COLLABORATIVE DESIGN OF MATHEMATICAL ACTIVITIES FOR LEARNING IN AN OUTDOOR SETTING

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In recent years, teaching mathematics in an outdoor setting has become popular among teachers, as it seems to offer alternative ways to motivate children's learning. These new learning possibilities pose crucial questions regarding the nature of how mathematical activities should be designed for outdoors settings. In this paper we describe our current work related to the design and implementation of mathematical activities in this particular environment in which a specific mathematical content was used as the central component in the design. We illustrate our collaborative design approach and the results from observations of two activities. Our initial results provide us with valuable insights that can help to better understand how to design and implement this kind of educational activities.

INTRODUCTION

A recent trend in Swedish elementary schools is an increasing interest to teach mathematics in an outdoor setting. Teachers believe that this particular approach motivates the children more than solving problems in textbooks, thus offering new ways to introduce and work with mathematical concepts (Lövgren, 2007). Teaching mathematics in an outdoor setting usually refers to school children solving practical problems using whichever forms of mathematics they find appropriate (Molander, Hedberg, Bucht, Wejdmark, Lättman-Mash, 2007). The approach presented in this article is somewhat different. The paper describes our initial efforts with regard to an ongoing project in which a specific mathematical content within the field of geometry was used as the central component in the design of mathematical activities in an outdoor setting.

Our project involves a development team consisting of schoolteachers, university teachers and researchers, who collaborate to develop mathematical activities with the purpose of supporting students' processes of learning. The mathematical activity described in this paper was developed during a period of eight months, counting from the first meeting of the development team and until the completion of the activities involving students. The methodological approach used for developing the mathematical activity will be the central focus of our discussions.

Even if outdoors teaching of mathematics has got an increasing interest among teachers and teacher educators in recent years, we found few published materials with reference to outdoor environments in the research field of mathematics education. For instance, we found no results when searching on *outdoor, outdoors* or *embodied* in titles or keywords in *Educational Studies in Mathematics, Journal for research in Mathematics Education* and *The Journal of Mathematical Behaviour*. When we searched on the term *physical*, some results showed up. However, in a brief check on research methodologies adopted in these studies, no one was centred on an outdoor activity.

Against this background, the current (ongoing) project aims at investigating different possibilities to support students' processes of learning by designing mathematical activities for an outdoor setting. This approach does not aim at replacing traditional mathematics teaching. It should rather be interpreted as a complementary method to be used at the discretion of the mathematics teacher in combination with other teaching methods. In this paper, we particularly aim at discussing our method of design in connection to the principles of Design experiments (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). Throughout the discussions presented in this paper, special attention is paid to the constitution and the working conditions of the development team.

The rest of paper is organized as follows; in the next section we present the mathematical tasks that guided our design and activity while the subsequent section gives a brief overview on the concept of design experiments. The preceding sections illustrate the results from observations of two activities followed by discussions on the notions of group and individual mathematical understanding and practices. The last two sections conclude this article by providing a description of current and coming directions of our work together with a discussion about future challenges.

DEVELOPMENT OF ACTIVITIES

In this section we describe, both the content of the proposed activities as well as the approach taken while designing the different tasks. The driving force in the design process has been experience-based suggestions from the schoolteachers. Each meeting of the development team has involved four to six teachers and two to three university researchers. The first meeting of the development team focused on identifying mathematical content and learning objectives for an outdoor activity suitable for beginners at lower secondary school. We soon agreed to focus on geometry. Aspects that were discussed dealt with the problems students have on understanding geometrical concepts such as area and perimeter. An early idea was to produce a series of activities showing progression from length to area and then to volume, using physical objects close to the school yard. The university representatives suggested utilizing non-standard measurements (sticks, steps and squares) to be used in relation to triangles, rectangles and polygons defined by trees or within the school soccer field. The school teachers instead suggested to focus on four aspects of the selected domain, namely the following learning objectives; comparison of figures, making own estimates, constructing figures with given measures and, specifically, discovering that a doubling of lengths makes the area four times larger.

It was decided that the university teachers should work on designing a task incorporating as many as possible of the agreed suggestions and present it to the whole group after the summer 2007. The proposed mathematical task, as described in figure 1, aimed at having the students construct the following sequence of figures using ropes and metal hooks to be fastened in the ground.

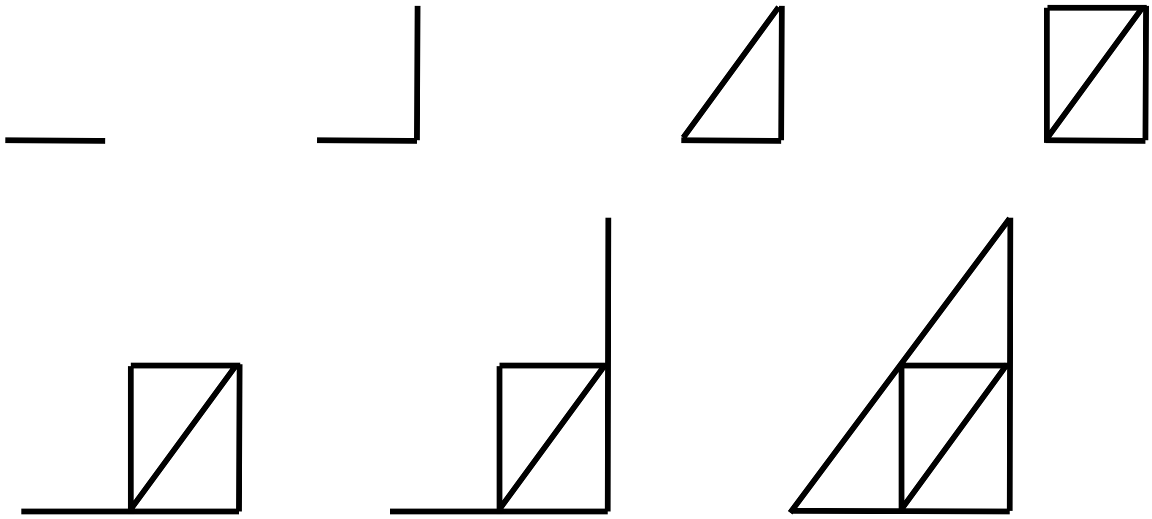

Figure 1: Intended sequence of figures to be constructed by the students.

Shortly after the summer, Växjö University hosted Professor Matthias Ludwig from *Pädagogische Hochschule Weingarten* in Germany, who offered to give two one-hour lectures at our department. One of these discussed outdoor geometrical tasks and tools used in connection with the tasks. Inspired by his lecture we decided to suggest construction of two tools; one for producing a right angle and one for measuring arbitrary angles, both based on making judgments by eyesight. The planned right angle tool consisted of a wooden square with markers at the middle of each side, as shown to the left in Figure 2.

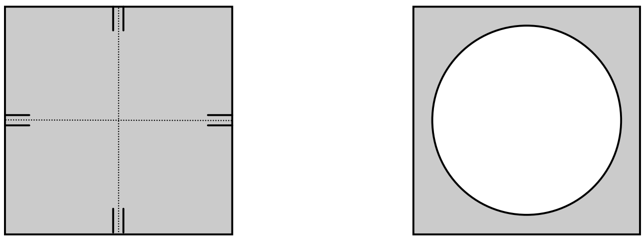

Figure 2: Ludwig's tool to the left, our tool to the right.

The woodwork teacher at the school prepared a number of square boards and also prepared a number of round boards intended for use in another activity. The square shaped tool could also be used to represent a square meter since its side was exactly one meter. However, we identified several disadvantages of this tool with respect to the intended task: it could not be used while placed on the ground, it was quite heavy, and the handling required several people operating close to the tool. We later chose the tool shown to the right in the figure above, which was actually what was left over after the round boards had been cut out. This second tool had several advantages. It could be used while placed directly on the ground, it was easy to carry due to the hole in the middle, and could be used at a distance. The right angle was aimed at the sides of the tool.

In the first proposal, the lengths for the catheti (that were to be doubled during the task) were 3 meters and 4 meters. In the construction, metal hooks and flag lines were used. While trying out the task on the (grass-covered) school yard we all agreed that larger measures were needed, to give the students a better overview of the construction and to give them reason to move within the figure. The first suggestion was to double the lengths to 6 meters and 8 meters, but we also agreed to avoid an exact measure for the hypotenuse and ended up choosing 5 meters and 8 meters as lengths for the catheti.

The task was communicated to the students through written instructions on paper. The first page of the instructions described the tools the students were supposed to bring to the school yard (3 flag lines, 6 metal hooks, roll-out length measure, right-angle tool, paper and pen). Three separate tasks were described on the following three pages.

Each task was divided into three subtasks in the same way (construct a figure, determine perimeter, determine area). This was done for several reasons. Since the students were not used to this kind of activity, we wanted to restrict the content in each subtask. We also wanted to encourage the students to discuss their conclusions on each subtask as a group, especially to verify that the construction was made according to the descriptions as we suspected that they otherwise might focus only on calculations. Also, since the written instructions were not supported by figures, we found it reasonable to restrict each subtask in order not to make it too difficult for the students to interpret the task. Our aim was to let the students work on the tasks without the support from the teacher; thereby inviting them to take on different roles and take more own initiatives than they were used to in their usual mathematics classroom. Another important aspect was that the tasks should allow for applying different solution strategies, such as measuring, calculation, and comparison.

DESIGN EXPERIMENTS

The methodology used in this project is founded on the principles of Design experiments (Cobb et al., 2003). Cobb and colleagues (2003) summarize Design experiments (DE) in five crosscutting features. The first feature, *develop theories*, concerns understanding processes of learning and the means that are designed to support that learning. The second feature, which concerns *control*, may be seen as the focus of the current project: “The intent is to investigate the possibilities for educational improvement by bringing about new forms of learning in order to study them” (Cobb et al., 2003, p. 10). To develop theories about learning processes, and to try to exert control of such processes, implies the need for *prospective* and *reflective* analyses. Prospective and reflective work is the third feature of DE. On the prospective side, our designs have been implemented with a hypothesized learning process in mind. The activity has been carried out with students and the following reflective work has been based on observations of students' actions. The prospective and reflective aspects come together in a fourth characteristic of DE, *iterative design.* Iterations are carried out with the modification and development of explaining learning and the means of supporting learning. The project so far has included only two iterations which have been based on informal observations with a rather weak theoretical base. Our strategy has been to let the preliminary informal observations guide us toward relevant learning theories to support later iterations. The fifth feature refers to the *pragmatic roots* of DE. As school teachers take active part in the design process, we feel confident that the activities are relevant for teachers' practice.

OBSERVATIONS FROM TWO ACTIVITIES

Two activities involving students have been carried out in the project. The two activities included two different groups of four students (14-15 years old). The activities were neither videotaped nor audiotaped. Instead, two researchers and two teachers observed the activities. The researchers were the same both times.

During the activities, the students were very eager to start working with the lines and hooks. We feel that the division of each task into subtasks made it possible for them to interpret the subtask while arranging lines and hooks. On a few occasions, when they were getting lost in the construction, we had to intervene and ask them to read the instructions again. We also observed that some of the students had problems handling the instruction papers. These problems concern locating and returning to the instructions after they have been left on the ground, as well as documenting answers to the questions.

One specific observation concerned the change in social behaviour. One of the teachers commented on a female student who was busy constructing sides by pulling flag lines:

> Look at her. She seldom takes initiatives in the classroom; she is very quiet and rarely shows interest. Here she is, pulling flag lines, talking to her classmates and really enjoying what she is doing.

Another notable observation can be seen as relating to gender issues. In a group of two boys and two girls, the boys were trying to solve the problem of extending the catheti, seemingly ignoring the girls. As the boys got stuck, one of the girls walked up to the (female) teacher and whispered her solution. The teacher encouraged her to talk to the boys, and the whole group ended up producing the intended construction.

One specific topic of discussion concerning mathematics emerged in our follow-up meetings. To recall, one of our intention with the design was to encourage different solution strategies, such as measuring, calculation and comparison. What was noticed however, was that measuring took a rather dominant role in the activity. Moreover, since the students were not familiar with the Pythagorean Theorem we did not expect them to calculate the hypotenuse of the first triangle, in order to determine its perimeter. However, when the students were asked to determine the perimeter of the larger triangle, i.e. after the catheti of the first triangle being doubled, they also now measured the hypotenuse. None of the students reflected on or argued that also the hypotenuse was doubled. The students did not even reflect on this after the three sides were measured. The data they used for determining the perimeter was the measured data.

During the first activity, the students quickly turned to calculating the area of the larger triangle by the rule; base times altitude divided by two. No attempt was made to compare the larger triangle with the smaller triangle, even if the construction supported looking four smaller triangles within the larger (see Figure 1). In the instructions for the second activity, we therefore explicitly asked the students if they could find out from the constructions any relation between the area of the larger triangle and the smaller triangle. After some discussion and guidance the students at least articulated that the area of the larger triangle was four times the area of the first triangle. However, we were not comfortable that the activity did not by itself provoke the students to involve principles and relations in their discussions.

We observed that the students solved the tasks rather pragmatically and routinely, in terms of measuring and applying rules for calculation. However, we do not have evidence that the students' behaviour depended on conceptual limitations. In the follow-up discussions within the development team we identified possible explanations in terms of the design of the activity and the students' history of being part of a certain educational system. Therefore, to develop the activity and to understand students' actions and potential, we have reached a point where we find it necessary to deepen the theoretical approach of our work, taking into account analytical constructs on several levels of interaction. In the next section we describe principles of the emergent perspective (Cobb et al., 2001), which we find suitable for our purposes.

CONCEPTUALIZING GROUP AND INDIVIDUAL MATHEMATICAL UNDERSTANDING

In Cobb, Stephan, McClain and Gravemeijer (2001) terms, the evolution of mathematical learning in classrooms constitutes of *social* as well as *psychological* structures of behaviour and reasoning. Within the social structure, they identify three analytical categories: *Classroom social norms*, *Sociomathematical norms* and *Classroom mathematical practices*. Examples of Classroom social norms can be for instance; that students collaborate to solve problems, that meaningful activity is valued more than correct answers, and that partners should reach consensus as they work on activities. With reference to our observations, Classroom social norms may have been in play when the quiet girl had to be encouraged by her teacher to communicate with her team members. *Sociomathematical* norms are defined as social constructs specific to mathematics. These are the norms in play when explanations and justifications are made acceptable (Hershkowitz and Schwarz, 1999). When applying the analytical construct of *classroom mathematical practices* the analytical lens is closer to a certain instructional activities. It concerns regularities of the collective engagement in a specific situation in terms of symbolizing, arguing and validating.

A student may experience a study activity in different ways, as compared to the teacher's and to other students' interpretations (Wistedt, 1987; Iversen and Nilsson, 2007). The psychological perspective concerns the nature of individual students' reasoning. It brings attention to the diversity in students' ways of interpreting and acting in mathematical activities (Cobb et al., 2001).

It is crucial to understand that the relation between the social and the psychological perspective is considered to be reflexive (Cobb et al., 2001): "…neither perspective exists without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective" (p. 122).

An implicit assumption of the current project has been that an unfamiliar teaching arrangement might encourage students to act beyond previously established Classroom social and Sociomathematical norms, with the possibility that these new actions may be more mathematically productive than their correlates of ordinary classrooms. The initial results of our observations, specifically the two separate incidents involving girls, support this assumption.

THE ORGANIZATION OF MATHEMATICAL PRACTICES

Weber, Maher, Powell, and Lee (2008) summarize some important ways in which discussions may establish opportunities for the learning of mathematics. Discussion can objectify students' experiences, thereby making these experiences the subject of analysis, encourage students to take a more reflective stance on their mathematical reasoning, require students to consolidate their thinking by verbalizing their thoughts, and help students learn to communicate mathematically and participate in a wider range of mathematical argumentation. Weber et al., (2008) also contend that group discussion can facilitate learning by inviting students to be explicit both about the ways in which they make new claims from previously established facts and about the standards they are using in deciding whether an argument is acceptable. Challenges from classmates can encourage students to debate whether a particular method of argumentation is appropriate and provide students with the opportunity either to justify their methods when their reasoning is sound or revise or abandon their methods when their reasoning is flawed.

In the organization of group discussions, Cobb et al., (2001) distinguish between three specific structures: taken-as-shared purposes, taken-as-shared ways of reasoning with tools and symbols, and taken-as-shared forms of mathematical argumentation. A taken-as-shared purpose is what the students and the teachers are trying to achieve together mathematically. The second structure is concerned with the ways in which tools and symbols are used and given taken-as-shared meanings. To account for taken-as-shared forms of argumentation Toulmin's (1969) analytical model of argumentation has proven useful (Cobb et al., 2001). According to Toulmin (1969), an argumentation consists of at least three core components: *the claim, the data*, and *the warrant*. When a speaker makes a claim he or she may be challenged to present evidence or data to support that claim. The data typically consist of facts that lead to the conclusion that is made. If a listener does not understand why the data justify the conclusion that was drawn she may challenge the presenter to clarify why the data led to the conclusion. When this type of challenge is made and a presenter clarifies the role of the data in making her claim the presenter is providing a warrant. A warrant can of course be questioned, thus obligating the presenter backing up the warrant.

DISCUSSION ON OUR METHOD OF DESIGN

Our choice of method has been influenced by the constitution and working conditions of the development team. The main focus has been on collaborative development of the mathematical activity. The project emphasizes the potential benefits of collaborative development in close interaction with stakeholders. There has been a very open climate of discussion where teachers' knowledge and experiences have been given equal attention as input from the researchers. The teachers have been very active providing ideas and reacting on suggestions from the researchers, both during physical meetings and through e-mail communication. We argue that this way of collaboration differs from the approach usually used by DE practitioners. In DE, theories are usually introduced in early stage of the design process (diSessa & Cobb, 2004). From the observations of two activities, we have been identified a need for supporting theories. The interpretative frameworks outlined above will enable us to strengthen our design and to better understand our observations. However, we have found it fruitful to use an experienced based approach. No theories have been explicitly communicated during the initial work of the development team. Particularly, we believe that introducing abstract theories early in the discussions would have reduced the teachers' interest and possibilities to communicate empirically grounded ideas, thereby limiting the pragmatic root of the project. Our approach may therefore serve as a reasonable model for others, who wish to engage in collaborative activities in order to enhance school teaching. On account of this, we suggest that researchers in collaboration with teachers should take seriously the role of theories, particularly when to introduce them in the project at hand.

We suggest a balance between theories and practice, where practice takes on a rather dominant role in the early work. As the project and iterations proceed, the role of theories may be increased in order to enhance control of the learning activity. The analytical categories argued by Cobb et al., (2001), and Toulmin's (1969) model of argumentation, offer instruments both for supporting the design process and for serving as tools for analysis of observed actions.

Finally, one can question the validity of our approach in relation to the pedagogical implementation and learning outcomes of these activities but the main point here is not assess the effectiveness of the learning materials neither the mathematical content but instead to explore how to design and organize the flow of pedagogical activities in an outdoor learning setting. Our initial impressions indicate that this kind of learning activities seem to encourage discussions and new collaboration patterns, thus promoting deeper understanding among students. Therefore, we believe that a major challenge for the mathematics education community is to create new possibilities for learners to understand complex mathematical concepts, as well as to develop new analytical tools and theories in order to facilitate our understanding on how learning takes place under these new circumstances.

FUTURE EFFORTS

Based on the discussions presented in this paper, the following suggestions appear to be relevant for the design of the next iteration. The design of the next activity should take into consideration how:

- collective understanding can be provoked by encouraging students to make claims and be explicit about the warrants on which the claims rest,
- collective discussion can capitalize on individual variations (implying that the activity should encourage a variation in reasoning and solution strategies),
- norms and structures of mathematical practices may support or limit students' behaviour.

The last aspect specifically refers to the observation of how measuring took on a rather dominant role in the activities, narrowing the students' conceptual structures. On account of these guidelines we suggest to follow up the described activity with a second activity, where the students are not allowed to use a measuring tool. Instead they start with a triangle with given perimeter and given area and whose sides are not known. The triangle will be marked with flag lines and the students will be asked to continue the construction of the same pattern as in the previous construction and will be asked to determine the perimeter and the area of the larger triangle. We conjecture that such a setup will provoke the students to reflect on conceptual aspects, by comparing features of the triangles. Another suggestion is to let the students choose their own measures and construct a triangle which will be extended to a rectangle, with the aim that they discover the connection between the areas of the two figures.

An obvious next step of the project is to investigate how the described outdoor activity can be followed up in the regular classroom. Earlier mentioned shortcomings concerning students' documentation may be overcome by using mobile technologies. According to Spikol and Milrad (2008), mobile technologies offer the potential for a new phase in the evolution of technology-enhanced learning, marked by a continuity of the learning experience across different learning contexts. In particular, we propose to let students use mobile technology in order both to communicate the tasks and to support the documentation of their solutions. Moreover, offering the students possibilities to videotape and taking pictures during the activity will support them in recalling and sharing experiences when they return to their regular classroom. We believe moreover that interesting applications may be developed in additional fields such as arithmetic and statistics, and even in algebra and functions. Our ambition is to invite students from the teacher training program at our university, so they can participate in widening our design approach to the above mentioned fields.

REFERENCES

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences*, *10*(1/2), 113–163.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003), Design Experiments in Educational Research. *Educational Researcher, 32* (1), 9-13.

diSessa, A., & Cobb, P. (2004). Ontological Innovation and the Role of Theory in Design Experiments. *Journal of the Learning Science, 13*(1), 77-104.

Hershkowitz and Schwarz, B. (1999). The Emergent Perspective in Rich Learning Environments: Some Roles of Tools and Activities in the Construction of Sociomathematical Norms. Educational Studies in Mathematics, 39, 149-166,

Iversen, K., & Nilsson, P. (2007). Students' Reasoning About One-Object Stochastic Phenomena in an ICT-Environment. *International Journal of Computers for Mathematical Learning*, *12*(2), 113-133.

Lövgren, A. (2007). Laborativ matematik – ger böckernas innehåll ny mening [Explorative mathematics – gives new meaning to book content]. *Lika värde, 4*, 6-7. Specialpedagogiska institutet [Institute for special education], Härnösand.

Molander, K., Hedberg, P., Bucht, M., Wejdmark, M., & Lättman-Mash, R. (2007). *Att lära in matematik ute [Learning mathematics outdoors]*. Falun: Research Centre.

Spikol, D., & Milrad, M. (2008). Combining Physical Activities and Mobile Games for Designing Novel Ways of Learning. Proceedings of the *IEEE WMUTE 2008,* Beijing, China.

Toulmin, S. (1969). The uses of arguments. Cambridge: Cambridge University Press.

Weber, K., Maher, C., Powell, A., & Lee, H. S. (2008). Learning opportunities from group discussions: warrants become the objects of debate. *Educational Studies in Mathematics*, *68*, 247-261.

Wistedt, I. (1987). *Rum för lärande [Latitude for learning]*. PhD Thesis, Stockholm University, Department of Education.