REALISATION OF MERS (MULTIPLE EXTERN REPRESENTATIONS) AND MELRS (MULTIPLE EQUIVALENT LINKED REPRESENTATIONS) IN ELEMENTARY MATHEMATICS SOFTWARE

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Assumptions of multiple mental representations lead to the presumption of an enhanced mathematical learning, especially of the process of internalization, due to MERs (Ainsworth 1999) and MELRs (Harrop 2003). So far, most educational software for mathematics at the primary level aims to help children to automatize mathematical operations, whereby symbolical representations are dominating. However, what is missing is software and principles for its design that support the process of internalization and the learning of external representations and their meaning themselves – in primary school these are in particular symbols. This paper summarizes the current state of research and presents a prototype that aims to the above-mentioned purpose.

INTRODUCTION

In this article we describe the theory and new achievements of a prototypical educational software for primary school arithmetic. After developing the guiding principles that are based on multimedia learning models, we present DOPPELMOPPEL[1], a learning module for doubling, halving and decomposing in first grade.

THE COGNITIVE THEORY OF MULTIMEDIA LEARNING (CTML)

In the 1970s and 80s it was assumed that comprehension is limited to the processing of categorical knowledge that is represented propositionally. Nowadays, most authors assume the presence of multiple mental representation systems (cp. Engelkamp & Zimmer 2006; Schnotz 2002; Mayer 2005) – mainly because of neuro-psychological research findings. With regard to multimedia learning the Cognitive Theory of Multimedia Learning (CTML) of Mayer is to emphasize (Fig. 1).

[1] see http://kortenkamps.net/material/doppelmoppel for the software

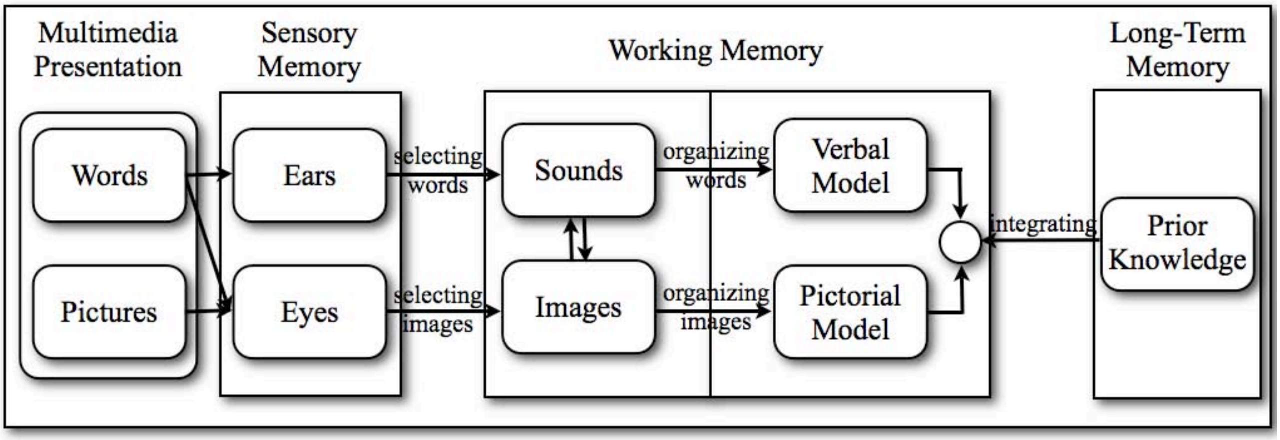

Figure 1: The Cognitive Theory of Multimedia Learning (CTML) of Mayer

Mayer (2005) acts on the assumption of two channels, one for visually represented material and one for auditory represented material. The differentiation between the visual/pictorial channel and the auditory/verbal channel is of importance only with respect to the working memory. Here humans are limited in the amount of information that can be processed through each channel at a time. Besides the working memory Mayer assumes two further types: the sensory memory and the long-term memory. Furthermore, according to Mayer humans are actively engaged in cognitive processing. For meaningful learning the learner has to engage in five cognitive processes:

(1) Selecting relevant words for processing in verbal working memory

(2) Selecting relevant images for processing in visual working memory

(3) Organizing selected words into a verbal model

(4) Organizing selected images into a pictorial model

(5) Integrating the verbal and pictorial representations, both with each other and with prior knowledge (Mayer 2005, 38)

Concerning the process of internalization the CTML is of particular importance. The comprehension of a mathematical operation is not developed unless a child has the ability to build mental connections between the different forms of representation. According to Aebli (1987) for that purpose every new and more symbolical extern representation must be connected as closely as possible to the preceding concrete one. This connection takes place on the second stage of the process of mathematical learning where the transfer from concrete acting over more abstract, iconic and particularly static representations to the numeral form takes place (Fig. 2). A chance in the use of computers in primary school is seen in supporting the process of internalization by the use of MELRs. This is the main motivation for the research on how the knowledge about MERs and MELRs in elementary mathematics and educational software is actually used and how it can be used in the future.

TO THE REALISATION OF MERS AND MELRS IN ELEMENTARY MATHEMATICS SOFTWARE

Despite the fact that computers can be used to link representations very closely, it is hardly made use of in current educational software packages. Software that offers MERs and MELRs with the aim to support the process of internalization is very rare. This is also the reason why tasks are mainly represented in a symbolic form (Fig. 2).

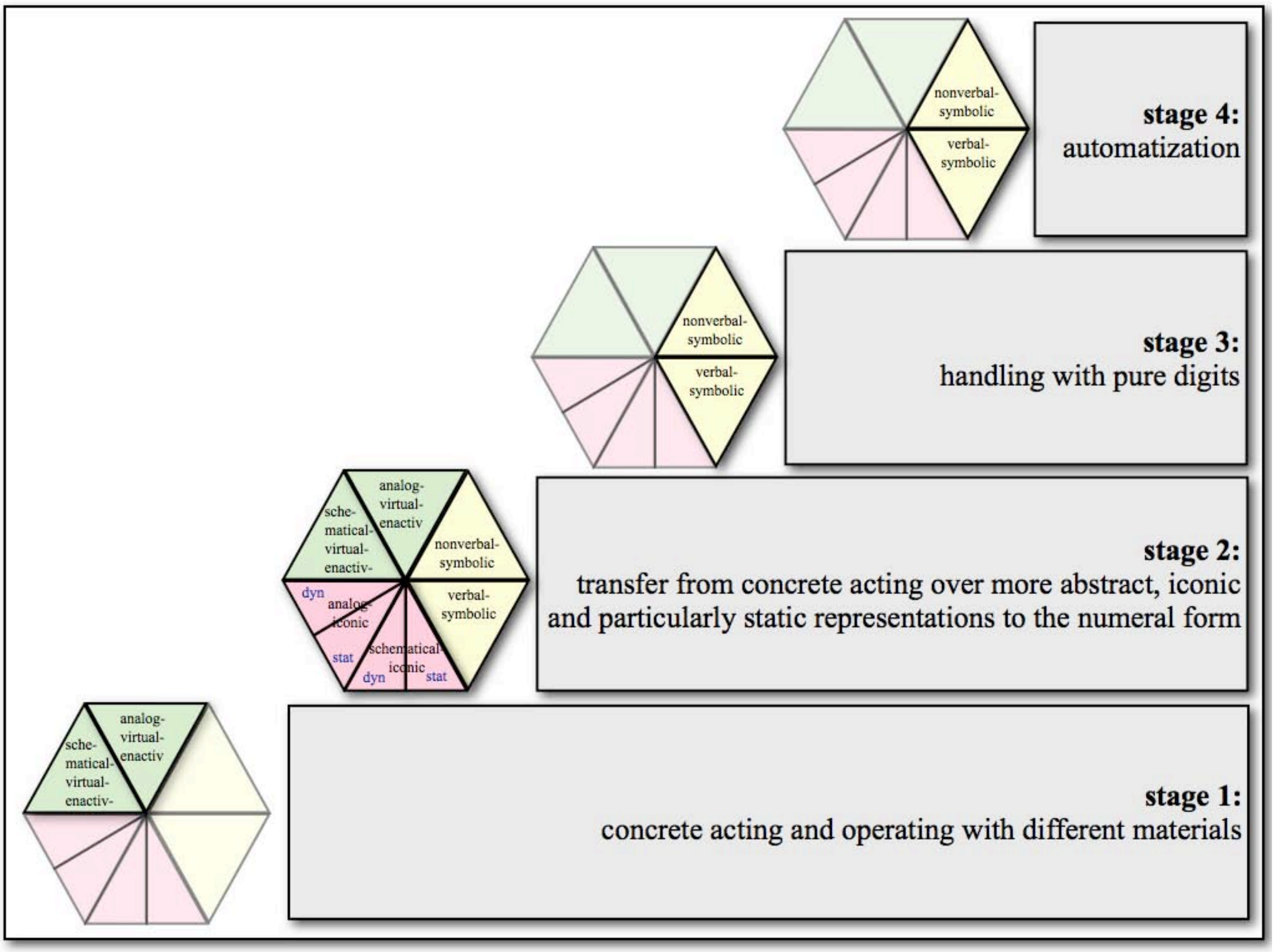

Figure 2: Forms of external representations combined with the four stages of the process of mathematical learning

Nevertheless, most software offers *help* in form of visualizations and thereby goes backward to the second stage. This is realised in different ways, which is why a study of current software was done with regard to the following aspects:

- Which forms of external representations are combined (MERs) and how are they designed?
- Does the software offer a linking of equivalent representations (MELRs) and how is the design of these links?

After this analyse, a total of sixty 1^{st}- and 2^{nd}-grade-children at the age of six to eight years were monitored in view of their handling of certain software (BLITZRECHNEN 1/2, MATHEMATIKUS 1/2, FÖRDERPYRAMIDE 1/2). Beside this own exploration – which will not be elaborated at this point - there is only a small number of studies that concentrates on MERs and MELRs on elementary mathematics software. In 1989, Thompson developed a program called BLOCKS MICROWORLD in which he combined Dienes blocks with nonverbal-symbolic information. Intention was the support of the instruction of decimal numeration (kindergarten), the addition, subtraction and division of integers (1^{st} – 4^{th} grade) as well as the support of operations with decimal numbers (Thompson 1992, 2). Compared to activities with "real things", there were no physical restrictions in the activities with the virtual objects to denote. Furthermore the program highlighted the effects of chances in the nonverbal-symbolic representation to the virtual-enactive representation and reverse. In his study with twenty 4^{th}-grade-children Thompson could show that the development of notations has been more meaningful to those students who worked with the computer setting compared to the paper-pencil-setting. The association between symbols and activities was established much better by those children than by the others.

Two further studies that examined multi-representational software for elementary mathematics are by Ainsworth, Bibby and Wood (1997 & 2002). The aim of COPPERS is to provide a better understanding of multiple results in coin problems. Ainsworth et al. could find out, that already six-years-old children do have the ability to use MERs effectively. The aim of the second program CENTS was the support of nine- to twelve-years-old children in learning basic knowledge of skills in successful estimation. There were different types of MERs to work with. In all three test groups a significant enhancement was seen. The knowledge of the representations themselves as well as the mental linking of the representations by the children were a necessary requirement. The fact that a lot of pupils weren't able to connect the iconic with the symbolic representation told Ainsworth et al. (1997, 102) that the translation between two forms of representations must be as transparent as possible.

The opinions about an automatic linking of multiple forms of representations vary very much. Harrop (2003) considers that links between multiple equivalent representations facilitate the transfer and thus lead to an enhanced understanding. However, such an automatic translation is seen very controversial. Notwithstanding this, it is precisely the automatism that presents one of the main roles of new technologies in the process of mathematical learning (cf. Kaput 1989). It states a substantial cognitive advantage that is based on the fact that the cognitive load will be reduced by what the student can concentrate on his activities with the different forms of representations and their effects. An alternative solution between those two extremes – the immediate automatic transfer on the one hand and its non-existence on the other hand – is to make the possibility to get an automatic transfer shown to a decision of the learner.

PRINCIPLES FOR DESIGNING MERS

The initial point and justification of multimedia learning is the so-called multimedia principle (cf. Mayer 2005, 31). It says that a MER generates a deeper understanding than a single representation in form of a text. The reason for this is rooted in the different conceptual processes for text and pictures. In being so, the kind of the combined design is of essential importance for a successful learning. The compliance of diverse principles can lead to an enhanced cognitive capacity. Thus Ayres & Sweller (2005) could find a *split-attention-effect* if redundant information is represented in two different ways because the learner has to integrate it mentally. For this more working space capacity is required, and this amount could be reduced if the integration were already be done externally. Mayer (2005) diversifies and formulates besides his *spatial contiguity principle* the *temporal contiguity principle*. According to this principle, information has not only to be represented in close adjacency but also close in time. If information is also redundant, the elimination of the redundancy can lead to an enhanced learning (*redundancy-effect*). The *modality principle* unlike the split-attention principle does not integrate two external visual representations but changes one of it into an auditory one. Hence an overload of the visual working memory can be avoided.

In addition to the modality principle Mayer recommends the segmenting principle as well as the pretraining principle to enhance essential processes in multimedia learning. As a result of the *segmenting principle* multimedia information is presented stepwise depending on the user so that the tempo is decelerated. Thus the learner has more time for cognitive processing. The *pretraining principle* states that less cognitive effort will be needed if an eventual overload of the working memory is prevented in advance through the acquisition of previous knowledge. Finally, the abidance of the *signaling principle* allows a deeper learning due to the highlighting of currently essential information. Extraneous material will be ignored so that more cognitive capacity is available and can be used for the essential information.

In elementary instruction the children first of all have to learn the meaning of symbolic representations and how to link them with the corresponding activities. So the above-described principles cannot be adopted one-to-one. Based on an empirical examination of the handling of six- to eight-years-old pupils with MERs and MELRs in chosen software, we could identify new principles and the above-described ones could be adapted, so that their compliance supports the process of internalization. These principles are demonstrated and realized in the following example of the prototype DOPPELMOPPEL.

THE PROTOTYPE DOPPELMOPPEL

Didactical concept and tools

The function of the ME(L)Rs in DOPPELMOPPEL is the construction of a deeper understanding through abstraction and relations (fig. 3). The prototype was built using the Geometry software Cinderella (Richter-Gebert & Kortenkamp 2006) and can be included into web pages as a Java applet.

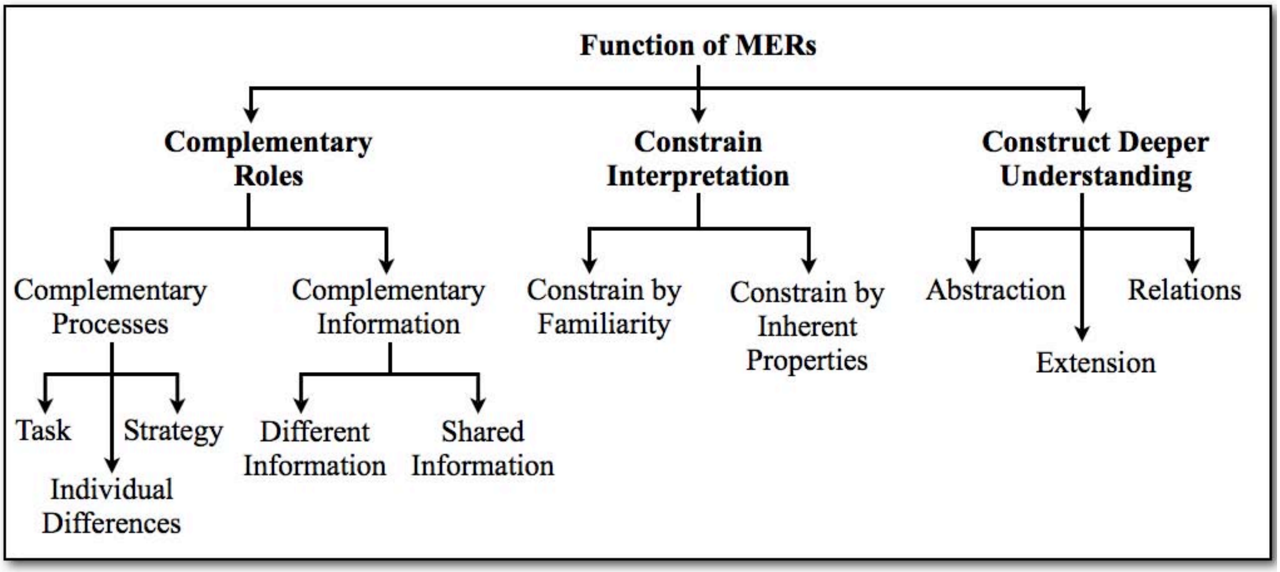

Figure 3: Functions of MERs according to Ainsworth (1999)

Using the example of doubling and halving the children shall – in terms of internalization – link their activities with the corresponding nonverbal-symbolic representation and they shall figure out those symbols as a log of their doing. The mathematical topic of doubling and halving was chosen because it is a basic strategy for solving addition and subtraction tasks. In addition, DOPPELMOPPEL offers to do segmentations in common use.

The main concern of the prototype is to offer a manifold choice of forms of representations and their linking in particular (MELRs). Two principles that lead the development are the *constant background principle* and the *constant position principle*. The first one claims a non-alteration of the design of the background but an always-constant one. Furthermore the position of the different forms of representations should always be fixed and visible from the very beginning so that they don't constrict each other.

DOPPELMOPPEL provides the children with the opportunity to work in many different forms of representations. On the one hand there is a zone in which the children can work **virtual-enactive**. Quantities are represented through circular pads in two colours (red and blue). To enable a fast representation (*easy construction principle*) and to avoid "calculating by counting" there are also stacks of five next to the single pads. According to our reading direction the five pads are laid out horizontally. The elimination of pads happens through an intuitive throw-away gesture from the "desk" or, if all should be cleaned, with the aid of the broom button. A total of maximal 100 pads fit on the table (10x10). The possible activities of doubling, halving and segmenting are done via the two tools on the right and the left hand side of the desk (fig. 4).

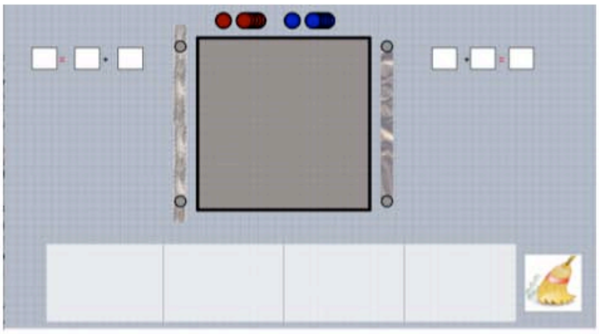

Figure 4: Screenshot of the prototype DOPPELMOPPEL

The doubling-tool (to the right) acts like a mirror and doubles the laid quantities. The saw (to the left) divides the pads and moves them apart. Both visualisations are only shown for a short time after clicking on the tools. Afterwards, the children only see the initial situation and have to imagine the final situation (mirrored resp. divided) themselves. The pupils can use the mouse to drag the circular points on the doubling-tool and the saw to move them into any position. A special feature of the saw is that it also can halve pads. At this point the program is responsive to the fact that already six-years-olds know the concept of halves because of the common use in everyday life.

The children can do **nonverbal-symbolic** inputs themselves in the two tables on the right and the left hand side. The left table enables inputs in the form _=_+_, the right one in the form _+_=_. The table on the right is only intended for doubling and halving tasks. That's why the respectively other summand appears automatically after the input of one. In the table on the left any addition task can be entered.

If the pupils don't fill in the equation completely they have the possibility to get their input shown in a **schematic-iconic** representation. Depending on the entered figures, the pads appears in that way that the children can't read the solution directly by means of their colour. The doubling-tool respectively the saw are placed according to the equation so that the children – like in the virtual-enactive representation – are able to act with the tools (fig. 5).

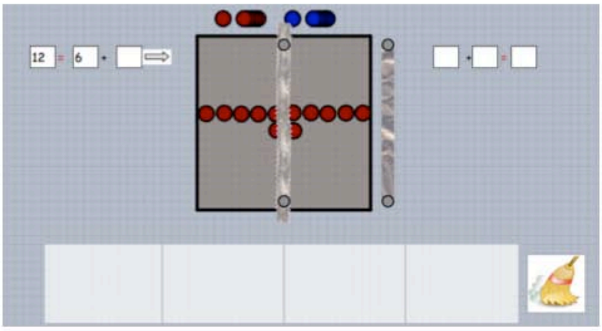

Figure 5: Schematic-iconic representation of a task

According to the *signaling principle* an arrow is highlighted when the pupils enter numbers in the free boxes. A click on this arrow initiates the **intermodal transfer**. A similar arrow appears below the desk after every activity done by the children (click on the doubling-tool respectively the saw). Here, the pupils have the possibility to let the software perform the intermodal transfer from the virtual-enactive and the schematic-iconic representation to the nonverbal-symbolic one. This is another special feature of DOPPELMOPPEL that is rarely found in current educational software. If external representations are linked, the linking is mostly restricted to the contrary direction. Depending on the activity the equation appears again in the form _=_+_ or _+_=_. Those equations aren't separated consciously, however a coloured differentiation of the equal and the addition sign (as in the tables above) point to pay attention.

Besides the forms of representations there are two more functions available. Both – the broom to clean the desk and the exclamation mark for checking answers – take some time in order to encourage considerate working and to avoid a trial-and-error-effect. If the equation is false the program differentiates on the type of error. In case of an off-by-one answer or other minor mistake the boxes are coloured orange otherwise red. If the equation is correct a new box appears below.

This prototype doesn't already respond to *modalities* but the concept already incorporates auditory elements.

Testing of DOPPELMOPPEL

For the testing of DOPPELMOPPEL four versions of the prototype were created. Two of those feature multiple representations; the other two only offer single representations. One of the multiple representations provides an additional linking, that is an intermodal transfer in both directions (fig. 6).

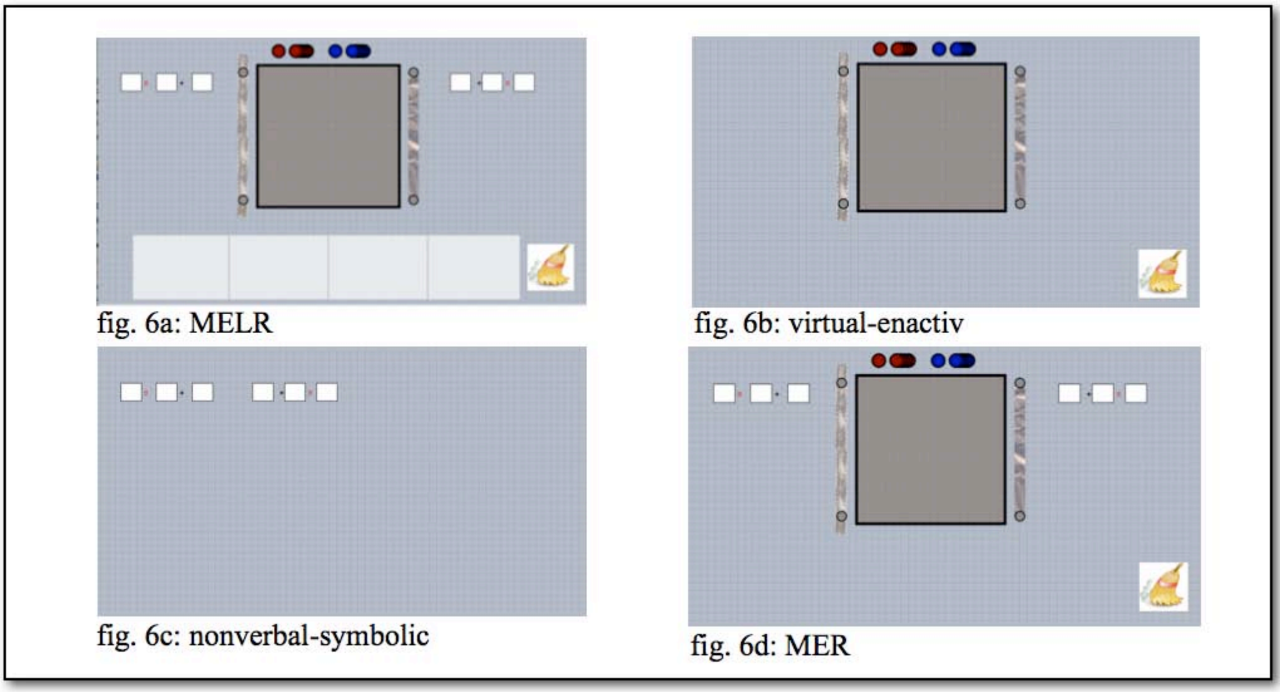

Figure 6: 4 versions of the prototype

The dedication of those four versions is to make sure that it is neither the medium computer nor the method of instruction that causes results of the testing.

28 pupils of a 1st class worked about 20 minutes per five terms with the program. During their work there was one student assistant who observed and took care of two children. In addition, the activities of the children were recorded with a screencorder-software. Furthermore a pre- and a posttest were done.

To the current point of time the data interpretation is still in progress but first results should be available to the end of January.

CONCLUSION

Educational software that is based on the primacy of educational theory, as claimed by Krauthausen and others, has to take both mathematics and multimedia theory into account. Carefully crafted software however, is very expensive in production. We hope to be able to show with our prototype that this investment is justified.

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