GESTALT CONFIGURATIONS IN GEOMETRY LEARNING

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ABSTRACT

The treatment of geometric diagrams requires the handling of the figural aspects of the drawing as much as the conceptual aspects contained in the figure[1]. In geometry we use the figural aspects of diagrams as symbols to prove or resolve problems. When we interpret figural information, what we call Gestalt configurations emerge: auxiliary figural configurations, real or virtual, that give meaning and substance to an idea that facilitates the proof or solution to the problem. In this work we give arguments to acknowledge the existence of these resources, identify their symbolic nature and consider the reasons behind their existence, sometimes ingrained, sometimes superficial.

INTRODUCTION

To conceive representation as "one thing in place of another, for someone" Pierce (1903) allows us to interpret it as a semiotic mediator between the abstract object of study and the cognizant individual.

In this sense the symbolic aspect in terms of the syntax of the representation must be considered as much as its semantics. The semantics are grasped by the individual through meaningful problematic practices.

In this work our aim is to identify the role played by the auxiliary constructions related to the use of diagrams, which we call Gestalt constructions and which are built by the users when they figural manipulate drawings in order to treat them as figures, Laborde and Caponni[2], (1994).

We hold that these configurations are profoundly ingrained in our students, that they are intentional but often unstable. They can be a particularly valuable resource in heuristic tasks of figural investigation.

THEORIC FRAMEWORK

From the point of view of Duval (1995):

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[1] In the sense of Laborde and Caponni

[2] The treatment of the graph as a drawing or figure, is based, firstly, on observing its properties as an actual pictorial representation or, secondly, considering the mathematical properties associated with the graphical representation.

> **One figure[3] is an organization in marked contrast to the shine. It emerges from the background through the presence of lines or points, governed by Gestalt law and perceptual indications** p.142

In terms of the Gestalt relationship the figure has “form, contour, and organization,” while its preceding appears as an “amorphous and infinite continuity”, Guillaume (1979) p. 67.

Pictorial representations may be considered external and iconic, Mesquita (1998); they are also defined as inscriptions, Roth & McGinn (1998); or diagrams, Pyke (2003). The unifying idea is that the graph is an external representation that is materialized through the use of pencil and paper, the computer or other means and is, therefore, available through these means, in contrast to mental representations which are not accessible, op cit.

Below we consider the graphic representation as a diagrammatic representation or diagram that preserves the relationships of the objects involved. Diagrams from the viewpoint of sense will be observed in themselves and interpreted from the point of view of the reference between them.

On the other hand, diagrams are figural concepts that, in the words of Fischbein (1993) can be thought of as concepts and as objects: this duality emphasizes the different interpretations associated with graphic representations.

Thinking of a diagram as an object means associating specific figural properties with it, such as position or form. These considerations on what thinking about it as an object means, in Fischbein’s way, refer to a mathematical object, this is abstract. The dichotomy between object and concept is related more to a theory need to include non-formalized mathematical aspects, such as position or form, than to the mathematical objects in themselves.

For the purposes of this work we refer to the treatment of representations in geometry based on their iconic or figural properties centered on visual image and to their external nature as embodied materially on paper or other support.

The nature of diagrams in geometry learning is ruled by two types of properties as Laborde (2005), observes:

> **Diagrams in two-dimensional geometry play an ambiguous role: on one hand they refer to theoretical geometrical properties, while on the other, they offer spatial-graphical properties that can give rise to a student’s perceptual activity** p. 159

The treatment given to the diagram as an object in geometry learning is closer to that given to a drawing as a current instance, and not as an

[3] The word “figure” in this quote has a meaning close to diagram, distinct from how we use it in the rest of the work.

abstract mathematical object in the concept-object duality. It takes students some time, in fact, to incorporate the idea that drawn objects (representations) have properties which are distinct from those of real life objects.

In terms of learning, Laborde op cit. warns:

> **The distinction of the two domains, the spatial-graphical domain and the geometrical one, allowed us to show that the intertwining of the spatial aspects of diagrams with the theoretical aspects of geometry is especially important at the beginning of learning geometry** op. cit. p. 177.

It is in the spatial-graphical domain where spatial and figural relations are developed that give shape to the thought structures that are developed around the Gestalt. First, as a relation between the background and the form and later, as resources in the explanation, construction or solution of problems, they give rise to Gestalt configurations.

Studies related to visualization and, most recently, visual perception, have addressed the role played by Gestalt relations between background and form in the pictorial representation that accompanies the mathematics, and the importance of considering it on a certain type of perceptive perception, Duval (1995)

In the work of Nemirovsky and Tierney (2001), regarding spaces of representation, we observe a special interest in establishing the existence of distinct ways of interpreting the same space of representation based on its use and meaning relative to the objects represented.

From the above we can say that the use of diagrams depends not only on what is represented in them, but also on the relations we can establish from them, including spatial information which includes Gestalt relations.

Gestalt configurations

In the work of Dvora and Dreyfus (2004) we have unjustified assumptions based on diagrams in geometry due to students confusing a mathematical motive and a purely visual motive. In addition, when problem solving they base themselves more on their beliefs about the topic in question than on the available propositions. The authors find that diagrams affect students' way of thinking because, among other things, they use diagrams as evidence.

The Gestalt configurations dealt with here have no evidential connotation, they are, instead, auxiliary constructions that complete or give shape to an idea and have their origin in the need to solve problems which involve a diagram.

Gestalt configurations are not related to all the possible pictorial tests that claim to find a solution helped by the drawing, whether the lead is promising or not.

A Gestalt-type configuration, as well as the intentionality of solution, should contain a reference to the relation between background and form, that is, Gestalt configuration "adjusts" to the general composition of the diagram. In other words, Gestalt configuration manifests as a cognitive resource to give substance to a thought and is distinguished by its figural relation between the background and the form of the diagram in question.

The symbolic relations of a Gestalt configuration are determinant: it is dependent on them whether this configuration can be built or not. By way of example, Acuña (2004), we have the case in which without the presence of a graphic reference the very existence of the geometric or graphic object is in doubt, as in the following cases:

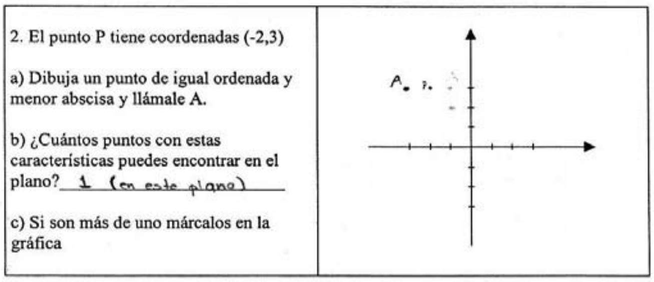

Fig. 1 Point A is the only one with equal ordinate and smaller abscissa than P, in this plane

In the student's answer to the question about the number of points that have an equal ordinate and smaller abscissa than the point (-2,3) in which he (or she) affirms: 1 on this plane*,* we can see that he is trapped by the actual representation since the picture offers only one unit mark on the abscissa axis. The student does not consider alternative solutions other than that point located above the mark of the whole abscissa unit. The absence of the mark combines with the idea that a point should have a whole abscissa unit. This student was unable to build neither of a suitable configuration for the solution or a Gestalt configuration.

In the following case, Acuña (1997) we have (see Figure 2) a question about whether the suggested points are on the drawn straight lines or not. If we look at the point (-2, 3) we see that the straight line proposed does not reach the position where a perceptive solution could be given, that is, one perceived "by eye". This fact makes the student doubtful and answers that if we lengthen the straight lines, the point is on it, otherwise it isn't.

Our student is unsure of the existence of the point in spite of knowing its coordinates, thus the Gestalt configuration cannot be built because of the absence of the graphic reference that gives it substance. In this case, if the straight line does not reach the indicated place, there is no security about its existence, which impedes the acceptance of the relation between the straight line and the point.

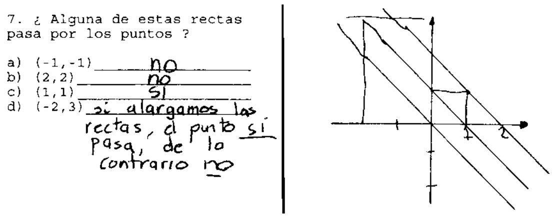

Fig. 2 Problem of points on the straight lines

Constructions with appropriate Gestalt configuration

In relation to the construction and use of geometric figures, Maracci (2001) has observed that students insist on making constructions that possess certain, from their point of view, appropriate aspect.

This insistence is accompanied by the preference for the horizontal-vertical position, or the choice of graphs that appear to be, for example, a straight line Mavarech and Kramarsky, (1997) or a segment of a straight line with an slope equal to 1, Acuña (2001), as well as students' penchant for using prototypes[4] Hershkowitz (1989), or the use of the "best" examples from among one same category of possible cases, Mesquita (1998).

This phenomenon can be explained by the students' need to find a good orientation and familiar representation. In other words, they prefer to build "appropriate" configurations in general and Gestalt configurations in particular that give meaning to the actual figural relation.

In some tasks with qualitative instructions, as in figure 3, we have identified a tendency to recognize and build graphs in a certain position and with a certain peculiarity, forming prototypes, Acuña (2001). A large part of the students surveyed with the question for draw straight line with only points with positive abscissa, responded with a half-line that reaches the origin, with a slope of 1. This answer was more frequent than any other, correct or incorrect, in high school students.

[4] We call prototypical figures those which correspond to a regular organization of contour, orientation and form; prototype figures tend to respect laws of enclosure (closed limits are preferably perceived), favoring some directions (such as horizontal and vertical) and forms (that tend to be regular, simple, and symmetrical); the components of the figure (sides, angles for example) have approximate dimensions.

5. Draw a straight line where all the points have a positive abscissa, that is, where $x > 0$ is true for all points on the line.

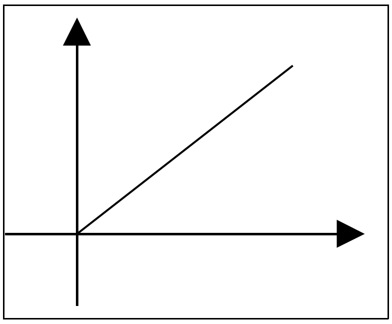

Fig. 3 Answer to a qualitative-type construction task

The students' answer presupposes that the straight line built does not cross to the other side of the vertical axis, as if it were a barrier, so that it will not take negative values for the abscissa.

The non-ostensive nature of the straight line related to the infinite extension of its extremes contributes to the incorrect interpretation of the answer that, in strictly figural terms, has a plausible logic, especially since it is not possible have a representation of a straight line, only parts of it.

The non-ostensive aspect on the infinite extension of the line can be accepted theoretically by the students, but the impossibility of building theoretical straight lines leads them to accept the segments of a straight line as if they were straight lines themselves.

In figure 4, Acuña, (2002) students are asked to draw the graph of the straight line that would have an ordinate equal to the origin of the original straight line that appears on the left.

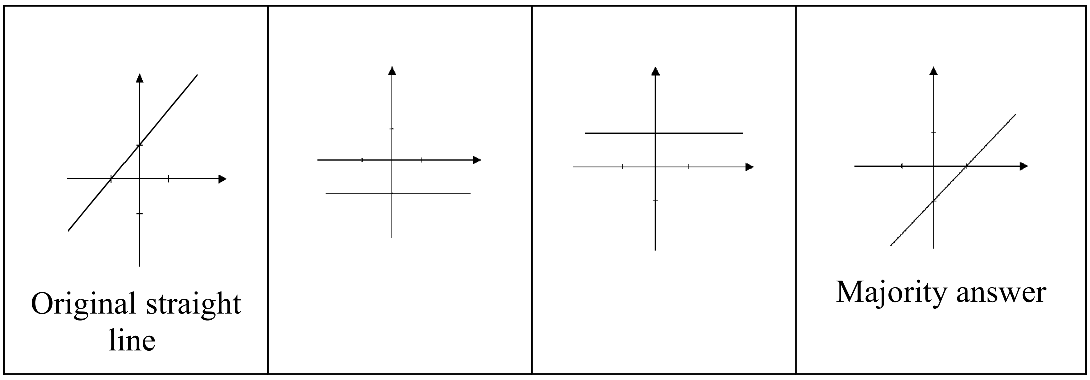

Fig. 4 Gestalt configuration combining figure and form

The majority of our students drew the graph on the far right. Many of them had correctly recognized the ordinate of the origin in straight lines given earlier; nevertheless, here they choose to conserve the "triangular" image formed in both graphs, preferring to relate the two graphs with a similar Gestalt.

This type of answer is strongly conditioned by the situation of the exercise, in particular given that this perception is unstable, as we can see in other exercises.

In the following exercise, Sosa (2008) two high school students have been asked to build the height corresponding to the side marked with X in each case.

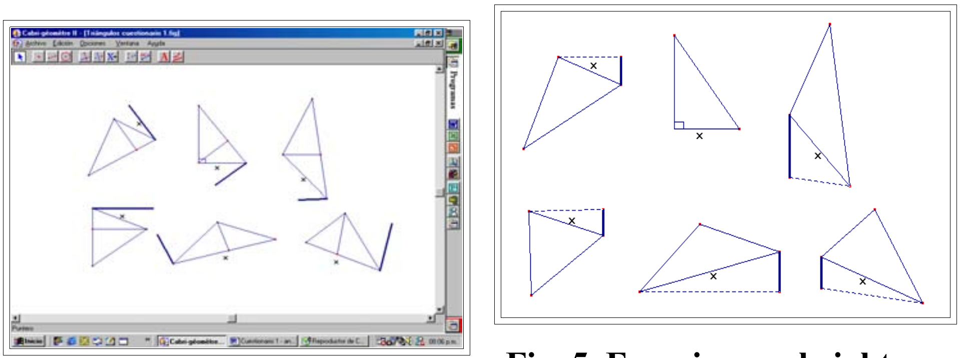

Fig. 5 Exercises on height construction

In these two cases, we have the application of a Gestalt configuration to solve the problem of the construction of the height of the marked side. In the answer on the left, the height is thought of as a conformation formed by the vertex of the obtuse angle, or what looks like it. The student also uses an auxiliary parallel line which we suppose was in the image the student recalled.

In the case of the constructions on the right (see figure 3) we have an auxiliary construction that includes the line marked with X but where this is a part of another auxiliary construction that presents a right-angle triangle where we observe some of the characteristics relevant to height, but its construction is unknown. The marked line is included in his construction, but its role in the construction is reinterpreted and he does everything he can to make it look good.

In the following case we ask students to mark the straight lines with a different slope to that of the one given.

The formation of this configuration not only appears when the definitions of the geometric objects are unknown or is recalled inexactly, but also when globalizing an idea of position, as in the following example. In the case of figure 6 and 7, we ask high school students to choose from the lower graphs that which have a different slope to the one proposed initially.

The results allow us to see their idea of a slope in this exercise. Despite having correctly compared, based on perception, the slope of the given lines, here they conceive it as the Gestalt configuration formed by the position of the straight line relative to the axes, that is, the line is positioned from left to right and from up to down.

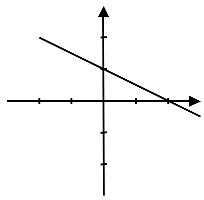

Fig. 6 Straight line with given slope

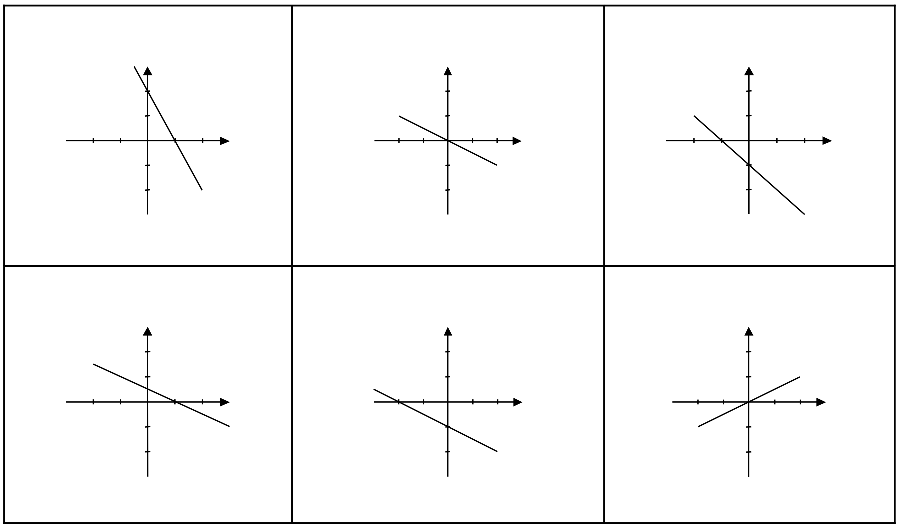

Fig. 7 Gestalt configuration on a slope

The 19.3 % of our sample only marked the straight line that is positioned from left to right, leaving aside the idea of slope that they used before.

The preference towards a "good" Gestalt appears to impose itself in tasks of identification of figural properties. This recourse may signify an advance or a backward step for solution strategies. What does appear to be constant is the use of this type of configuration to test solutions to problems with diagrams.

These configurations may disappear quickly with better instruction, but they also have aspects of profound rooted as in the case of Moschkovich's (1999) investigation, regarding the use of the y-intercept. She finds that when observing the graph of a straight line students may expect the x-intercept to appear in the equation because on the graph it is a salient as y-intercept although this is not necessarily convenient in the case of the equation $y = m\,x + b$ however, they are important for the equation that considers two points on the straight line. The appeal of the x-intercept is so big than could think it as a preconception; in her investigation she affirms that:

> **The use of x-intercept is not merely the result of choosing or emphasizing the form y = m x + b over other forms but is instead an instance of students making sense of the connection between the two representations and reflection on the conceptual complexity of this domain** p. 182

We believe from the above that it is possible to suppose the existence of figural resources that take the form of Gestalt configurations that respond on one hand, to the necessity of giving substance to figural ideas, and on the other, that these configurations are ruled by the relations between background and form on which rests the figural representation of mathematical and, more concretely, geometric diagrams.

CONCLUSIONS

A Gestalt configuration is a mental or real construction utilized by the user to resolve, complete or give meaning to a given problem through a diagram that can be treated as a drawing or figure.

Gestalt configurations have a personal character, but on occasions reflect epistemological obstacles that are supported by the non-ostensive nature of the properties of the objects represented by the diagrams, as in the case of the infinite character of some of these representations.

The formation of some Gestalt configurations is characterized by having an ephemeral life, although there are some that persist; as they are personal productions of the user. In general, they are considered productive and reliable for confronting familiar graphic settings towards resolving problems that include diagrams.

In all cases, the construction of the Gestalt configurations is intentional in spite of the inability to ensure its pertinence. Gestalt configurations do not only appear as visual traps but as a diversity of resources to solve figural problems or proving.

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