mathematics and instrumentation, one difficulty appears to be especially visible in this situation: measures and computations in the geometry application are dealt with in approximate mode. Thus, when testing the validity of the construction proposed by Descartes for the quotient for instance, the students did not get exactly what they expected and were puzzled. Very interesting classroom discussions emerged from this situation which attest the intertwining of mathematical and instrumental issues. Students had limited familiarity with the tool, and had to understand that exact calculations are restricted to the Calculation application. The problem nevertheless was not solved just by giving this technical information, showing that this was not enough for making sense of such information, rather related to the idea of number itself, the distinction between a number and its diverse possible representations, the notions of exact and approximate calculations.

Students and the *Equal area* resource

As already explained, this resource is quite different from the previous one and students had been using the TI-*n*spire for more than 6 months. It has been experimented several times with different scenarios, and the analysis of the data collected is still ongoing. Some instrumentalization difficulties were still observed, even when students worked with an improved version of the artifact. These often concerned the spreadsheet application, less frequently used, but the main difficulties involved tightly intertwined mathematics and instrumental issues as in the previous example. We will illustrate this point by the use of a spreadsheet for finding and refining intervals including the solution.

Students used the spreadsheet application after a geometrical exploration of the problem. This convinced them of the existence and uniqueness of the solution, provided its approximate value and showed that the geometrical application could not provide exactly equal values for the two areas. The use of the spreadsheet application generally raised a lot of difficulties linked to the syntax for defining the content of the successive columns, for refining the step taking into account the existing limitation in the number of lines available. Students often tried to refer to spreadsheet files used in previous problems to solve them. Some could be helpful (another functional problem), some were problematic (a probabilistic situation recently studied). Choosing an appropriate file required an ability to see the similarities and differences between the mathematical problems at stake. Benefiting from an adequate file required the matching of the two mathematical situations, establishing correspondences between the data and variables involved, and understanding how these reflected in the syntax of the commands. The use of the generated tables, once obtained, also raised many difficulties. Students tried to get the same values for the two areas or to find the closest ones. This was not at all easy, and very few of them were spontaneously able to create a new column for the difference. Moreover, when asked to find an interval for the solution, they were unable to exploit the table in a successful way. The idea that the solution of the problem corresponded to an inversion in the order of the two areas, and that they had thus to look at the two successive lines showing this inversion for getting the limits of the interval asked for was not a natural idea. The screen copies and discussions between students or/and with the teacher of this episode clearly illustrates to what extent mathematics and instrumentation are intertwined.

In these two examples, we have focused on the mathematical/instrumental connection through the analysis of students' difficulties but the observations also show episodes where an original mathematical/instrumental synergy is at stake, made possible by the students' joint mathematical and instrumental progression. We will illustrate this by examining students' activity when working on the previous problem, but with greater autonomy. A group of two students had begun with a geometrical exploration, then defined the two functions expressing the areas and moved to a graphical exploration, selecting an appropriate window for the problem ($0 \leq x \leq 4$). They carried out this exploration cleverly, created the intersection point of the two curves to get its coordinates and found numerical values with only 6 decimals. This fact associated with the visual evidence of the intersection point convinced them that they had got the exact solution. They came back to the geometry page and checked that this solution was coherent with the approximate value with 2 decimals they had already got. They then moved to the calculation application (exact mode) and asked for the solution of the equation. They obtained 2 irrational values and were puzzled. The screen captures show several quick shifts between the graphic and calculation pages, before one of the boys decided to ask for an approximate value of the two solutions. Once obtained, they came back to the graph page, changed the window so to visualize the 2^{nd} intersection point, seemed satisfied, went back to the geometry page and discarded the 2^{nd} solution as non relevant. Once more, we cannot enter into more details, but the productive interplay here is evident. Let us just add that there has been an interesting collective discussion about the conviction of obtaining an exact solution in the graph page and the rationale underlying it. Linked with a deep mathematics discussion, the way TI-*n*spire manages approximations in the different applications and the way the user can fix the number of decimals was clarified.

For making sense of such synergies and instrumented practices, there is no doubt in our opinion that a semiotic approach limited to the identification of treatments inside a given semiotic register of representation or conversions between such registers is not fully adequate. What we observe indeed is a sophisticated interplay between different instruments belonging to the students' mathematical working space and a swing between these certainly supported by technological practices developed out of school. These are efficiently put at the service of mathematical activity and part of their efficiency also results from their kinesthetic characteristics.

Beyond that, there is no doubt that the work performed by the students in this task, through the diversity of perspectives developed around the same mathematical problem, and the small group and collective discussion raised about the potential and limits of these different perspectives and their global coherence, corresponds to a quality of mathematical activity hardly observed in most grade 10 classes.

CONCLUSION AND PERSPECTIVES

Due to its specific features which distinguish TI-*n*spire from other calculators and as it had been envisaged *a priori*, the introduction of this new tool was not without difficulty and required considerable initial work on the part of the teachers, both to allow rapid familiarisation on their part and those of the pupils but also to actualize the potentials offered by this new tool in mathematics activities. When examining both the design of the resources created by the pilot teachers and the work performed by students, as we have tried to show in this contribution, we grasp how delicate and somehow frail the harmony between the mathematical and instrumental activity is, and how the semiotic games underlying it are complex. We also see the impact of new kinds of instrumental distances (Haspekian & Artigue, 2007) and closeness that shaped teachers' and students' activities: on the one side, distance from more familiar mathematical tools and especially graphic and even symbolic calculators, on the other side closeness with technological artifacts on offer out of school (computers, IPods, etc…). These characteristics affect teachers and students differently, and individuals belonging to the same category differently, according to their personal characteristics and experience. They can have both positive and negative influences on teaching and learning processes and need to be better understood. For that purpose, beyond the theoretical constructs we have used in this study, we consider it interesting to extend the tool/object dialectics (Douady, 1986) to the instrumental component of the activities. By choosing to closely articulate mathematical and instrumental knowledge, the latter is inevitably introduced within a specific mathematical context. Reinvesting instrumental knowledge also requires students, even implicitly, to decontextualise and to a certain extent generalize what has been acquired.

NOTES

1. A more general overview of the project as well as other findings can be found elsewhere (see Aldon et al., 2008).

2. Some resources can be found at: http://educmath.inrp.fr/Educmath/partenariat/partenariat-inrp-07-08/e-colab/

ACKNOWLEDGEMENTS

We would like to thank all e-CoLab team and in particular D. Baroux, M. C. Combes, F. Hérault and J. Salles who have elaborated the resources mentioned in this text, experimented with them in their classrooms, and contributed to the analysis developed here.

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