NATURAL DIFFERENTIATION IN A PATTERN ENVIRONMENT (4 YEAR OLD CHILDREN MAKE PATTERNS)[1]

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Manipulation in learning geometry is a disputable topic because of different theoretical bases for creation of geometrical concepts. Some theories underline a great importance of visual information in forming the first level of understanding geometry. For children, such visual geometrical information could be provided by patterns. Assuming that visual information gives the first stimulus for creation of geometrical concept, I undertook the experiment to observe the possibility of going beyond visual states in early geometry, towards its dynamic images.

INTRODUCTION

Many children have a well-developed, spontaneous and intuitive mathematical competence before their school education (Clarke, Clarke, Cheeseman, 2006). Researches in this field put a great emphasis on early numeracy and competence in counting, although in some articles the topic of "spatial and geometrical competence and concepts" is described as well. In these attempts, "spatial development" is described by relations like: behind, beside, in front of…; concepts are usually limited to the basic geometrical shapes: triangles, squares, circles.

I strongly believe that guasi – geometrical activities can develop widely understood children's mathematical competence. On one hand, since geometrical approach to mathematics is closer to children than arithmetical one, geometry can open doors to a world of mathematics. Geometrical cognition starts from a reflection upon the perceived phenomena and in this way correlates with the basic ways of learning among children. On the other hand, it gives a chance to develop such ways of thinking, that are typical for mathematical thinking. Skills like generalization, abstraction, perceiving relations, understanding rules are the base for this aim. Early geometry is in-between physical and abstracts worlds. By this, it enables to mathematize this world.

By stating an issue of enriching children's mathematics by adding geometrical activities, we simultaneously pose a question: what such activities should include? Should they be focused on geometrical figures, or should they go beyond traditionally understood areas of children's geometry? It seems, that geometrical regularities (patterns) are unexploited areas for such goals.

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Many educators are in opinion, that during the work with patterns, elements of mathematical thinking occur. A pattern is a form, a template, a model (or, more abstractly, a set of rules). It is a well-known fact that geometrical regularities rooted in patterns can be described by the language of geometrical transformations. My previous research confirm, that 4-7 year old children are capable of organizing the space and arranging it accordingly to geometrical relations in a spontaneous way (Swoboda, 2006). But these are static relations, represented visually, and connections between such grasping of relations and their dynamic representations are not scientifically proven.

PERCEPTION VERSUS ACTION IN EARLY GEOMETRY

Some theories stress the fact that geometrical knowing and understanding is created in a specific way. In those theories, the priority is given to perception.

The most popular theory of forming the geometrical concept comes from P.van Hiele. He describes the first level of understanding as "visual", connected with non-verbal thinking. The emphasis is placed on the ability of recognizing shapes, which are judged by their appearance as the 'whole'. Not much concerning the role of action is spoken, although a didactics conceptions suggest activities based on the action with objects. In J. de Lange's opinion (who comments van Hiele's theory), *a pupil who is on the visual level can obtain the first level of thinking when s/he is able to manipulate in domain of regularities.* (1987, p.78).

Some very interesting depictions related to geometrical understanding are present in conceptions worked out in Czech Republic by M. Hejný and P. Vopěnka. In their opinion, geometrical world is hidden in the real world, and it is emerging from the surroundings through the special intellectual activity which can be called "the geometrical insight" (Hejný, M. 1993, Vopěnka, P. 1989). At the beginning, there is no geometrical world nor geometrical object in a child's mind. Only objects from the real world exist. But we focus our attention on those objects in various ways. Sometimes we perceive „something". Vopěnka (1989, p. 19) describes such a situation in the following way: *To see „this", means to focus attention on "this", to distinguish "this" from the whole rest. This, what can absorb the whole attention on itself, we call „phenomenon"*. Perceiving „something" creates the <u>first understanding.</u> For example, a child can focus his or her attention on a shape of an object or on a specific position of one object in relation to another. *Phenomena* open the geometrical world to a child. In spite of the fact that our attention is attracted by these phenomena, this first understanding is passive: stimulus goes from the phenomenon. In this depiction, the role of perception is large – the perception of „something" is the first step to creation of the child's own geometrical world.

In these depictions, the role of an action is lost. Results of psychological researches confirm that in understanding of shapes, the great importance lays upon the pictorial designate. But the next stage is needed. Acts of perception are important but are not

a sufficient source of geometrical cognition. Szemińska (1991, p.131) states that: *perception give us only static images; through these, we can catch only some states, whereas by actions we can understand what causes them. It also guides us to possibilities of creating dynamic images.*

Szemińska has worked very closely with Piaget and, widely known his results show that children (on the pre-operational level) have great difficulties in movements reproduction – they are not able to foresee a movement of an object in a space. The process of acquisition of such skills is lengthy and gradual. During manipulations, child's attention should be focused on *action*, not on the very *result of action.* It requires a different type of reflection than the one that accompanied his or her perception.

This short juxtaposition above shows that the relation between visual recognition of geometrical objects and actions which can lead to creation of dynamic images of those objects, need further investigations. They are still not recognized as an educational problem. For this reason I undertook the experiment to observe the role of manipulation in early geometry.

EXPERIMENT

In my experiment, as the basis I took Vopěnka' and Hejný's theories about the opening of the geometrical world. First of all, I based on the assumption, that the first understanding takes place when a child turns its attention on any geometrical phenomenon. I was interested in situations where children can manipulate. Results of my previous experiments showed that making patterns (arranging them out of blocks, folding out of puzzles, drawing), can fulfill our expectations.

In order to test the possibilities of creating a "path" from perception to manipulation, I prepared an experiment, which took place in March - April 2008. Children from a nursery school, aged 4, 5, 6 , were the subject of the series of observations. Clinical observation an interview with a small group of children was chosen as a methods.

Children were tested individually. As a research tool we used „tiles" (two types), shown on the right (Fig.1). The whole investigation of one child consists of two parts.

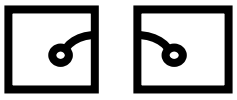


Fig. 1 – research tool

Part I, Stage I: A teacher makes a segment of the pattern (Fig.2).

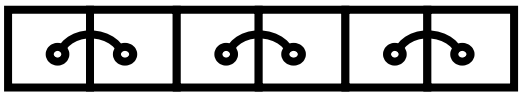


Fig.2 – a segment of the pattern prepared by a teacher

On the table, there are also tiles arranged into two separate piles. Teacher says: Look carefully at this pattern and try to continue it. If a child doesn't undertake the task, the teacher will say: look how I do it. After that you will continue. If a child undertakes a task, then after having finished making the pattern, he/she will take part in the next stage of an investigation.

Part I, Stage II: Teacher says: Now, please close your eyes, and I will change something in your pattern. After that, you will say what has been changed. (Teacher exchanges one tile in the pattern, so that the regularity is distorted). Then, the teacher shows the pattern and asks a child: Is there something wrong here? Why? Regardless of the answer received from the child, the teacher says: and now try to correct the mistake I have just done.

Part II, Stage I: Teacher says: some days before we made a pattern by using these tiles. Do you remember? Now, try to build it again. If a child does not remember, the teacher starts to create the pattern and invites the child to cooperate.
Part II, Stage II: Teacher says: and now, I will invite your colleague and you will be the teacher for her. Firstly, you will show her how to work to make the pattern, and after that you will play with her in correcting it. You will do it just like we did it some days ago.

General aims of the experiment were to observe the possibility of awareness of results of different types of movements: translations and rotations (possible by using only one type of tiles) and mirror symmetry (which requires reverse copy of the shape). Additionally, for group of 4 year old children, I tried to find answers on these questions:

- How do children understand the task presented visually,
- How do they understand a verbal instruction related to the given task,
- How do they act by making and retrieving patterns.

RESULTS OF THE EXPERIMENT

In this paper I will present some results gathered in a group of 4 year old children and only from Part I. This educational and developmental level, in each of investigated domain, turned out very diverse. Children demonstrated both: various understanding of the task and various ways of its realization.

1. Reflection upon the visual information

Many children started to work spontaneously, just after hearing the command: *take a careful look at this pattern and try to continue arranging it.* From the command they depicted only the words: *try arranging it.* It is also possible that they acted in a spontaneous way: while seeing the fragment of the pattern and material for manipulations they started to play with them. The other group observed all that used to be on a table for a long time. Sometimes, they were taking and analyzing separate tiles. Therefore, different strategies were possible. It is showed by the following examples:

Strategy „helpless”. Here, a child did not actually know how to create motifs. It could act only when guided by the teacher. Left alone, the child could not follow these guidelines. According to Vygotski’s theory, the creation of the whole motif is beyond the zone of proximal development.

Example: Kaja (girl)

Teacher: Look carefully at this pattern and try to continue it …..5 seconds break… you can take it into hands.

Pupil: She takes one tile, keeps it for 8 seconds without any movement. Finally she says: I don't know.

Teacher: Look, put this tile here (the one in your hands), take another tile from the second pile, connect them – and what do you obtain? (a girl acts according to teacher's instructions). Could you continue your work in the same way? …(10 sec. girl does not do anything). Take one from this pile, ….. and from the second one … (girl connects the motif in an upside-down position).

Strategy „trials and errors". The beginning of work can be based on „blind" experiments: child has some materials for manipulation, but she/he doesn't know how to use it in order to obtain the aim. A child decides „to do something". Manipulations can lead to interesting findings and frequently a child can draw conclusions from previous experiences.

Example: Oliwka (girl)

Pupil: Quickly reaches for two tiles from one pile and tries to create a motif above the pattern. Although she manipulates and does not succeed, she accepts the arrangement consisting of two tiles of the same type, placed in an opposite way. She continues her work by taking tiles from the same pile again. This time she is not satisfied with the outcome so she takes two different tiles and creates a motif, which is upside-down. The last one she created was correct so she finished her work (Fig.3).

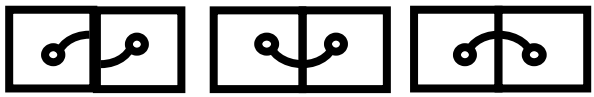


Fig. 3.

Strategy of a conscious creation of one motif by using two different types of tiles. Before starting the work, a child visually analyzed the whole pattern prepared by the teacher, as well the manipulative material. He/she could perceive the relation which enables them to continue the work without any trials proceeding the right action. Sometimes only few manipulations support his/her decisions.

Example: Kuba (boy)

Pupil: Observes…18 second motionless.

Teacher: Go on. If you have any questions, you can ask. You can do whatever you want.

Pupil: Takes one tile in his left hand, arranges it in a certain distance from the pattern as if he was planning to place the second one to match them. 10 sec break.

Teacher: You started well.

Pupil: 8 seconds. He takes a tile from the second pile and connects it to the motif. Then, he takes two tiles from the left pile, places them close to each other.

He manipulates them for a while but quickly puts them back and reaches for the other tile from the right pile. Next couples of tiles are arranged well. He continues the pattern from both sides.

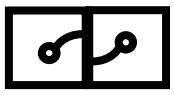
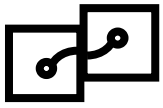
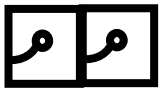
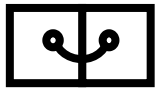
Commentary: On this level actions from two distinct areas of activity exist: primal instinctive actions stimulated by a visual impulse and actions preceded by a reflection and a visual analysis of shape. Observations confirm that visual information is very important and many children can use it in a way, which is significant for 'geometrical seeing'. This means that children have the ability to analyze shapes, create a visual relation between the whole and the part, and perceive the relation of mirror reflection.

2. Various understandings of the instruction: try to continue.

Strategy „any nice motif". In this situation, 4 year old children understand that tiles are a means to create a motif. They reach for them eagerly, and observe configurations of two tiles. Every interesting arrangement is a good solution for them.

Example: Stasiu (boy)

Pupil: He takes two tiles from one pile and he manipulates them in the corner of the table. He arranges them in a way which is shown at fig. 4 and, with satisfaction, looks at them.
Teacher: Is this like in our pattern?
Pupil: He puts tiles crookedly, trying to connect the line from tiles (fig.5).
Teacher: It is nice, but does it fit into our pattern?
Pupil: He manipulates again, exchanges a tile for another one but still of the same type. Then, he creates a configuration like shown at the fig.6. Very satisfied, he looks at the teacher.
Teacher: And again you have something different than we have here (the teacher shows the pattern). I will give you a small hint: try to take a tile from this pile.
Pupil: Quickly he reaches to the second pile and connects the motif (fig.7).
Teacher: So. …. And what do you think?
Pupil: He moves his motif to the pattern and says: this is a happy face.

Fig.4.  Fig.5  Fig.6  Fig.7 

Strategy „one, identical motif". Among 4 year old children continuity does not necessarily mean infinity. This may mean that a child will create just one, identical motif. A child notices a rule but it is realized only by a simple duplication. This is rather a manifestation of the noticed rule than its continuity.

Example: Roksana (girl)

Pupil: reaches for one of the motifs that were previously created by the teacher. She puts her hands on her knees, sits still and looks at the teacher.

Teacher: So you moved one motif towards you. Now let us do the same with the second and the third one. And now try to continue. Try to make the pattern longer.

Pupil: Simultaneously, she reaches for tiles from both piles, takes one out of each, checks the motif in the air and connects it to the pattern. She looks at the teacher.

Strategy „a lot of identical motifs".

In this case, a child sees that the pattern consists of certain motifs and there is a large number of them. They do not necessarily have to match one another.

Example: Zuzia (girl)

Pupil: First, she decides to arrange a motif using the same type of tiles but quickly she changes her strategy. She takes tiles from two piles, arranges a couple of separately placed motifs.

Strategy „one-dimensional continuation". A child demonstrates the awareness that a pattern can be continued in both directions – to the right and to the left.

Example: Tomek (boy)

Pupil: Immediately reaches for separate tiles from piles and correctly, in turns, he continues his work. Seeing that the space on the right side of table is finished, he continues his work on the left side.

Strategy „two-dimensional continuation". A child wants to arrange tiles for as long as it is possible. If there is not enough space in a horizontal direction then it starts to build the next level, a vertical one. Nevertheless, the relation between the tiles is maintained.

Example: Ola (girl)

Pupil: Immediately takes two different tiles in both hands and she places the connected motif close to the pattern. Without any hints she continues work in both directions – left and right. When there is no empty place in the line she asks: also here? (she shows the place over the pattern). She continues work as long as she has tiles.

Commentary: The possibility of manipulation may create occasions for something which P. Vopěnka calls 'the first geometrical recognition' - focusing attention on geometrical phenomena and specific relations of one object to another. A child may find satisfaction in searching for different configurations of two identical objects. But children at this age usually analyze patterns, search for repeated motifs. Finding and constructing motifs indicates a certain developmental level. In the framework of this period we may find examples of children that can spontaneously receive information from the pattern as an encouragement and challenge for making a whole

series of repeated motifs, for continuing them both in one and two-dimensional space. It is an action aimed at a rhythmical organization of infinite space.

Ad.3. Various methods of retrieving the „destroyed" pattern.

The correction of regularities progressed in two different ways:

A. A child rejected a „wrong tile" immediately and replaced it with the correct tile, taken from the proper pile – "replaced strategy".

B. A child started to manipulate the „wrong tile", trying at all costs to obtain the mirror position – "manipulative strategy". Despite of his previous experience gathered while making the pattern, children undertook attempts of matching up two tiles of the same type. The strategy can be divided into three subcategories:

B1. A blind manipulation, simultaneous rotation of one or two tiles. Here, a child is convinced that two tiles don't match each other but through a certain movement they could fit.

B2. A feeling that one tile is right but the second one is somehow wrongly placed. Therefore, manipulations, mainly rotations, are made with only one tile. Frequently a change order of tiles and their places occur.

B3. Going to the reverse side of the tile. Initial manipulations (rotations and translations) occur only in the area of a one-side oriented plane. After this stage, a child reverses the tile to its other side and checks the possibility of placing it in a different orientation.

Commentary: The occurrence of manipulation strategy suggest that there is a big conceptual gap between a static understanding of axis relation and its dynamic depiction. In the observed age group there was no crucial connection between the stage of making the pattern and the stage of correcting it. It seems that children treated the tasks as two totally different activities. As the children could not see any relation, they did not use the experience from the first stage. The first stage required only visual information. If they used it, they succeeded. The second stage introduced a false suggestion. Children recognized that the motif on the exchanged tile consists of a circle and arch configuration but they could not recognize the mirror symmetry in it. Because of obvious reasons, this manipulation strategy could not lead to success, but it seems that by these actions children gained many important experiences. For example, they became convinced that certain movements on a plane lead only to a limited range of final configurations. This type of movements will probably have a great significance for creating concepts of geometrical transformations or dynamic visual imaginations of geometrical objects.

The action, where a child uses a 'replaced strategy" could be interpreted dually. It is very probable that a child is well capable of benefitting from visual information. It is possible that a child sees the connections between two separate piles with tiles and the whole motif and can analyze shapes. In this case, when a child decides to replace

a tile, he/she chooses the „strategy of certainty". The other interpretation is that a child knows only that two different piles exist, and by using tiles from both it is possible to be successful in some way. Those two interpretations do not give any answer about children's intuitive knowledge regarding mirror symmetry as a transformation. The fact that some children immediately exchanged tiles for the proper ones does not necessarily mean that they were aware of the relation type or the type of the movement which is required for mirror translation. Such intuitions could only emerge during manipulations.

The table below contains the quantitative specification which shows the presence of these strategies in children's work.

Replaced strategy (A)	Manipulative strategy (B)	Helpless	Other
4	13	1	1^2

Table 1. Pattern correction strategies

SUMMARY

In the research, which I partially describe in this article, educational level of four year old children came out to be diverse. The results of investigations show different phases, activity levels in the framework of geometrical regularities.

Psychologists underline the great importance of visual information in early childhood. It is important for thinking development as perceived objects provoke a closer active recognition. Such direction should be obligatory when we speak about geometrical objects. The perceived geometrical phenomenon should be investigated by means of a spontaneous manipulation. Therefore, the direction should be as follows: phenomenon -> manipulation.

At this stage, manipulations are evoked by perception and are subordinated to perception. The manipulation itself is only a tool which enables to reach the aim. A child has a vague feeling that some kind of manipulations can establish an expected relation between objects, but has no idea what kind of movement is needed. While solving the problem, child does not consider what kind of manipulation he/she makes. In spite of this, these manipulations are important for further discoveries. The research showed that in this age group beginnings of behaviors that may be treated as a good basis for creating geometrical concepts in the future (dynamic images of geometrical transformations) take place.

Educational level of four year old children in this field may prove to be important. Observations in older age groups indicate a loss of dominance of a manipulative strategy to the advantage of a replaced strategy. Does it mean that the awareness of axis-symmetrical transformation increase? In my opinion, no. To my mind, it is the outcome of a higher ability to analyze shapes, to decompose a whole object into its attendants. A symmetrical object consists of two 'identical' halves, and older children find it easier to recognize them. But static relation of axis symmetry does

not mean that children understand transformations that change one half into the other.

A question arises: are these the following developmental steps of understanding these regularities or maybe they are the outcome of different relations between visual representations and actions? An overall glance on the course of individual children's work confirm that actions in the first phase do not give any reasons to forecast the way in which children will work in the second one. These problems require further investigations.

On the other hand – in this case, immaturity in visual analysis of shapes can be beneficial. Children do not make decisions on the basis of visual recognition of differences among tiles. They make most of their manipulations in a spontaneous way, and by this they gain experience which activates a dynamic understanding of geometrical relations.

The level of work with 4 year old children, for various reasons, is a very promising one. Every time, when a child is able to start the work, the outcome of undertaken actions can be treated as a springboard for a further discussion. None of chosen approaches towards the task can be understood as wrong and by this children do not suffer from the feeling of defeat. It gives a chance to compare results, discussion. It give a chance to function in the world of regularities, which is crucial for general mathematical understanding.

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