THE STRUCTURE OF PROSPECTIVE KINDERGARTEN TEACHERS' PROPORTIONAL REASONING

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Lamon (1997) claimed the development of proportional reasoning relies on different kinds of understanding and thinking processes. The critical components she suggested are: understanding of rational numbers, partitioning, unitizing, relative thinking, understanding quantities and change, ratio sense. In this study we empirically tested a theoretical model based on Lamon's model, with data collected from 244 prospective kindergarten teachers. The analysis of the data provided support to this theoretical model and revealed that rational number, reasoning proportionally up and down and relative thinking are statistically significant predictors of proportional reasoning. These findings allow us to make some first speculations of which type of processes should be emphasized for the development of proportional reasoning in early years.

Key words: proportional reasoning, rational number

INTRODUCTION

Ratio, proportional thinking and reasoning abilities are seen as a corner stone of school mathematics; this observation is reflected in current syllabus documents, (e.g., National Council of Teachers of Mathematics, 2004) and by educators (e.g., Nabors, 2002). Researchers have often noted that the topic of proportional thinking can be challenging for schoolchildren (Fuson, 1988; English & Halford, 1995; Gelman, 1991; Steffe & Olive, 1991; Kilpatrick, Mack, 1995; Swafford, & Findell, 2001). Proportional reasoning is in essence a process of comparing one relative amount with another. From a psychological perspective, proportional reasoning is a late accomplishment developmentally because it entails second-order reasoning; inasmuch as proportions are relations between two quantities, comparisons between proportions entail considering relations between relations (Piaget & Inhelder, 1975). However, although there is indeed considerable evidence that a full understanding of proportional relations develops slowly (e.g., Moore, Dixon, & Haines, 1991; Noelting, 1980), the notion that reasoning about relations among relations is intrinsically beyond the capabilities of young children has been strongly questioned (Spinillo & Bryant, 1991). To develop young students' understanding, teachers should be aware of the critical components of understanding proportions. Thus, the main focus of the present study is to shed some light on the structure of kindergarten prospective teachers' understanding of proportional problems.

Until recently, we have had little understanding of how proportional reasoning develops. Based on previous research, we will develop and validate a framework of kindergarten pre-service teachers' thinking while they work on representations of proportional problems. Lamon (1999, 2007) asserted that understanding rational numbers marks the beginning of the process of proportional reasoning. Thus, in the proposed framework we will articulate the understanding of kindergarten prospective teachers' on rational numbers, and related concept such as unitizing, partitioning, relative thinking, understanding quantities and change, ratio sense.

Specifically, in this study, we will propose a conceptual framework, which is mostly based on previous research on rational numbers (Kieren, 1988) and on the features of Lamon's (1999) model of proportional thinking. This framework constitutes an attempt to encompass the whole spectrum of kindergarten prospective teachers' understanding of proportional situations and problems. Furthermore, the study provides an empirical verification of the proposed model and traces the different types of thinking projected by kindergarten prospective teachers in the context of rational number and proportional tasks.

THEORETICAL BACKGROUND

Components of proportional reasoning

Lamon (1999, 2007) suggested that proportional reasoning is complex and to achieve it one has to master different kinds of understanding, thinking processes and contexts. Specifically, she proposed six areas that contribute to proportional reasoning: partitioning, unitizing, quantities and change, rational numbers, relative thinking and rate. Kieren (1988) claimed that the concept of rational number consists of four interrelated subconstructs, ratio, operator, quotient and measure, and part-whole permeates these four subconstructs. A short description of each proportional reasoning components and a brief definition of each subconstruct are provided below:

Relative thinking is a cognitive function which describes the ability to analyze change in relative terms. It is also called multiplicative thinking (Lamon, 1999).

Unitizing is the cognitive process of mentally chunking or restructuring a given quantity into familiar or manageable or conveniently sized pieces in order to operate with that quantity (Lamon, 2007).

Quantitative reasoning in visual and verbal situations is the ability to interpret and operate on changing quantities. Quantitative reasoning may or may not involve numbers. It may involve the comparison of numbers in standard form or qualitative judgments (such as more, less, etc) without actually having a quantity (Lamon, 1999).

The partitioning and part-whole subconstruct of fractions is defined as a situation in which a continuous quantity or a set of discrete objects are partitioned into parts of equal size (Lamon, 1999).

The ratio subconstruct of rational numbers is regarded as a comparison between two quantities. Thus, it is considered as a comparative index, rather than as a number (Carraher, 1996).

In the operator interpretation, rational numbers are viewed as functions applied to some number, object, or set (Behr, Harel, Post, Lesh, 1993; Marshall, 1993). One could conceive operator either as a single composite function that results from the combination of two multiplicative operations or as two discrete, but related functions that are applied consecutively.

The quotient subconstruct can be seen as the result of a division situation. In particular, the fraction x/y indicates the numerical value obtained when x is divided by y, where x and y represent whole numbers (Kieren, 1993).

In the measure subconstruct, a fraction is associated with two closely interrelated and interdependent notions. First, it is considered as a number, which conveys the quantitative personality of fractions, its size. Second, it is associated with the measure assigned to some interval. For example, 2/3 corresponds to the distance of 2 (1/3-units) from a given point. This is the reason that this subconstruct is associated with the use of number lines.

Prospective teachers' subject matter and pedagogical knowledge

Although previous studies have examined teachers' abilities to solve proportionality problems (Post, Harel, Behr, & Lesh, 1991) and their ability to distinguish between proportional and non proportional situations (Simon & Blume, 1994) until now, no studies have described teachers' understanding of all the above mentioned components of proportional reasoning and whether they actually contribute to proportional reasoning. Since we encourage teachers to aim to a more conceptual understanding of mathematical concepts, we need to determine whether they have the necessary understanding of the concept and certainly its related components (Cramer, Post, & Currier, 1993).

There is no doubt that teachers' understanding of proportional reasoning also affects the way that they will present this topic to their students. In other words, the way in which a teacher will present proportional activities in her classroom is an indicator of what she believes to be more important and appropriate for students to learn, and hence, affects the way that their students understand mathematics (Thompson, 1992). The fact that mathematics in kindergarten may appear to some individuals as simple or trivial can be very misleading. Kindergarten teachers must know the mathematical concepts that students need to master and facilitate them to build necessary knowledge that these children are capable of, in those early years.

Proportional reasoning is a topic often introduced in the last years of primary school. Still, it is believed that it is not an all-or-nothing affair but various dimensions contribute to its construction which grows over a period of time (Lamon, 1999). During students' kindergarten years some of these dimensions may be addressed. It is important to clearly identify the contribution of these various dimensions to proportional reasoning and find ways that these may be introduced and addressed in the kindergarten classroom. It is very likely that the exposure to one or some of these dimensions may provide a better in-road to proportional reasoning.

The Proposed Model

The model proposed in this article is based on Lamon's (1999) conceptualisation of different kinds of understanding and thinking process necessary for the development of proportional reasoning and Kieren's (1988) theory on the multifaceted personality of rational number (see Figure 1). Two modifications were made to Lamon's model. Firstly, we added the dimension "reasoning proportionally up and down". Reasoning proportionally up and down, involves students' ability to analyse the quantities in a given situation to determine that they are related proportionally and that it is appropriate to scale them up or down (Lamon, 1999). We felt that this dimension was necessary and was missing from the Lamon's model. Secondly, the rate dimension was taken as one of the four subconstructs of rational number and not an isolated dimension (Kieren, 1988).

The proposed model consists of nine first-order factors as shown in Figure 1. Figure 1, makes easy the conceptualisation of the way in which the nine first order factors are: unitizing, understanding quantities and change, relative thinking, ability to reason proportionally up and down, partitioning/part-whole, ratio, operator, quotient and measure. There are also two second order factors, rational number and proportional reasoning. The model suggests that proportional reasoning is related to students' abilities in unitizing, quantities and change, relative thinking, reasoning proportionally up and down and rational number. Rational number is presented as a multi-dimensional factor which is composed of four subconsturcts: ratio, operator, quotient and measure, with partitioning/part-whole being the basis for the development of these four subconstructs.

METHODOLOGY

Purpose of the study

Drawing on Lamon's (1999) and Kieren's (1988) theoretical models and employing tasks used in previous studies, the present study aimed to examine prospective kindergarten teachers' proportional reasoning. In particular, the study aims to investigate the relationship amongst: partitioning, unitizing, understanding quantities and change, relative thinking, reasoning proportionally up and down, measure, rate, operator and quotient with proportional reasoning as they will be projected through prospective kindergarten teachers' responses.

Participants and tasks

To answer our research questions, a test on proportional reasoning was constructed guided by the criteria regarding the development and the measurement of the concepts embedded in the theoretical models described earlier. The test included 31 items measuring the participants' abilities in part-whole, unitizing, quantities and change, rational numbers, relative thinking and reasoning proportionally up and down. For the measurement of rational number, the test included tasks on its four interrelated subconstructs: ratio, operator, quotient and measure. Most of the tasks that were used were taken from previous studies such as Lamon's (1999) and Charalampous and Pitta-Pantazi (2007).

The test was administered to 244 kindergarten pre-service teachers studying at three universities in Cyprus.

Scoring and Analysis

Students' fully correct responses were marked with 1 and the incorrect responses with 0. If a student gave a partly correct response, for example if s/he gave a correct answer but wrong justification, this again was marked with 0. The confirmatory factor analysis (CFA), which is part of a more general class of approaches called structural equation modeling, was applied in order to assess the results of the study. CFA is appropriate in situations where the factors of a set of variables for a given population are already known because of previous research. In the case of the present study, CFA was used to test hypotheses corresponding to Lamon's theoretical conceptualization of what constitutes proportional reasoning and Kieren's model of rational number subconstructs. Specifically, our task was not to determine the factors of a set of variables or to find the pattern of the factor loadings. Instead, our purpose of using CFA was to investigate whether proportional reasoning is a composite function of various types of understanding presented by previous research (Kieren, 1988; Lamon, 1999, 2007).

One of the most widely used structural equation modeling computer programs, MPLUS (Muthen & Muthen, 1998), which is appropriate for discrete variables, was used to test for model fitting in this study. In order to evaluate model fit, three fit indices were computed: The chi-square to its degree of freedom ratio (x^2/df), the comparative fit index (CFI), and the root mean-square error of approximation (RMSEA) (Marcoulides & Schumacker, 1996). The observed values of x^2/df should be less than 2, the values for CFI should be higher than .9, and the RMSEA values should be close to zero.

RESULTS

The results are presented in relation to the aim of the study. Figure 1, represents the model which best describes the theoretical model we proposed for proportional reasoning. More specifically, it illustrates that proportional reasoning is a result of abilities in partitioning, unitizing, understanding quantities and change, relative thinking, reasoning proportionally up and down and rational number. From a structural point of view, nine first order factors were included: unitizing, understanding quantities and change, relative thinking, reasoning proportionally up and down, part-whole, measure, rate, quotient and operator. Each of these factors involved three to six tasks. There were also two second order factors: rational number and proportional reasoning.

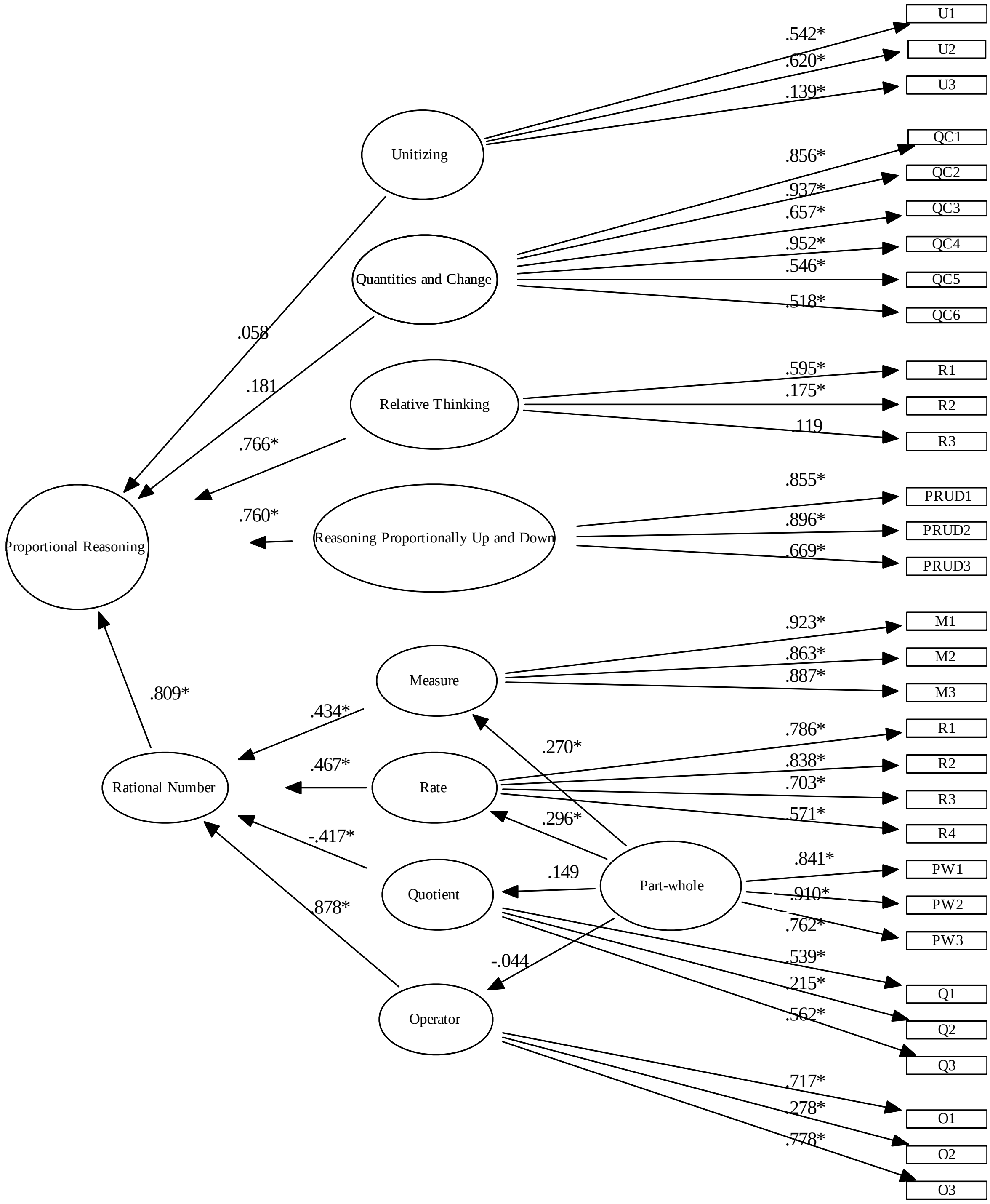

Figure 1: Model for proportional reasoning.

The numbers in the diagrams indicate the factor loadings and the * the values that are statistically significant

Confirmatory factor analysis (CFA) was used to evaluate the construct validity of the model. CFA showed that 30 out of the 31 tasks employed in the present study significantly correlated on each factor, as shown in Figure 1. It also showed that the observed and theoretical factor structures matched the data set of the present study and determined the "goodness of fit" of the factor model (CFI=0.933, x^2= 641.330, df= 418, x^2/df=1.53, RMSEA=0.047), indicating that, unitizing, understanding of quantities and change, relative thinking, reasoning proportionally up and down and rational number can represent distinct function of prospective kindergarten teachers' proportional reasoning.

The structure of the proposed model also addressed the predictions of unitizing, understanding of quantities and change, relative thinking, reasoning proportionally up and down and rational number, in proportional reasoning. First, the results obtained confirmed Kieren's (1988) conceptualisation, that the concept of rational number is comprised by four subconstructs: ratio (r=.467 $p<0.05$), operator (r=.878 $p<0.05$), quotient (r=-.417 $p<0.05$) and measure (r=.434 $p<0.05$). The three subconstructs, ratio, operator and measure correlated significantly with rational number whereas the quotient subconstruct had a negative significant correlation with rational number (r= -.417 $p<0.05$). This may be due to the fact that the quotient task required division, a reverse type of thinking. It was also confirmed that the part whole/partitioning interpretation of rational number is related to the four subconstructs, ratio (r=.296 $p<0.05$), measure (r=.270 $p<0.05$), operator (r= -.044 $p>0.05$), and quotient (r=.149 $p>0.05$). However, only the relationships to ratio and measure subconstructs were statistically significant.

Second, the results obtained showed that to develop proportional reasoning different kinds of understanding, thinking processes and contexts are essential. The analysis revealed that the critical components of proportional reasoning are: unitizing, understanding of quantities and change, relative thinking, reasoning proportionally up and down and rational number. The loadings of each of these factors on proportional reasoning indicated that rational number (r=.809 $p<0.05$), reasoning proportionally up and down (r=.760 $p<0.05$) and relative thinking (r=.766 $p<0.05$) significantly predicted students' performance in proportional reasoning. Performance in rational number was the strongest predictor for success in proportional reasoning. Unitizing (r=.058 $p>0.05$), and understanding of quantities and change (r=.181 $p>0.05$) although appeared to predict abilities in proportional reasoning, did not significantly contribute to proportional reasoning.

DISCUSSION

The present study aimed to empirically test a theoretical model based on Lamon's (1999) conceptualisation of proportional reasoning, with prospective kindergarten school teachers. The results of this study confirmed the theoretical model and also indicated the extent of the impact that different components have in proportional reasoning. It was confirmed that part-whole, unitizing, understanding of quantities and change, relative thinking, reasoning proportionally up and down and rational number predicted prospective teachers' abilities in proportional reasoning, with rational numbers, relative thinking and reasoning proportionally up and down being the most significant predictors. The results of the study also lend support to Kieren's (1988) conceptualisation of the multifaceted construct of rational number, since this construct was significantly related to all four subordinate constructs measure, rate, operator and quotient. As a whole, these findings suggest that a profound understanding of rational number, unitizing, relative thinking, thinking about quantities and change, reasoning proportionally up and down are related to students' performance in proportional reasoning.

The findings of the study suggest that different thinking processes and contexts are necessary for the teaching of proportional reasoning. For instance, teachers may present children with situations which require relative thinking or scenarios where quantities and change need to be discussed. Students may be asked to compare extensive (the length of two ribbons) or intensive quantities (the sweetness of a drink when adding sugar) (Nunes, Desli, & Bell, 2004). Other teachers may decide to start with partitioning tasks, by asking students to share one item or a set of items to two or more individuals. Another possibility is to introduce activities where reasoning proportionally up and down is required. Previous research (Sophian & Madrid, 2003) has shown that young students are capable of this type of thinking. Such reasoning can be introduced through activities where students are required to carry out many-to-one correspondence. These processes allow young students to build an understanding of composite units, provide additive solutions which may later be linked to multiplicative solutions (Sophian & Madrid, 2003).

Obviously, designing instruction that will develop young students' proportional reasoning requires an understanding of young students' intuitive knowledge. It is very likely that from their everyday life, young students may develop a tendency towards certain ways of thinking which may make one of the abovementioned approaches to proportional reasoning more effective. It still needs to be investigated which teaching approach and emphasis on which one of these proportional reasoning dimensions can be more effective for students development of proportional reasoning in their early years of schooling.

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